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In-depth analysis of the Naming Game dynamics:
the homogeneous mixing case

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Language emergence and evolution has recently gained growing attention through multi-agent models and mathematical frameworks to study their behavior. Here we investigate further the Naming Game, a model able to account for the emergence of a shared vocabulary of form-meaning associations through social/cultural learning. Due to the simplicity of both the structure of the agents and their interaction rules, the dynamics of this model can be analyzed in great detail using numerical simulations and analytical arguments. This paper first reviews some existing results and then presents a new overall understanding.

Keywords: Cultural evolution; Language self-organization; Social interaction; Emergence of consensus; Statistical physics

1. Introduction

Language is based on a set of cultural conventions socially shared by a group. But how are these conventions established without a central coordinator and without

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telepathy? The problem has been addressed by several disciplines, but it is only in the last decade that there has been a growing effort to tackle it scientifically using multi-agent models and mathematical approaches (cfr.[1,2,3] for a review). Initially these models focused on the emergence of a shared vocabulary, but increasingly attempts are made to tackle grammar [1,4,5,6,7].

The proposed models can be classified as defending a sociobiological or a sociocultural explanation. The sociobiological approach [8], which includes the evolutionary language game [1], is based on the assumption that successful communicators, enjoying a selective advantage, are more likely to reproduce than worse communicators. If communication strategies are innate, then more successful strategies will displace rivals. The term strategy acquires its precise meaning in the context of a particular model. For instance, it can be a strategy for acquiring the lexicon of a language, i.e., a function from samplings of observed behaviors to acquired communicative behavior patterns [8,9,10], or it can simply coincide with the lexicon of the parents [1] or with some strong disposition to acquire a particular kind of syntax, usually called innate Universal Grammar [11].

In this paper we discuss a model, first proposed in [12], that belongs to the sociocultural family [13,14,15]. Here, good strategies do not necessarily provide higher reproductive success, but only higher communicative success and greater expressive power, and hence greater success in reaching cooperative goals, with less effort. Agents select better strategies exploiting cultural choices, feedback from communication, and a sense of effort. Agents have not only the ability to acquire an existing system but to expand their rules to deal with new communicative challenges and to adjust their rules based on observing the behavior of others. Global coordination emerges over cultural timescales, and language is seen as an evolving and self-organized system [16]. While the sociobiological approach emphasizes language transmission following a vertical, genetic or generational line, the sociocultural approach emphasizes peer-to-peer interaction [17].

A second, fundamental distinction among the different models concerns the adopted mechanisms of social learning describing how stable dispositions are exchanged and coordinated between individuals [18]. The two main approaches are the so called observational learning model and the reinforcement model [19]. In the first approach [8,9,10,1], observation is the main ingredient of learning and statistical sampling of observed behaviors determines their acquisition [8,9,10,1]. The second emphasizes the functional and inferential nature of conventional communication, the scaffolding role of the speaker, the restrictive power of the joint attention frame set up in the shared context, and the importance of pragmatic feedback in language interaction. Here we adopt the reinforcement learning approach as in [13,14,15].

In this paper we shall discuss a recently introduced model [12], inspired by one of the first language game models known as the Naming Game [14]. It is able to account for the emergence of a shared set of conventions in a population of agents. Central control or co-ordination are absent, and agents perform only pairwise interactions following straightforward rules. Indeed, due to the simplicity of the in-

teraction scheme, the dynamics of the model can be studied both with massive simulations and analytical approaches. By doing so we import a pre-existing model into the statistical mechanics context (as opposed to the reverse which is often the case).

In past work, sociocultural investigations largely focused on computational issues and the application for emergent communication in software agents or physical robots [20], resulting in a lack of quantitative investigations. For instance, we shall discuss in detail later how the main features of the process leading the population to a final convergence state scale with the population size, whereas earlier work has concentrated on studying very small populations [21]. The price to pay for quantitative comprehension is a reduction in the number of aspects of the phenomena we can treat. Thus, the agent architectures we shall describe are indeed very basic and stylized, and are much too simple compared to the cognitive mechanisms humans employ, but on the other hand they allow us to study much more clearly what is crucial to obtain the desired global co-ordination based on only local interaction. The present paper shows that the crucial features are in fact simple and we consider this to be one of our major contributions. Despite simplifying the original Naming Game [14], we retained however its most important properties so that the interaction scheme could still be ported to real world robots or be used to explain the behavior of biological agents.

The paper is organized as follows. In Sec. 2.1 we present the Naming Game model and discuss its basic phenomenology. Sec. 3 is devoted to the study of the role of population size. We investigate the scaling relations of some important quantities and provide analytical arguments to derive the relevant exponents. In Sec. 4 we look in more detail at the mechanisms that give rise to convergence, deepening the analysis presented in [12]. In particular, we identify and explain the presence of a hidden timescale that governs the transition to the final consensus state. In Sec. 5 we focus on the relation between single simulation runs and averaged quantities, while in Sec. 6 we investigate the properties of the consensus word. We then analyze, in Sec. 7, a controlled case that sheds light on the nature of the symmetry breaking process leading to lexical convergence. Finally, in Sec. 8, we discuss the most relevant features of the model and present some conclusions concerning particularly its connections with the fields of Opinion Dynamics on one hand and Artificial Intelligence on the other.

2. The model

2.1. *Naming Game*

We present here the version of the Naming Game introduced in [12] (see also [22] for a comprehensive analysis of the model). The game is played by a population of N agents in pairwise interactions. As a side effect of a game, agents *negotiate* conventions, i.e., associations between forms (names) and meanings (for example individuals in the world), and it is obviously desirable that a global consensus emerges.

Because different agents can each independently invent a different name for the same meaning, synonymy (one meaning many words) is unavoidable. However we do not consider here the possibility of homonymy (one word many meanings). In the invention process, in fact, we consider the situation where the number of possibly invented words is so huge that the probability that two players will ever invent the same word at two different times for two different meanings is practically negligible. This means that the dynamics of the inventories associated to different meanings are completely independent and the number of meanngs becomes a trivial parameter of the model. As a consequence we can reduce, without loss of generality, the environment as composed by one single meaning and focus on how a population can establish a convention for expressing that meaning. In a generalized Naming Game, homonymy is not always an unstable feature and its survival depends in general on the size of the meaning and signal spaces [23]. Homonymy becomes crucial if, during a conversation, agents do not get precise feedback about the meaning. If there is more than one possible meaning compatible with the current situation (for example if the word expresses a category but we do not know which one) then homonymy would be unavoidable. This is not the case for the Naming Game while it becomes crucial for the so-called Guessing [2] and Category Game [6].

The model definition can be summarized as follows. We consider an environment composed by one single object to be named, the extension to many different objects being trivial if one neglects homonymy. Each individual is described by its *inventory*, i.e., a set of form-meaning pairs (in this case only names competing to name the unique object)) which is empty at the beginning of the game ($t = 0$) and evolves dynamically in time. At each time step ($t = 1, 2, ..$) two agents are randomly selected and interact: one of them plays the role of *speaker*, the other one that of *hearer*. The interactions obey the following rules (Fig. 1):

- The speaker transmits a name to the hearer. If its inventory is empty, the speaker invents a new name, otherwise it selects randomly one of the names it knows;
- If the hearer has the uttered name in his inventory, the game is a <u>success</u>, and both agents delete all their names, but the winning one;
- If the hearer does not know the uttered name, the game is a <u>failure</u>, and the hearer inserts the name in its inventory.

Another important assumption of the model is that two agents are *randomly* selected at each time step. This means that each agent in principle can talk to anybody else, i.e., that the population is completely unstructured (homogeneous mixing assumption). The role of different agent topologies has been discussed extensively elsewhere [24,25,26,27,28,29,22]. A generalized model of the Naming Game has also been proposed, in which agents do not update their inventories deterministically after a success, but rather do that according to a certain probability [30]. Generalized models exhibit interesting phenomenologies, including a non-equilibrium phase transition, but we do not consider them here.
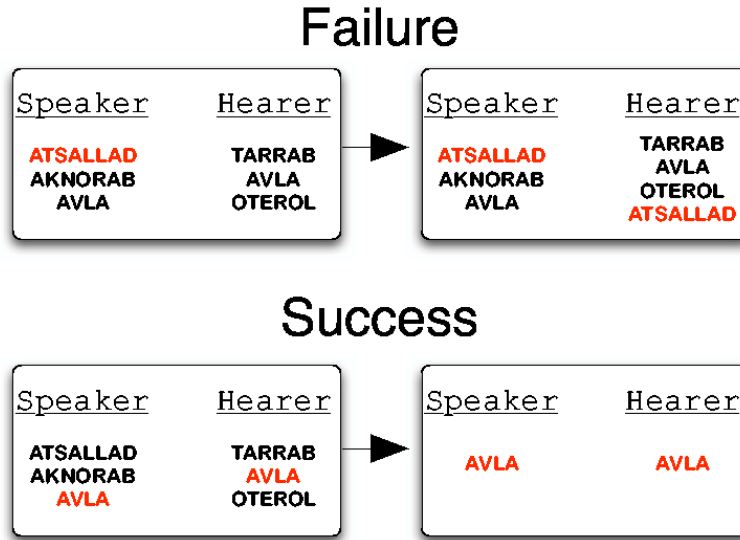
Failure

Speaker	Hearer
ATSALLAD	TARRAB
AKNORAB	AVLA
AVLA	OTEROL

→

Speaker	Hearer
ATSALLAD	TARRAB
AKNORAB	AVLA
AVLA	OTEROL
	ATSALLAD

Success

Speaker	Hearer
ATSALLAD	TARRAB
AKNORAB	AVLA
AVLA	OTEROL

→

Speaker	Hearer
AVLA	AVLA

Fig. 1.   **Naming game interaction rules.** The speaker selects randomly one of its names, or invents a new name if its inventory is empty (i.e., we are at the beginning of the game). If the hearer does not know the uttered name, it simply adds it to its inventory, and the interaction is a *failure*. If, on the other hand, the hearer recognizes the name, the interaction is a *success*, and both agents delete from their inventories all their names but the winning one.

