

Emergence of fast agreement in an overhearing population: the case of naming game

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The Naming game (NG) describes the agreement dynamics of a population of N agents interacting locally in pairs leading to the emergence of a shared vocabulary. This model has its relevance in the novel fields of semiotic dynamics and specifically to opinion formation and language evolution. The application of this model ranges from wireless sensor networks as spreading algorithms, leader election algorithms to user based social tagging systems. In this article, we introduce the concept of overhearing (i.e., at every time step of the game, a random set of N^δ individuals are chosen from the population who overhear the transmitted word from the speaker and accordingly reshape their inventories). When $\delta = 0$ one recovers the behaviour of the original NG. As one increases δ , the population of agents reaches a faster agreement with a significantly low memory requirement. Remarkably, the convergence time to reach global consensus scales as $\log N$ as δ approaches 1.

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I. INTRODUCTION

The naming game (NG) [1] is a simple multi-agent model that employs local communications which leads to the emergence of shared communication scheme in a population of agents. The game is played by a group of agents in pairwise interactions to negotiate conventions, i.e., associations between forms (names) and meanings (for example individuals in the world, objects, categories, etc.). The negotiation of conventions is a process through which one of the agents (i.e., the speaker) tries to draw attention of the other agent (the so-called hearer) towards the external meaning by the production of a conventional form. For example, the speaker might be interested to make the hearer identify an object through the production of a name. The hearer may be able to express the proper meaning and the speaker-hearer pair meet a local consensus in which case we call it a “success”. The other side of the coin is the hearer producing a wrong interpretation in which case the hearer takes lesson from the meeting by updating its meaning-form association. Thus, on the basis of success and failure of the hearer in producing meaning of the name, both the interacting

agents reshape their internal meaning-form association. Through successive interactions, the local adjustment of individual meaning-form association leads or should lead to the emergence of a global consensus.

The model represents one of the simplest example leading progressively to the establishment of human-like languages. It was expressly conceived to explore the role of self-organization in the evolution of language [2, 3] and it has acquired, since then, a paradigmatic role in the novel field of semiotic dynamics which studies how language evolves through invention of new words and grammatical constructions, adoption of new meaning for words.

Implementing the naming game with local broadcasts, serves as a model for opinion dynamics in large-scale autonomously operating wireless sensor networks. In [4], it is pointed out that NG can be used as a leader-election model among a group of sensors where one does not intend to disclose information as to who the leader is at the end of the agreement process. The leader is a trusted agent having possible responsibilities ranging from routing coordination to key distribution and the NG identifies the leader which is hardly predictable from outside resulting in highly secure systems.

The creation of shared classification schemes by the NG in a system of artificial and networked autonomous agents can also be relevant from a system-design viewpoint, e.g., for sensor networks [5, 6]. Imagine a scenario where mobile or static sensor nodes are deployed in a large spatially extended region exploring an unknown and

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possibly hostile environment. One of the important tasks would be to convey information to the agents about their discoveries, in particular they should be able to agree on the identification of the new objects with no prior classification scheme or language to communicate regarding detecting and sensing objects. Since subsequent efficient operation of the sensor network inherently depends on unique object identification, the birth of a communication system among the agents is crucial at the exploration stage after network deployment. Besides artificial systems where it is obvious that the agreement has to take place rapidly, it concerns social dynamics too [7]. In particular, as an example, one can think of the emergence of shared lexicon inside social groups and communities. When a new concept is introduced, different people name it differently. These words spread among the population, competing against each other, until the choice of one of them is taken and everybody uses the same word [8–10]. This type of dynamics has become a broad interest of social groups and communities with the inception of user-based tagging systems (such as flickr.com or del.icio.us) [11, 12], where users manage tags to share and categorize information as well as the “likes” of Facebook [13] and Twitter [14].

The minimal NG has diverse applications in many fields [4–6, 11–14]. Here we shall reshape the model in a “multi-party” communication framework. In particular, this involves conversations between two parties and plays a significant role in the formation of shared mental model [15]. Parties involved in a multi-party dialogue can assume roles other than the speaker/addressee roles in traditional two-party communication. One of the most important roles is that of the *overhearer*. Overhearing involves monitoring the routine conversations of agents, who know they are being overheard, to infer information about the agents. The overhearers might then use such information to assist themselves, assess their progress or suggest advice to the others. When an agent ‘overhears an interaction’, she receives information about something that is not primarily addressed to him. For instance, one can listen to a conversation between two friends without being part of their dialogue. Multi-party discourse analysis shows that overhearing is a required communication type to model group interactions and consequently reproduces them among artificial agents [16]. Various applications are known to employ the concept of overhearers [17–22]. Novick and Ward [17] have employed overhearing to model interactions between pilots and air traffic controllers. Kaminka et al. [18] have developed a plan-recognition approach to overhearing in order to monitor the state of distributed agent teams. Aiello et al. [19] and Bussetta et al. [20, 21] have investigated an architecture that enables overhearing, so that domain experts can provide advice to problem-solving agents when necessary. Legras [22] has examined the use of overhearing for maintaining organizational awareness. Recently, Komarova et al. [23] have studied the effect of eavesdropping in the evolution of language.

Motivated by the above literature and diverse applications of overhearer, we review the naming game for the emergence of a communication system in the presence of overhearers and attempt to investigate its global properties. To the best of our knowledge, NG has not been studied in this perspective of multi-party communication. The basic activity of the overhearers in the naming game is as follows: when a conversation between two parties is going on, the third party (i.e., the overhearers) may eavesdrop the conversation and reshape their meaning-form association. As we shall see in this article that the introduction of the concept of overhearing leads to much faster convergence than traditional NG [1] coupled with a low memory requirement per agent.

An alternative but closely related approach for opinion (rumor) spreading has been introduced in [24] where the authors investigated the problem on a fully connected network of N agents and showed that the rumor spreading takes $O(\log N)$ rounds. The same rumor spreading problem has been studied on networks with conductance ϕ in [25] and later thoroughly investigated and made more efficient in [26]. In particular, the authors achieve a tight bound on the number of rounds required in spreading a rumor over a connected network of N nodes and conductance ϕ which is $O(\frac{\log N}{\phi})$. We shall outline a detailed comparison between our approach and the above literature later in this article.

The rest of the article is organized as follows. Section II is devoted to the description of the basic naming game model in the presence of overhearers. In section III, we investigate the scaling relations of some important quantities and provide analytical arguments to derive the relevant exponents. In section IV, we discuss the state of the art and compare our findings with [26]. Finally, conclusions are drawn in section V.

II. THE MODEL DEFINITION

The model consists of an interacting population of N artificial agents observing a single object to be named, i.e., a set of form-meaning pairs (in this case only names competing to name the unique object) which is empty at the beginning of the game ($t = 0$) and evolves dynamically in time. At each time step ($t = 1, 2, \dots$) two agents are randomly selected and interact: one of them plays the role of *speaker*, the other one that of *hearer*. In addition, a set of N^δ individuals are randomly selected in each step who behave as *overhearers*. Note that δ is a parameter of the model.

In each game the following steps are executed:

- The speaker transmits a name to the hearer. If her inventory is empty, the speaker invents a new name, otherwise she selects randomly one of the names she knows.
- If the hearer has the uttered name in her inventory, the game is a success, and both agents delete all