Finally, it is worth stressing that the random selection rule adopted by the speaker to select the word to be transmitted, and the absence of weights to be associated with words, expressly violate the fundamental ingredients of earlier models [2]. Indeed, as we are going to show, they turn out to be unnecessary.

2.2. *Basic phenomenology*

The most basic quantities describing the state of the population at a given time t are: the total number of names present in the system, $N_w(t)$, the number of different names known by agents, $N_d(t)$, and the success rate, i.e. the probability of observing a successful interaction at a given time, $S(t)$. In Figure 2 we report data concerning a population of $N = 10^3$ agents. The process starts with a trivial transient in which agents invent new names. It follows a longer period of time where the $N/2$ (on average) different names are exchanged after unsuccessful interactions. The probability of a success taking place at this time is indeed very small ($S(t) \simeq 0$) since each agent knows only a few different names. As a consequence, the total number of names grows, while the number of different names remains constant. However, agents keep correlating their inventories so that at a certain point the probability of a successful interaction ceases to be negligible. As fruitful interactions become more

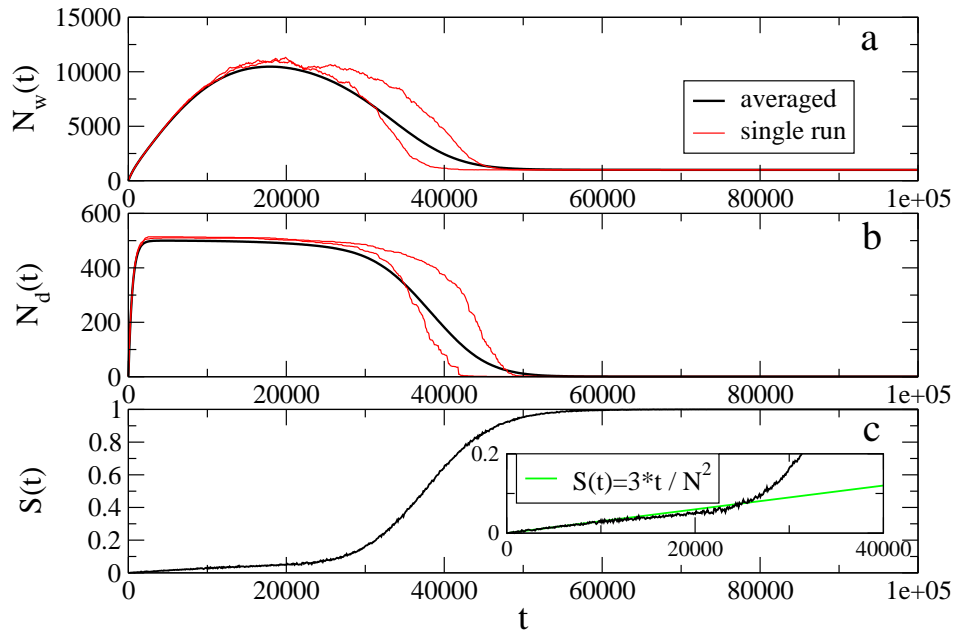6   *Andrea Baronchelli, Vittorio Loreto and Luc Steels*

Fig. 2.   **Basic global quantities.** a) Total Number of names present in the system, $N_w(t)$; b) Number of different names, $N_d(t)$; c) Success rate $S(t)$, i.e., probability of observing a successful interaction at a time t. The inset shows the linear behavior of $S(t)$ at small times. All curves concern a population of $N = 10^3$ agents. The system reaches the final absorbing state, described by $N_w(t) = N$, $N_d(t) = 1$ and $S(t) = 1$, in which a global agreement on the form (name) to assign to the meaning (individual object) has been reached.

frequent the total number of names at first reduces its growth and then starts to decrease, so that the $N_w(t)$ curve presents a well identified peak. Moreover, after a while, some names start disappearing from the system. The process evolves with an abrupt increase in the success rate, with a curve $S(t)$ which exhibits a characteristic *S-shaped* behavior, and a further reduction in the numbers of both total and different names. Finally, the dynamics ends when all agents have the same unique name and the system is in the desired convergence state. It is worth noting that the developed communication system is not only *effective* (each agent understands all the others), but also *efficient* (no memory is wasted in the final state).

From the inset of Figure 2 it is also clear that the $S(t)$ curve exhibits a linear behavior at the beginning of the process: $S(t) \sim t/N^2$. This can be understood

noting that, at early stages, most successful interactions involve agents which have already met in previous games. Thus the probability of success is proportional to the ratio between the number of couples that have interacted before time t, whose order is $O(t)$, and the total number of possible pairs, $N(N-1)/2$. The linear growth ends in correspondence with the peak of the N_w curve, where it holds $S(t) \sim 1/N^{0.5}$, and the success rate curve exhibits a bending afterward, slowing down its growth till a sudden burst that corresponds to convergence.

3. The role of system size

3.1. *Scaling relations*

A crucial question concerns the role played by the system size N. In particular, two fundamental aspects depend on N. The first is the time needed by the population to reach the final state, which we shall call the convergence time t_{conv}. The second concerns the cognitive effort in terms of memory required by each agent in achieving this dynamics. This reaches its maximum in correspondence of the peak of the $N_w(t)$ curve. Figure 3 shows scaling of the convergence time t_{conv}, and the time and height of the peak of $N_w(t)$, namely t_{max} and $N_w^{max} \doteq N_w(t_{max})$. The difference time $(t_{conv} - t_{max})$ is also plotted. It turns out that all these quantities follow power law behaviors: $t_{max} \sim N^{\alpha}$, $t_{conv} \sim N^{\beta}$, $N_w^{max} \sim N^{\gamma}$ and $t_{diff} = (t_{conv} - t_{max}) \sim N^{\delta}$, with exponents $\alpha \approx \beta \approx \gamma \approx \delta \approx 1.5$.

The values for α and γ can be understood through simple analytical arguments. Indeed, assume that, when the total number of words is close to the maximum, each agent has on average cN^a words, so that it holds $\alpha = a + 1$. If we assume also that the distribution of different words in the inventories is uniform, the probability for the speaker to play a given word is $1/(cN^a)$, while the probability that the hearer knows that word is $2cN^a/N$ (where $N/2$ is the number of different words present in the system). The equation for the evolution of the number of words then reads:

$$\frac{dN_w(t)}{dt} \propto \frac{1}{cN^a}\left(1 - \frac{2cN^a}{N}\right) - \frac{1}{cN^a}\frac{2cN^a}{N}2cN^a \tag{1}$$

where the first term is related to unsuccessful interactions (which increase N_w by one unit), while the second one to successful ones (which decrease N_w by $2cN^a$). At the maximum $dN_w(t_{max})/dt = 0$, so that, in the thermodynamic limit $N \to \infty$, the only possible value for the exponent is $a = 1/2$ which implies $\alpha = 3/2$ in perfect agreement with data from simulations.

For the exponent γ the procedure is analogous, but we have to use the linear behavior of the success rate and the relation $a = 1/2$ we have just obtained. The equation for $N_w(t)$ now can be written as:

$$\frac{dN_w(t)}{dt} \propto \frac{1}{cN^{1/2}}\left(1 - \frac{ct}{N^2}\right) - \frac{1}{cN^{1/2}}\frac{ct}{N^2}2cN^{1/2} \ . \tag{2}$$

If we impose $dN_w(t)/dt = 0$, we find that the time of the maximum has to scale with the right exponent $\gamma = 3/2$ in the thermodynamic limit.
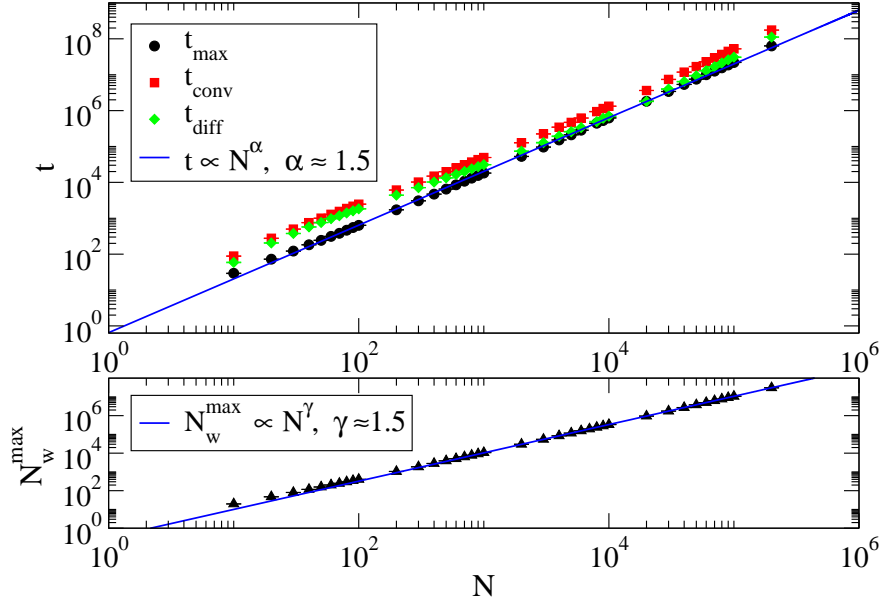
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Fig. 3.   **Scaling with the population size** N. In the upper graph the scaling of the peak and convergence time, t_{max} and t_{conv}, is reported, along with their difference, t_{diff}. All curves scale with the power law $N^{1.5}$. Note that t_{conv} and t_{diff} scaling curves present characteristic log-periodic oscillations (see Sec. 3.2). The lower curve shows that the maximum number of words (peak height, $N_w^{max} = N_w(t_{max})$) obeys the same power law scaling.

The exponent for the convergence time, β, deserves a more articulate discussion, and we can only provide a more *naive* argument, even though well supported by evidence from numerical simulations. We concentrate on the scaling of the interval of time separating the peak of $N_w(t)$ and the convergence, i.e., $t_{diff} = (t_{conv} - t_{max}) \sim t^\delta \sim N^{1.5}$, since we already have an argument for the time of the peak of the total number of words t_{max}. t_{diff} is the time span required by the system to get rid of all the words but the one which survives in the final state. The problem cast in such a way, we argue that a crucial parameter is the maximum number of words the system stores at the beginning of the elimination phase.

If we adopt the mean field assumption that at $t = t_{max}$ each agent has on average $N_w^{max}/N \sim \sqrt{N}$ words (see 28 for a detailed discussion of such a mean field approximation), we see that, by definition, in the interval t_{diff}, each agent must have won at least once. This is a necessary condition to have convergence, and it is interesting to investigate the timescale over which this happens. Assuming that $\overline{N}$ is the number of agents who did not yet have a successful interaction at time t, we have:

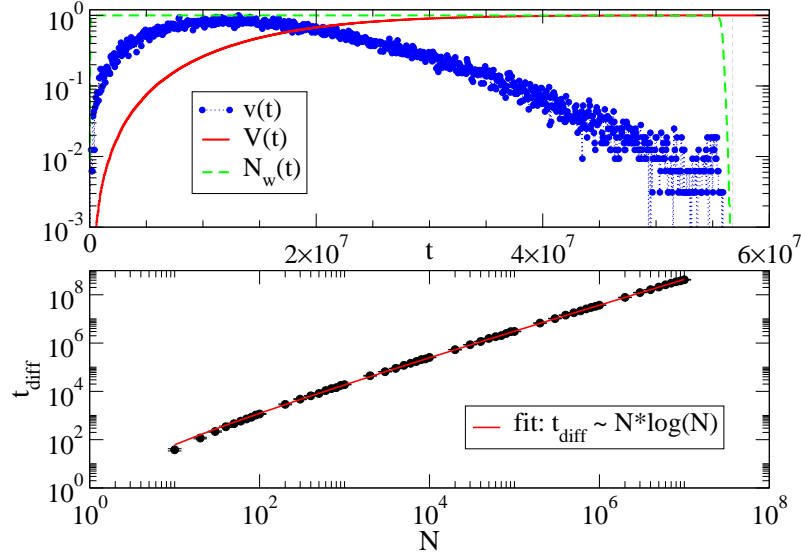$$\overline{N} = N(1 - p_s p_w)^t \tag{3}$$

Fig. 4.   **Evidences supporting the argument for the β exponent.** Top: $v(t)$ is the (non normalized) histogram of the times at which agents play their first successful interaction, while $V(t)$ is the cumulative curve. It is clear that up to a time very close to convergence there are still agents that have never won. Thus, the investigation of the first time in which $V(t) = 1$ provides a good estimate of t_{conv}. Data refer to a single run for a population of $N = 10^5$ agents. The $N_d(t)$ curve is also plotted, for reference, while the vertical dashed grey line indicates convergence time. Bottom: scaling of t_{diff} with N for a system in which, at the beginning of the process, half of the population knows word A and the other half word B. Thus, $N_d(t = 0) = 2$ and invention is eliminated. Experimental points are well fitted by $t_{diff} \sim N \log N$, as predicted by our argument (see text). A fit of the form $t_{diff} \sim N^\delta$, on the other hand, turns out to be less accurate (data not shown).

where $p_s = 1/N$ is the probability to randomly select an agent and $p_w = S(t)$ is the probability of a success. The latter is $O(1/N^{0.5})$ at t_{max}, and stays around that value for a quite long time span afterward. Indeed, as we have seen, the success rate $S(t)$ grows linearly till the peak, where $S(t) = ct_{max}/N^2 \sim 1/N^{0.5}$, and exhibits a bending afterward, before the final jump to $S(t) = 1$ (Fig. 2). If we insert the estimates of p_s and p_w in eq. (3), and we require the number of agents who have not yet had a successful interaction to be finite just before the convergence, i.e., $\overline{N}(t_{conv}) \sim O(1)$, we obtain $t_{diff} \sim N^{3/2} \log N$. Thus, the leading term of the difference time $t_{diff} \sim N^{1.5}$ is correctly recovered, and the necessary condition $\overline{N}(t_{conv}) \sim O(1)$ turns out to be also sufficient. The possible presence of the logarithmic correction, on the other hand, cannot be appreciated in simulations due also to logarithmic oscillations in the t_{diff} curve (see following Sec. 3.2). Finally, it is worth noting that the $S(t) \sim 1/N^{0.5}$ behavior can be understood also assuming that at the peak of $N_w(t)$ each agent has $O(N^{0.5})$ words (mean field assumption), and that the average number of words in common between two inventories is $O(1)$
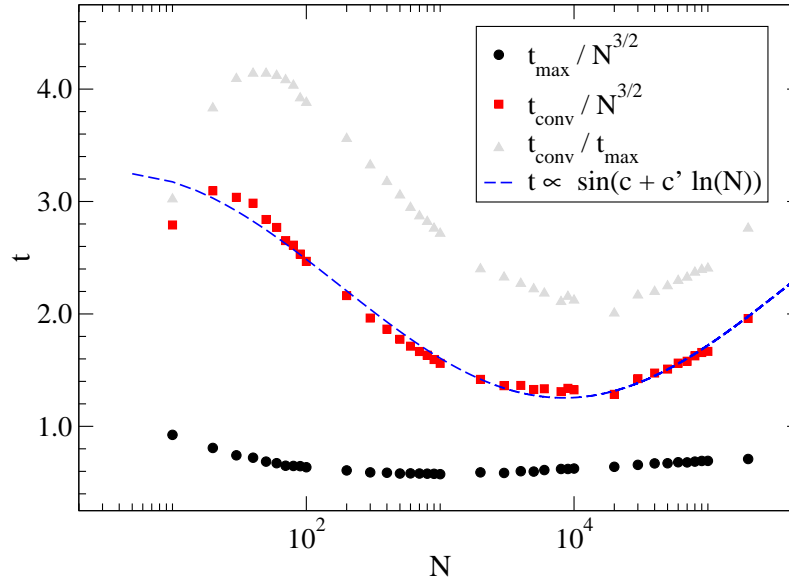
Fig. 5.   **Log-periodic oscillations for convergence times.** Rescaled values of t_{conv} and t_{max} are plotted along with their ratio. The rescaled convergence times exhibit global oscillations that are well fitted by the function $t \propto \sin(c + c' \ln(N))$, where c and c' are constants whose values are $c \approx 1.0$ and $c' \approx 0.4$.

(as confirmed by numerical simulations shown in Fig. 12).

We can test the hypothesis behind the above argument in two ways. First of all we can investigate the distribution $v(t)$ of the times at which agents perform their first successful interaction. Remarkably, Fig. 4 (top) shows that this distribution extends approximately up to t_{conv}, so that the time t^*, at which $V(t) \equiv \int_0^{t^*} v(t) = 1$, turns out to provide a good estimate for t_{conv}. Then, we can validate our approach studying a controlled case. Consider a simplified situation in which each agent starts the usual Naming Game knowing one of only two possible words, say A and B. Invention is then prevented, and for the peak of $N_w(t)$ it holds $N_w^{max} \sim N$. Noting that in this case we have $S(t_{max}) \sim O(1)$, and substituting this value in eq. (3), we obtain that $t_{diff} \sim N \log N$. Indeed, this prediction is confirmed by simulations also for what concerns the logarithmic correction (Fig. 4 (bottom)), and our approach is supported by a second validation.

3.2. *Rescaling curves*

Since we know that the characteristic time required by the system to reach convergence scales as $N^{1.5}$ we would expect a transformation of the form $t \rightarrow t/N^{3/2}$ to yield a collapse of the global-quantity curves, such as $S(t)$ or $N_w(t)$, relative to systems of different sizes. However this does not happen.

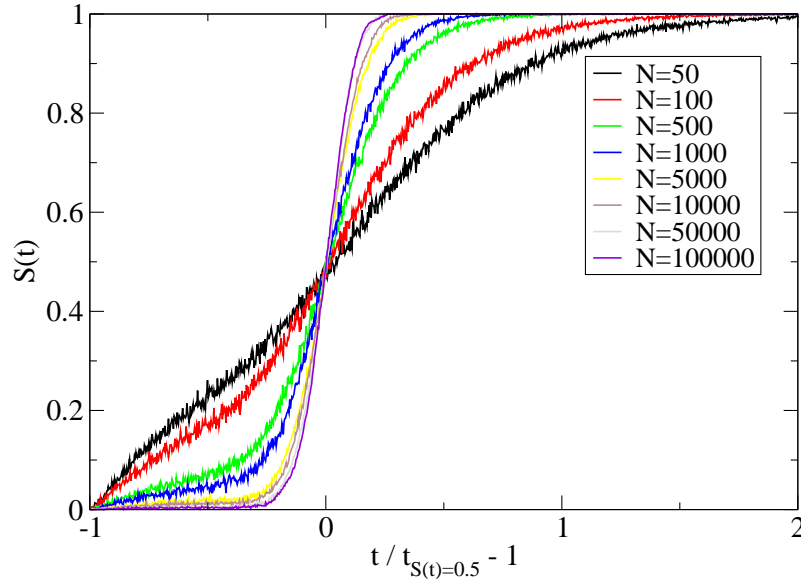The first reason is that the curve of the scaling of the convergence time with N

Fig. 6.   **Rescaling of the success rate curves.** Curves relative to different system sizes show different qualitative behavior if time is rescaled as $t \rightarrow t/t_{S(t)=0.5} - 1$, where $t_{S(t)=0.5} \sim N^{3/2}$. Indeed, on this timescale, the transition between the initial disordered state and the final ordered one where $S(t) \approx 1$ (i.e., the disorder-order transition) becomes steeper and steeper as N grows.

does follow a $N^{3/2}$ trend, but presents a peculiar, seemingly oscillatory, behavior in logarithmic scale. This is already visible from Figure 3, but is clearer in Figure 5, where it is shown that the curve $t_{conv}/N^{3/2}$ is well fitted by a function of the type $t \propto \sin(c + c' \ln(N))$, where c and c' are constants[a]. The same figure also shows that such oscillations are absent, or at are least very reduced, in the curve of peak times, t_{max}.

The deviations of the convergence time scaling curve from a pure power law have the effect of scattering rescaled curves, thus preventing any possible collapse. An easy solution to this problem is that of rescaling according to intrinsic features of each curve. In Figure 6, we have rescaled success rate $S(t)$ curves following the transformation $t \rightarrow t/t_{S(t)=0.5} - 1$, where $t_{S(t)=0.5}$ is the time in which the considered curve reaches the value 0.5 (with $t_{S(t)=0.5} \sim N^{1.5}$, not shown). Interestingly we note that the curves still do not collapse. In particular, the transition between a disordered state in which there is almost no communication between agents ($S(t) \approx 0$), to the final ordered state in which most interactions are successful ($S(t) \approx 1$)

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[a]It must be noted that, since the supposed oscillations should happen on logarithmic scale, it is hard to obtain data able to confirm their actual oscillatory behavior. Thus, the fit proposed here must be intended only as a possible suggestion on the true behavior of the irregularities of the t_{conv} scaling curve.
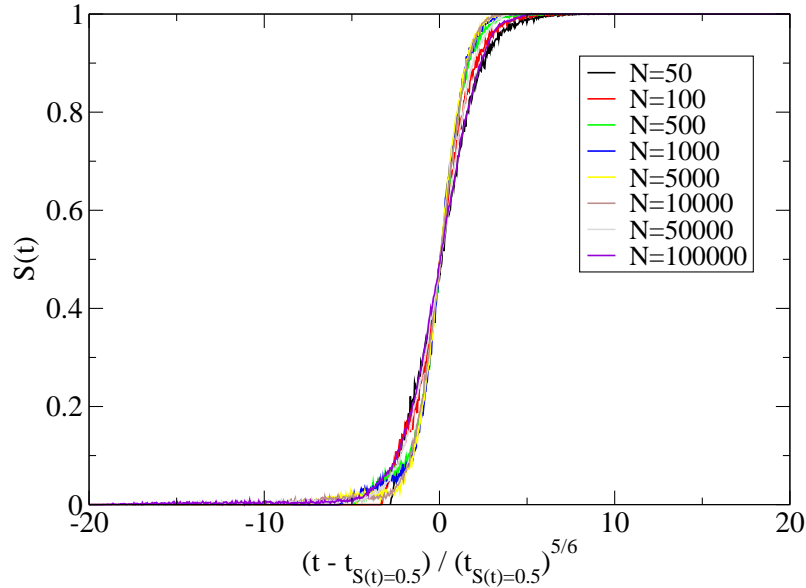
Fig. 7.   **Collapse of the success rate curves.** The time rescaling transformation $t \rightarrow (t - t_{S(t)=0.5})/t_{S(t)=0.5}^{5/6}$ makes the different $S(t)$ curves collapse. Since the time at which the success rate is equal to 0.5 scales as $N^{3/2}$ (data not shown), the transformation is equivalent to $t \rightarrow (t - \alpha N^{3/2})/N^{5/4}$. The collapse shows that the disorder-order transition between an initial disordered state in which $S(t) \approx 0$ and an ordered state in which $S(t) \approx 1$ happens on new timescale $t \sim N^{\theta}$ with $\theta \approx 5/4$.

becomes steeper and steeper as N becomes larger [12]. In other words, it is clear that the *shape* of the curves changes when we observe them on our rescaled timescale.

Figure 6 suggests that the disorder-order transitions happen on a new timescale $t \sim N^{\theta}$ with $\theta < \beta$, so that $N^{\theta}/t_{conv} \rightarrow 0$ when $N \rightarrow \infty$ and the transition becomes instantaneous, on the rescaled timescale, in the thermodynamic limit. Indeed this is exactly the case and, as shown in Figure 7, the value $\theta = 5/4$ and the transformation $t \rightarrow (t - \alpha N^{3/2})/N^{5/4}$ produces a good collapse of the success rate curves relative to different N. In the next section we shall show how the right value for θ can be derived with scaling arguments after a deeper investigation of the model dynamics [12].

4. The approach to convergence

4.1. *The domain of agents*

We have seen that agents at first accumulate a growing number of words and then, as their interactions become more and more successful, reduce the size of their inventories till the point in which all of them know the same unique word. More quantitatively, the evolution in time of the fraction of agents f_n with inventory sizes n is shown in Figure 8. The curves refer to a population of $N = 10^3$ agents
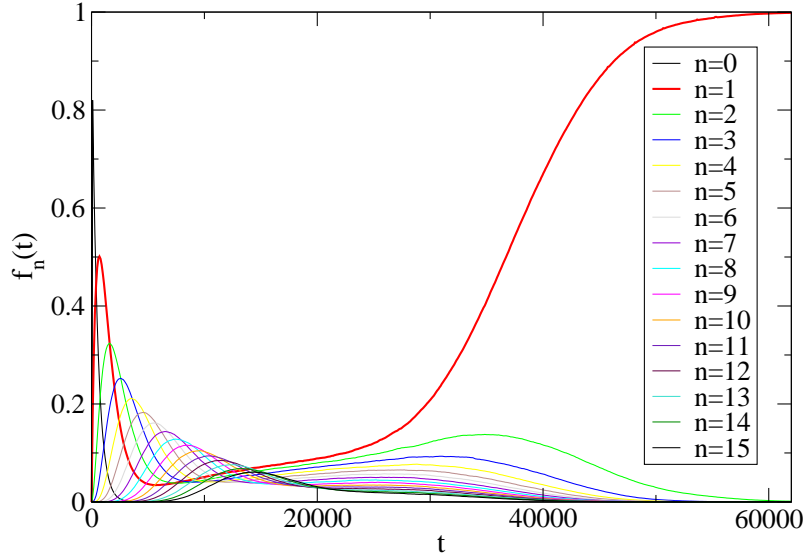
Fig. 8.   **Evolution in time of inventory sizes** n $(n = 1 \ldots 15)$. $f_n(t)$ is the fraction of agents whose inventory size is n at time t. In the right part, $f_n(t)$ decreases with increasing n. The process ends with all agents having the same unique word in their inventory, so that $f_1 = 1$. Curves obtained by averaging 500 simulation runs on a population of $N = 10^3$ agents.

and have been obtained averaging over several simulation runs. We see that the process starts with a rapid decrease of f_0 and a concomitant increase of the fraction of agents with larger inventories. After a while, however, successful interactions produce a new growth in the fraction of agents with small values of n. The process evolves until the point in which all agents have the same unique word and $f_1 = 1$.

Some of the initial-time regularities of the f_n curves can be easily described analytically. For instance, it is easy to write equations for the evolution of the number of species as long as $S(t) = 0$. We have:

$$df_0/dt = -f_0 \tag{4}$$

$$df_{n>1}/dt = f_{n-1} - f_n$$

$$\tag{5}$$

These trivial relations allow to understand some features of the curves, like the exponential decay of f_0, or the fact that, at early times, each f_n $(n > 0)$ crosses the correspondent f_{n-1} in correspondence of its maximum (as can be recovered imposing $df_n/dt = 0$). However, generalizing eq. (4) is not easy, since, as the dynamics proceeds, one should take into account the correlations among inventories to estimate the probability of successful interactions, and the analytical solution of our Naming Game model is still lacking.
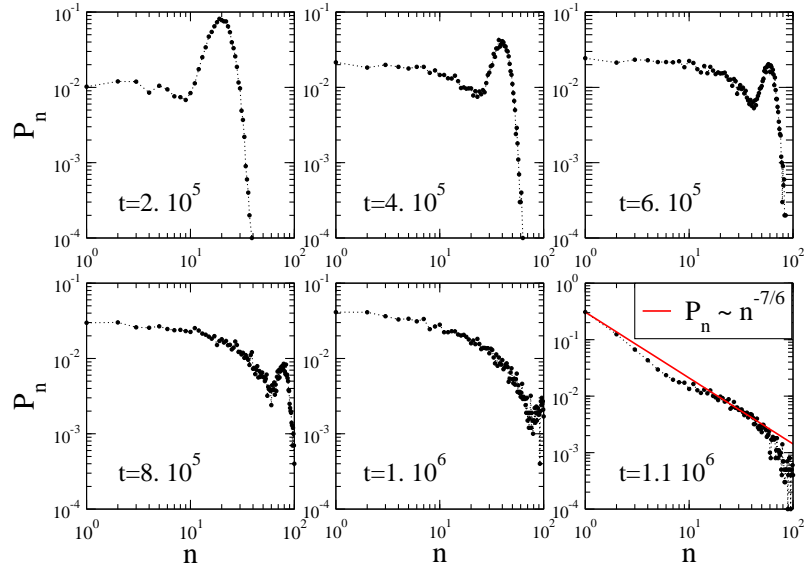
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Fig. 9.   **Distribution P_n of inventory sizes n.** Curves obtained by a single simulation run for a population of $N = 10^4$ agents, for which $t_{max} = 6.2 \times 10^5$ and $t_{conv} = 1.3 \times 10^6$ time steps. Close to convergence the distribution is well described by a power law $P_n \sim n^{-7/6}$.

More quantitative insights can be obtained looking at the distribution P_n of inventory sizes n at fixed times [12], reported in Figure 9 for the case $N = 10^4$ (see [28] for a detailed discussion of the P_n behavior in different temporal regions and different topologies). We see that in early stages most agents tend to have large inventories, thus determining a peak in the distribution. When agents start to understand each other, however, the peak disappears and large n values keep decreasing. Interestingly, in correspondence with the jump of the success rate that leads to convergence, the histogram can be described by a power law distribution:

$$P_n \sim n^{-\sigma} g(n/\sqrt{N}) \tag{6}$$

with the cut-off function $g(x) = 1$ for $x << 1$ and $g(x) = 0$ for $x >> 1$. Numerically it turns out that $1 < \sigma < 3/2$. To be more precise, in Figure 9 it is shown that the value $\sigma \approx 7/6$ allows a good fitting of the P_n at the transition, and from simulations it turns out that this is true irrespectively of the system size.

Finally, it is also worth mentioning that, well before the transition, the larger number of words in the inventory of a hearer increases (linearly) the chances of success in a interaction (data not shown). The number of words known by the speaker, on the other hand, basically plays no role until the system is close to the transition. Here, small inventories are likely to contain the most popular word, thus yielding higher probability of success [28].
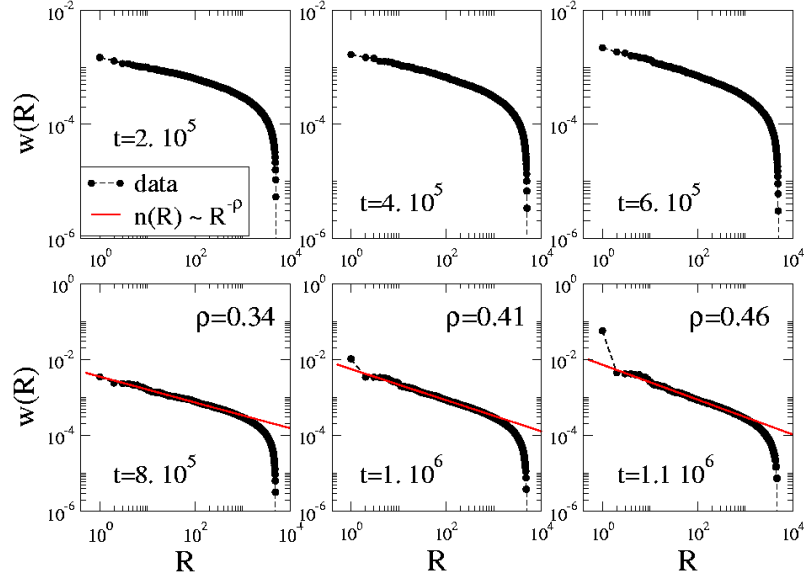
Fig. 10. **Distribution $w(R)$ of words of rank R.** The most popular word has rank $R = 1$, the second $R = 2$, etc. The distribution follows a power law behavior $w(R) \sim R^{-\rho}$ with an exponent that varies in time, while for high ranks it is truncated at $R \approx N/2$. Close to the disorder-order transition, however, the most diffused word abandons the distribution that keeps describing the less popular words. Data come from a single simulation run and concern a population of $N = 10^4$ agents.

4.2. *The domain of words*

While agents negotiate with each others, words compete to survive [12]. In Figure 10 the rank distribution of words at fixed times is reported. The most popular word is given rank 1, the second one 2 and so on. The first part of the distribution is well described by a power law function, with an exponent that decreases with time. In proximity of the disorder-order transition, however, the most popular word breaks the symmetry and abandons the power law behavior, which continues to describe well the remaining words. More precisely, the global distribution for the fraction of agents possessing the R-ranked word, $w(R)$, can be described as:

$$w(R) = w(1)\delta_{R,1} + \frac{N_w/N - w(1)}{(1 - \rho)((N/2)^{1-\rho} - 2^{1-\rho})}R^{-\rho}g(\frac{R}{N/2}), \tag{7}$$

where δ is the Kronecker delta function ($\delta_{a,b} = 1$ iff $a = b$ and $\delta_{a,b} = 0$ if $a \neq b$) and the normalization factors are derived imposing that $\int_1^\infty w(R)dR = N_w/N$ [b].

On the other hand from equation (6) one gets, by a simple integration, the

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[b]We use integrals instead of discrete sums, an approximation valid in the limit of large systems.

relation $N_w/N \sim N^{1-\sigma/2}$ which, substituted into eq. (7), gives:

$$w(R)|_{R>1} \sim \frac{1}{N^{\sigma/2-\rho}} R^{-\rho} f(\frac{R}{N/2}). \tag{8}$$

It follows that $w(R)|_{R>1} \to 0$ as $N \to \infty$, so that, in the thermodynamic limit $w(1) \sim O(1)$, i.e., the number of players with the most popular word, is a finite fraction of the whole population.

4.3. *Network view - The disorder-order transition*

We now need a more precise description of the convergence process [12]. A profitable approach consists in mapping the agents in the nodes of a network (see Figure 11). Two agents are connected by a link each time that they both know the same word, so that multiple links are allowed. For example, if m out of the n words known by agent A are present also in the inventory of agent B, they will be connected by m links. In the network, a word is represented by a fully connected sub-graph, i.e., by a clique, and the final coherent state corresponds to a fully connected network with all pairs connected by only one link. When two players interact, a failure determines the propagation of a word, while a success can result in the elimination of a certain number of words competing with the one used. In the network view, as shown in Figure 11, this translates into a clique that grows when one of its nodes is represented by a speaker that takes part in a failure, and is diminished when one (or two) of its nodes are involved in a successful interaction with a competing word.

To understand why the disorder-order transition becomes steeper and steeper, if observed on the right timescale, we must investigate the dynamics that leads to convergence. If we make the hypothesis that, when N is large, just before the transition all the agents have the word that will dominate, the problem reduces to the study of the rate at which competing words disappear. In different words, the crucial information is how the number of deleted links in the network, M_d, scales with N. It holds:

$$M_d = \frac{N_w}{N} \int_2^\infty w^2(R) N dR \sim N^{3-\frac{3}{2}\sigma} \tag{9}$$

where $\frac{N_w}{N}$ is the average number of words known by each agent, $w(R)$ is the probability of having a word of rank R, and $w(R)N$ is the number of agents that have that word (i.e., the size of the clique). On the other hand, considering the network structure, eq. 9 is the product of the average number of cliques involved in each deletion process $[\frac{N_w}{N}]$, multiplied by an integral stating, in probability, which clique is involved $[w(R)]$ and which is its size $[w(R)N]$. The integral on R starts from the first deletable word, i.e., the second most popular, because of the assumption that all the successes are due to the use of the most popular word.

In our case, for $\sigma \approx 7/6$, we obtain that $M_d \sim N^{5/4}$. Thus, from equation (9), we have that the ratio $M_d/N^{3/2} \sim N^{-\frac{3}{2}(\sigma-1)}$ goes to zero for large systems (since $\sigma \approx 7/6$, and in general $\sigma > 1$), and this explains the greater slope, on the system timescale, of the success rate curves for large populations (Figure 7).
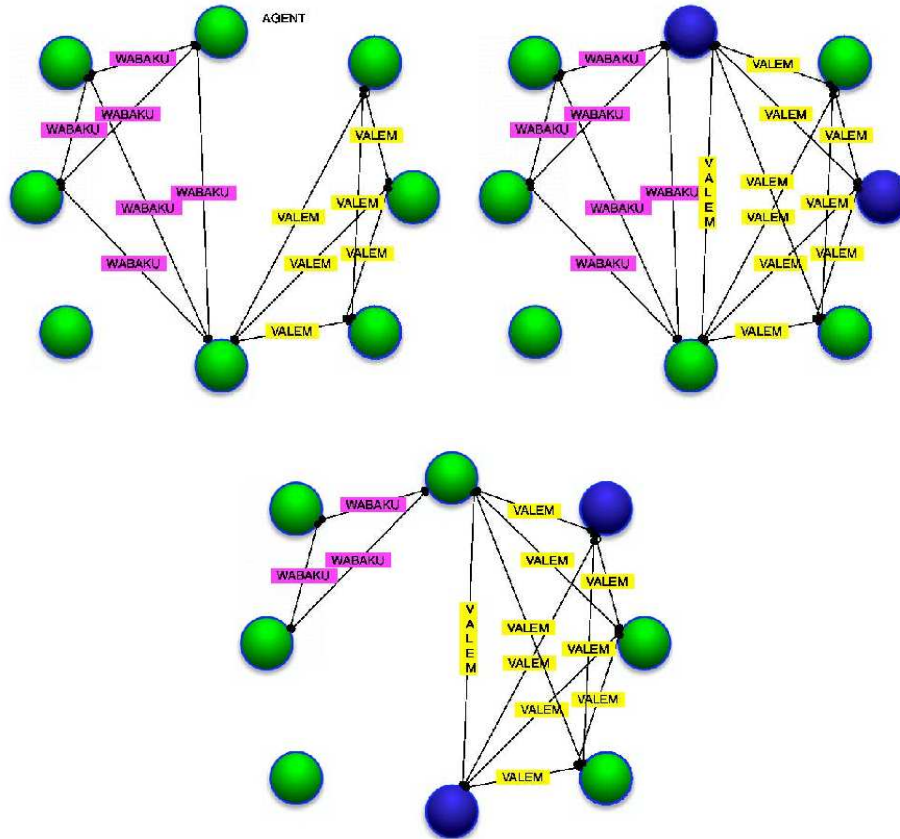
Fig. 11.   **Agents network dynamics.** Top Left: a link between two agents (i.e., nodes) exists every time they have a word in common in their inventories, so that multiple links are allowed. In this representation, a word corresponds to a fully connected (sub)set of agents, i.e., a clique; in Figure, the two cliques corresponding to words WABAKU and VALEM are highlighted. Top Right: the two highlighted agents have just failed to communicate, so that the word VALEM has been transmitted to the agent placed in the top of the graphical representation. It therefore enters into the enlarged clique corresponding to the transmitted word VALEM. Bottom: the two highlighted agents have just succeeded using word VALEM. The clique corresponding to the used word does not change in any respect, but the competing cliques (here that of WABAKU) are reduced.

4.4. *The overlap functional*

We have looked at all the timescales involved in the process leading the population to the final agreement state. Yet, we have not investigated *whether* this convergence state is always reached. Actually, this is the case, and trivial considerations allow to clarify this point. First of all, it must be noticed that, according to the interaction rules of the agents, the agreement condition constitutes the only possible absorbing state of our model. The proof that convergence is always reached is then straight-forward. Indeed, from any possible state there is always a non-zero probability to reach an absorbing state in, for instance, $2(N-1)$ interactions. For example, a possible sequence is as follows. A given agent speaks twice with all the other $(N-1)$ agents using always the same word A. After these $2(N-1)$ interactions all the agents have only the word A. Denoting with p the probability of the sequence of $2(N-1)$ steps, the probability that the system has not reached an absorbing state after $2(N-1)$ iterations is smaller or equal to $(1-p)$. Therefore, iterating this procedure, the probability that, starting from any state, the system has not reached an absorbing state after $2k(N-1)$ iterations, is smaller than $(1-p)^k$ which vanishes exponentially with k. The above argument, though being very simple and general, is exact. However, another perspective to address the problem of convergence consists in monitoring the lexical coherence of the system. To this purpose, we introduce the overlap functional O:

$$O(t) = \frac{2}{N(N-1)} \sum_{i>j} \frac{|a_i \cap a_j|}{k_i k_j}, \tag{10}$$

where a_i is the ith agent's inventory, whose size is k_i, and $|a_i \cap a_j|$ is the number of words in common between a_i and a_j. The overlap functional is a measure of the lexical coherence in the system and it is bounded, $O(t) \leq 1$. A the beginning of the process it is equal to zero, $O(t=0) = 0$, while at convergence it reaches its maximum, $O(t = t_{conv}) = 1$.

From extensive numerical investigations it turns out that, averaged over several runs, the functional always grows, i.e., $\langle O(t+1) \rangle > \langle O(t) \rangle$ (see Figure 12). Moreover, looking at the single realization, this function grows almost always, i.e., $\langle O(t+1) \rangle > O(t)$, except for a set a very rare configurations whose statistical weight appears to be negligible (data not shown). Even if it is not a proof in a rigorous sense, this monotonicity, combined with the fact that the functional is bounded, gives a strong indication that the system will indeed converge [12].

It is also interesting to note that eq. (10) is very similar to the expression for the success rate $S(t)$, which can formally be written as:

$$S(t) = \frac{1}{N(N-1)} \sum_{i>j} \left(\frac{|a_i \cap a_j|}{k_i} + \frac{|a_i \cap a_j|}{k_j}, \right) \tag{11}$$

where the intersection between two inventories are divided only by the inventory size of the speaker. Figure 12 shows that these two quantities exhibit a very similar
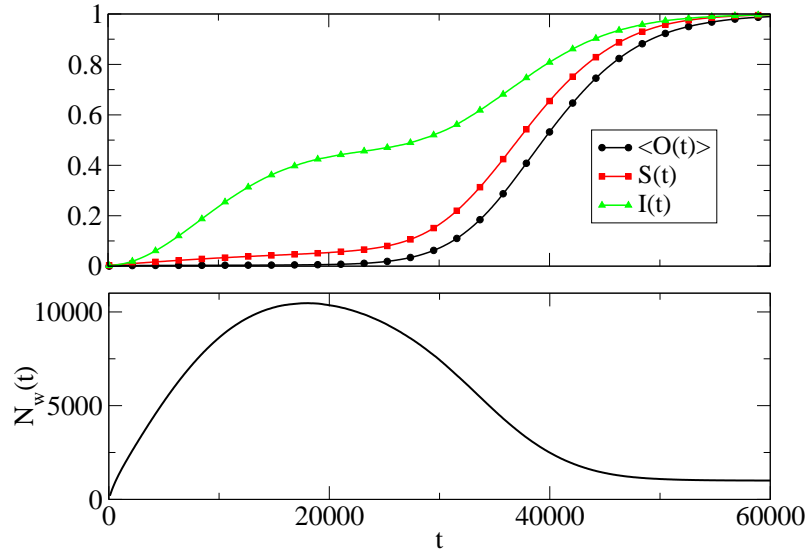
Fig. 12.   **Overlap functional** $O(t)$**.** Top: it is shown the evolution in time of the overlap functional averaged on 1000 simulation runs (for a population of $N = 10^3$ agents). Curves for the success rate, $S(t)$, and the average intersection between inventories, $I(t)$, are also included. By definition, $O(t) \leq 1$. It is evident that it holds $\langle O(t+1) \rangle > \langle O(t) \rangle$, which, along with the stronger $\langle O(t+1) \rangle > O(t)$ valid for almost all configurations (not shown), indicate that the system will reach the final state of convergence where $O(t) = 1$. Bottom: The total number of words $N_w(t)$ is plotted for reference.

behavior. However, while the overlap functional is equal to 1 only at convergence, this is not true for the success rate: if all agents had identical inventories of size $n > 1$ we would have $S(t) = 1$ and $O(t) = 1/n$. For this reason the success rate is not a suitable functional to prove convergence.

Finally, in Fig. 12 we have plotted also the average intersection between inventories, i.e.

$$I(t) = \frac{2}{N(N-1)} \sum_{i>j} |a_i \cap a_j|. \tag{12}$$

Remarkably, it turns out that $I(t) < 1$ during all the process, even if in principle this quantity is not bounded.

5. Single games

We know that single realizations have a quite irregular behavior and can deviate significantly from average curves (Fig. 2). It is therefore interesting to investigate to what extent *average* times and curves provide a good description of single processes.

In Figure 13(top) we have plotted the distribution of peak times for a population of $N = 10^3$ agents. It is clear that data cannot be fitted by a Gaussian distribution.
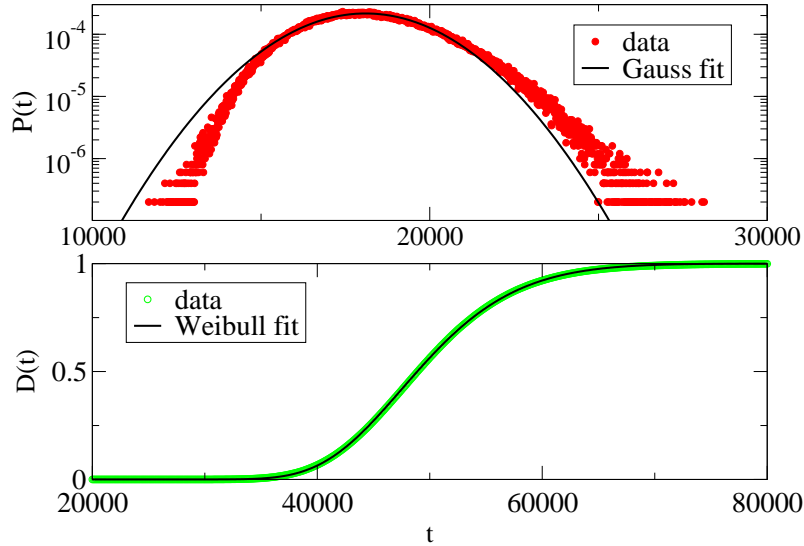
Fig. 13.   **Peak and convergence time distributions.** Top: the distribution of the peak times t_{max} clearly deviates from Gauss behavior. Bottom: the cumulative distribution of the convergence times t_{conv} is well fitted by a Weibull distribution $D(t) = (\exp{(\frac{t-g_0}{g_1})})^{g_2}$, with fit parameters $g_0 \approx 4.9 \times 10^4$, $g_1 \approx 7.9 \times 10^0$ and $g_2 \approx 9.6 \times 10^4$. The same function describes well also the peak time distribution (data not shown). Data refers to a population of $N = 10^3$ agents and are the result of 10^6 simulation runs.

The same peculiar behavior is shown also by the distribution of the convergence times (Fig 13(bottom)) and by that of the intervals between the time of the maximum number of words and the time of convergence (data not shown). Thus, the non-Gaussian behavior appears to be an intrinsic feature of the model. In fact, as shown in Figure 13(bottom) for the convergence times, all these distributions turn out to be well fitted (in their cumulative form) by an extreme value distribution:

$$D(t) = \exp{(\frac{t - g_0}{g_1})^{g_2}} \tag{13}$$

where g_0, g_1 and g_2 are fit parameters [31,32].

Extreme value distributions originated from the study of the distribution of the maximum (or minimum) in a large set of independent and identically distributed set of variables [31,32]. It turns out, however, that a generalization of these functions including a continuous shape parameter a, known as Gumbel distribution $G_a(x)$, has been observed in many models ranging from turbulence and equilibrium critical systems [33] to non-equilibrium models related to self-organized criticality [34], to $1/f$ noise [35] and many others systems (see [36] and references therein). The Naming Game model provides another example.

It must be noted, however, that there is no obvious theoretical explanation of the fact that extreme-value like distributions are found also in the study of the
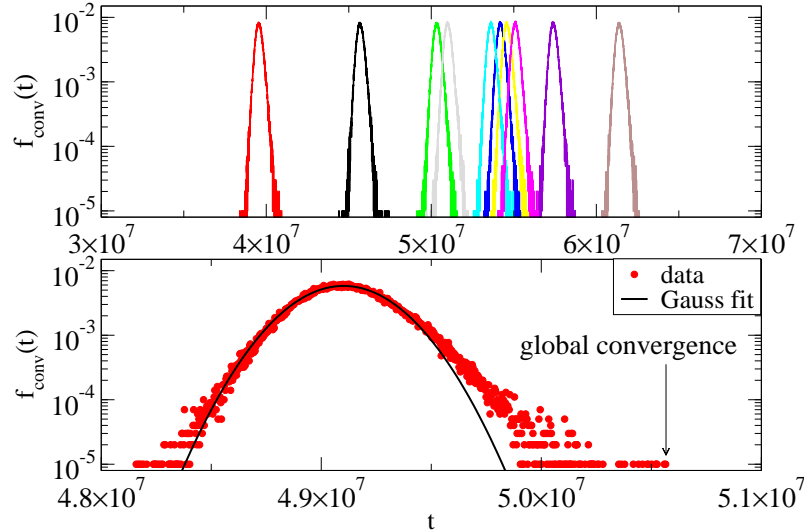
Fig. 14.   **Single agents convergence time distribution.** We define the convergence time of a single agent the last time in which it had to delete words after a successful interactions; $f_{conv}(t)$ is the fraction of agents who reach convergence at time t. Top: distributions coming from 10 simulation runs are plotted. It is clear that distributions coming from different runs can be non-overlapping, i.e., that the distance between the peaks of single curves can be much larger than the average width of the same curves (that does not exhibit any strong dependence on the single run). Bottom: a single distribution is analyzed, showing that it can not be described by a Gauss distribution. The last agent to converge determines the global convergence time. Curves are relative to a population of $N = 10^5$ agents.

fluctuations of global quantities. Yet, in many cases, these distributions are used simply like convenient fitting functions. Interestingly, it was recently shown that there is a connection between Gumbel functions and the statistics of global quantities expressed as sums of non identically distributed random variables, without the need of invoking extremal processes [36]. We can therefore argue that there is not necessarily a hidden extreme value problem in our model. In any case, a more rigorous explanation of the presence of Gumbel like distribution is left for future work.

In Figure 14(top) we show 10 single-run distributions of convergence times. Each curve illustrates the fraction of agents that converged at a given time in that run, $f_{conv}(t)$. We consider the single agent convergence time as the last time in which it had to delete words after a successful interaction. From Figure it is clear that the separation between the peaks of two different distributions can be much larger than the average width of a single curve. In other words, we see that the first moment of the distributions strongly depends on the single realization, while the second one does not. This information is crucial to interpret the curves shown in Figure 13 correctly. In fact, we now know that they are indeed representative of fluctuations
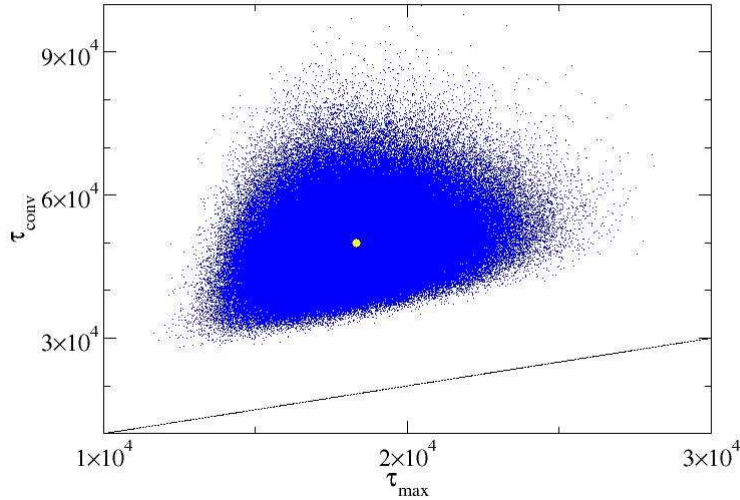
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Fig. 15.   **Correlation between peak and convergence times** (τ_{max} τ_{conv}, respectively). Each run is represented by a point in the scatter plot. The dashed line is $\tau_{conv} = \tau_{max}$ and therefore no points can lay below it. The average times t_{conv} and t_{max} are also shown with a clearer (yellow) point at the center of the distribution (statistical errors are not visible on the scale of the graph).

occurring among different runs, and do not describe simply the behavior of the last converging agent in a scenario in which most agents always converge, on average, at the same run-independent time. In Figure 14(bottom) it is shown that single run curves also deviate from Gauss behavior showing long tails for large times.

Given these distributions of convergence and peak times, and also that their difference t_{diff}, behaves in the same way, it is interesting to investigate whether there is any correlation between these two times. In Figure 15 we present a scatter plot in which the axis indicate τ_{conv} and τ_{max}, respectively the convergence and peak times for a single run (so that $t_{max} = \langle \tau_{max} \rangle$ and $t_{conv} = \langle \tau_{conv} \rangle$). It is clear that the correlation between this two times is very feeble. Indeed, the knowledge of τ_{max} does not allow to make any sharp predictions on when the population will reach convergence in the considered run.

Finally, Figure 16 shows that the relative standard deviation of all the relevant global quantities (t_{max}, t_{diff}, t_{conv} and N_w^{max}) decreases slowly as the system size N grows. In general, if the ratio $\sigma(x)/\langle x \rangle$ goes to zero as N increases, the system is said to exhibit self-averaging, and this seems to be the case for the Naming Game. However, it is difficult to draw a definitive conclusion, due to the large amount of time needed to perform a significant number of simulation runs for large values of
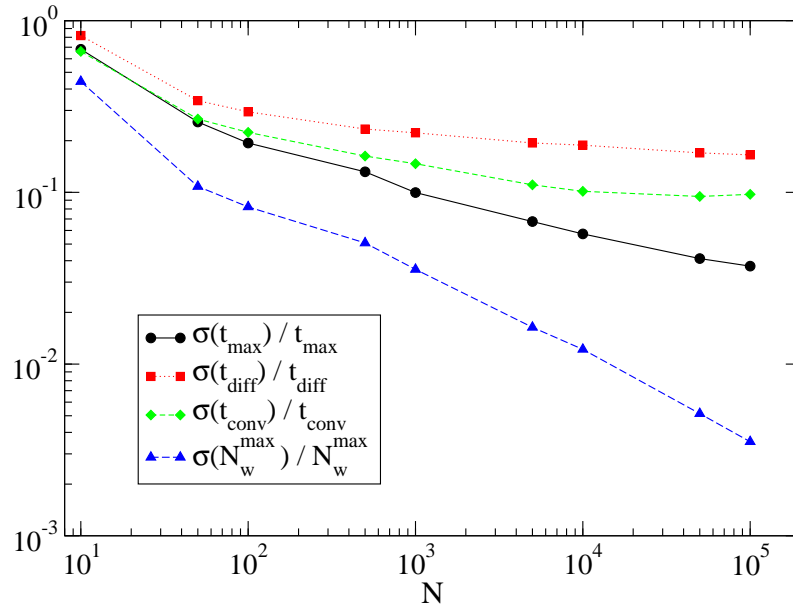
Fig. 16.   **Scaling of the relative standard deviation** $\sigma(x)/\langle x \rangle$**.** The ratio between the standard deviation σ and the corresponding (average) quantity is plotted as a function of the system size. In all cases the ratio decreases slightly, or stays constant, as the population size N grows. In particular, the decrease is more evident for N_w^{max} and t_{max}, while t_{conv} and t_{diff} curves are almost constant for large N. However, data from our simulations are not sufficient to conclude whether the Naming Game exhibits self-averaging. The standard deviation of x is defined as $\sigma(x) = \sqrt{\frac{1}{N_{runs}-1} \sum_{i=1}^{N_{runs}} (x_i - \langle x \rangle)^2}$, x_i is the i_{th} measured value, $\langle x \rangle$ is the average value, and N_{runs} is the number of simulation runs (here, $N_{runs} = 1000$).

N. Seemingly, the system seems to show self-averaging for what concerns the peak height and time, but this does not seem the case for the time of convergence. In any case, it is worth mentioning that Lu, Korniss and Szymanski [37] conclude that a slightly modified version of the Naming Game model does not display self-averaging when the population is embedded in random geometric networks.

6. Convergence Word

As we have seen, the negotiation process leading agents to convergence can be seen as a competition process among different words. Only one of them will survive in the final state of the system. It is therefore interesting to ask whether it is possible to predict, at some extent, which word is going to dominate.

According to the Naming Game dynamical rules, the only parameter that makes single words distinguishable is their creation time. Thus, it seems natural investigating whether the moment in which a word is invented can affect its chances of surviving. It turns out that this is indeed the case, as it is shown in Figure 17. The

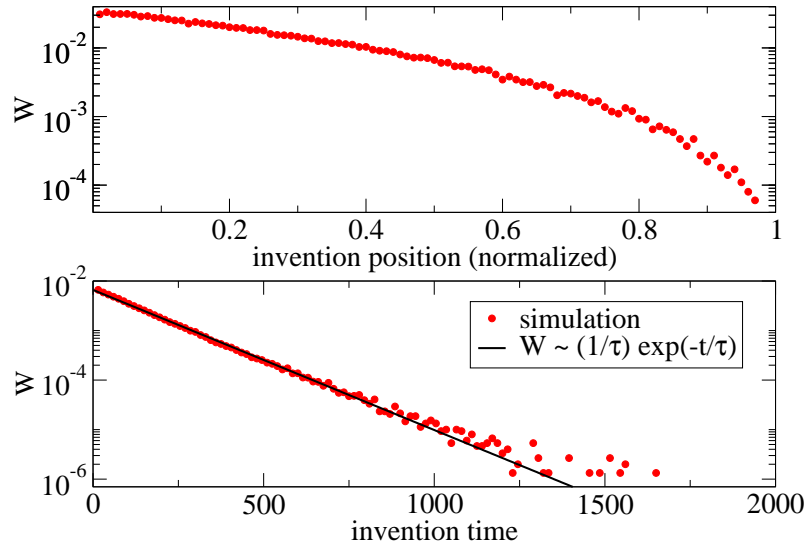24   *Andrea Baronchelli, Vittorio Loreto and Luc Steels*

Fig. 17.   **Word survival probability.** Top: The probability that a given word becomes the dominating one (i.e., the only one to survive when the system reaches the convergence state) is plotted as a function of its normalized invention position (see text for details). Early invention is clearly an advantageous factor. Bottom: the survival probability is now plotted in function of the invention time of words. The experimental distribution can be fitted by an exponential of the form $W \sim (1/\tau) \exp(-t/\tau)$, with $\tau \approx 150$. In both graphs, data have been obtained by 10^5 simulation runs of a population made of $N = 10^3$ agents.

upper graph plots the probability for a word to become the dominating one as a function of its normalized creation position. This means that each word is identified by its creation order: the first invented word is labeled as 1, the second as 2 and so on. To normalize the labels, they are then divided by the last invented word. From Figure it is clear that early invented words have higher chances of survival. The supremacy can be better quantified if we plot the winning probability of a word as a function of its invention time, as it is done in the bottom graph of Figure 17. We find that data from simulations are well fitted by an exponential distribution of the form $W = (1/\tau)exp(-t/\tau)$, indicating that the advantage of early invention is indeed quite strong.

Finally, an interesting question concerns the behavior of the winning probability distribution as a function of the system size N. In Figure 18 we show the distributions as a function of normalized labels described above for three different system sizes, $N = 10^2$, $N = 10^3$ and $N = 10^4$. The advantage of earlier creation increases with the system size, but our data do not allow clear predictions about the behavior of the distribution in the thermodynamic, $N \to \infty$, limit. We might speculate that the distribution collapses into a Dirac's delta of the first invented word [38].
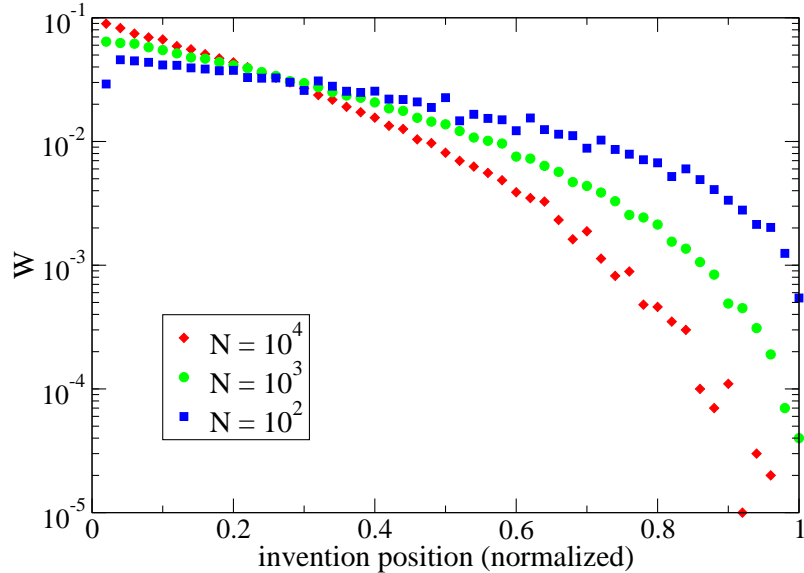
Fig. 18.   **Role of the system size on the distribution of the winning word.** The advantage of early invention increases in larger populations.

7. Symmetry Breaking - A controlled case

In the previous sections we have seen that the winning word is chosen by a symmetry breaking process (section 4.2). This is true even if, as we have seen in section 6, early invention increases the probability for a word to impose itself. Indeed, if we start with an artificial configuration in which each agent has a different word in its inventory, i.e., if we remove the influences of the invention process, the process still ends up in the usual agreement state (data not shown).

In particular, we can concentrate on the case in which there are only two words at the beginning of the process [30], say A and B, so that the population can be divided into three classes: the fraction of agents with only A, n_A, the fraction of those with only the word B, n_B, and finally the fraction of agents with both words, n_{AB} (see also [39] for a similar model). Describing the time evolution of the three species is straightforward:

$$
\begin{aligned}
dn_A/dt &= -n_A n_B + n_{AB}^2 + n_A n_{AB} \\
dn_B/dt &= -n_A n_B + n_{AB}^2 + n_B n_{AB} \\
dn_{AB}/dt &= +2n_A n_B - 2n_{AB}^2 - (n_A + n_B)n_{AB}
\end{aligned}
\tag{14}
$$

The meaning of the different terms of the equations is clear. For instance, for dn_A/dt we have that $-n_A n_B$ considers the case in which an agent with the word B transmits it to an agent with the word A, n_{AB}^2 takes into account the probability
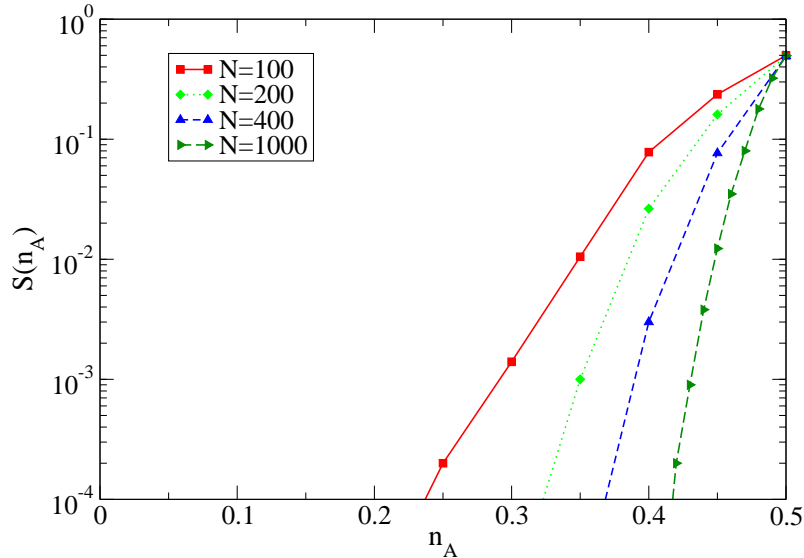
Fig. 19.   **Resistance against invasion.** Two populations that converged separately on conventions A and B merge. In Figure it is plotted the probability $S(n_A)$ that convention A becomes the final accepted convention of the new population, versus the normalized size n_A (where $n_A + n_B = 1$) of the original population of A spreaders. As the total population size increases, the probability for the initially less diffused convention to impose itself decreases, as predicted by equations (14).

that two more agents with only the A word are created if two agents with both words happen to have a success with A, and $n_A n_{AB}$ is due to the probability that an agent with only A has a success speaking to an agent with both A and B.

The system of differential equations (14) is deterministic. It has three fixed points in which the system can collapse depending on initial conditions. If $n_A(t = 0) > n_B(t = 0)$ $[n_B(t = 0) > n_A(t = 0)]$ then at the end of the evolution we will have the stable fixed point $n_A = 1$ $[n_B = 1]$ and, obviously, $n_B = n_{AB} = 0$ $[n_A = n_{AB} = 0]$. If, on the other hand, we start from $n_A(t = 0) = n_B(t = 0)$, then the equations lead to $n_A = n_B = 2n_{AB} = b$, with $b \simeq 0.18$. [30] The latter situation is clearly unstable, since any external perturbation would make the system fall into one of the two stable fixed points. Indeed, it is never observed in simulations due to stochastic fluctuations that in all cases determine a symmetry breaking forcing a single word to prevail.

Equations (14), however, are not only a useful example to clarify the nature of the symmetry breaking process. In fact, they also describe the interaction among two different populations that converged separately on two distinct conventions. In this perspective, eq. (14) predicts that the population whose size is larger will impose its conventions. In the absence of fluctuations, this is true even if the difference is very small: B will dominate if $n_B(t = 0) = 0.5 + \epsilon$ and $n_A(t = 0) = 0.5 - \epsilon$, for any

$0 < \epsilon \leq 0.5$ (we consider $n_{AB}(t = 0) = 0$). Figure 19 reports data from simulations in which the probability of success of the convention of the minority group n_A, $S(n_A)$, was monitored as a function of the fraction n_A (where $n_A + n_B = 1$). The absence of fluctuations is partly recovered as the total number of agents grows, and in fact it turns out that, for any given $n_A < 0.5$, the probability of success decreases as the system size is increased. Following eq. (14), in the thermodynamic limit ($N \to \infty$) this probability goes to zero.

8. Discussion and Conclusions

The Naming Game is the simplest model able to account for the emergence of a shared set of conventions in a population of agents. The main characteristics are [12,22]:

- The negotiation dynamics between individuals: the interaction rules are asymmetric and feedback is an essential ingredient to reach consensus;
- The memory of the agents: individuals can accumulate words, and only after many interactions they have to decide on the final word chosen;
- The absence of bounds to the inventory size: the number of words is neither fixed nor limited.

All these aspects derive from issues in Artifical Intelligence, namely to understand how an open population of physically embodied autonomous robots could self-organize communication systems grounded in the world [20]. The model is also relevant for all cases in which a distributed group of agents have to tacitly negotiate decisions, as in opinion spreading or market decisions [3]. Nevertheless the ingredients listed above are absent from most of the well known opinion-dynamics models. In the Axelrod model [40], for instance, each agent is endowed with a fixed-size vector of opinions, while in the Sznajd model [41] or the Voter model [42,43,44], the opinion can take only two discrete values, and an agent adopts deterministically the opinion of one of its neighbors. Deffuant et al. [45] model the opinion as a unique variable and the evolution of two interacting agents is deterministic, while in the Hegselmann and Krause model [46] opinions evolve as an averaging process. Most of these models include in some way the concept of bounded confidence, according to which two individuals do not interact if their opinions are not close enough, something which is entirely absent in the Naming Game. Interestingly, a recently proposed generalized version of the Naming Game, in which a simple parameter rules the consolidation behavior of the agents after a game, shows a non-equilibrium phase transition in which the final state can be consensus (as in the model we have analyzed in this paper), polarization (a finite number of conventions survives asymptotically) or fragmentation (the final number of conventions scales with the system size) [30], thus showing some phenomena also found for most opinion dynamics models.

Compared to earlier Semiotic Dynamics models of the Naming Game [47], this paper has made two contributions. The effort towards the definition of simple in-

teraction rules has helped to bring out the essential features needed to achieve a consensus state. Remarkably, we have shown that the weights typically associated with word-meaning pairs in all earlier Naming Game models are not crucial. The simplification does not impinge on the ability of the model to be used on embodied agents i.e., it does not introduce a global observer or other forms of global knowledge.

Next, because of the simplicity of the presented model, we have been able to perform a comprehensive analysis of its behavior which has never been done with earlier models due to their complexity. We have investigated the basic features of the process leading the population to converge, and how the crucial quantities scale with system size. In this context, we have also revealed a hidden timescale that rules the transition between the initial state, in which there is no communication among agents, and the final one, in which there is global agreement. Then we have analyzed several other aspects of the whole process, such as its properties of convergence, the relation between single runs and averaged curves, and the different probabilities for single words to impose themselves. We have also studied the elementary case in which only two words are present in the system, which can be interpreted as the merging of two converged populations, that clarifies the role of stochastic fluctuations in the convergence process. Although many of these results have been seen in numerical simulations, we have here been able to perform for the first time a mathematical analysis. In future work, the techniques we have used will be applied to more complex forms of communication including grammatical language for which some Artificial Intelligence models already exist [48].

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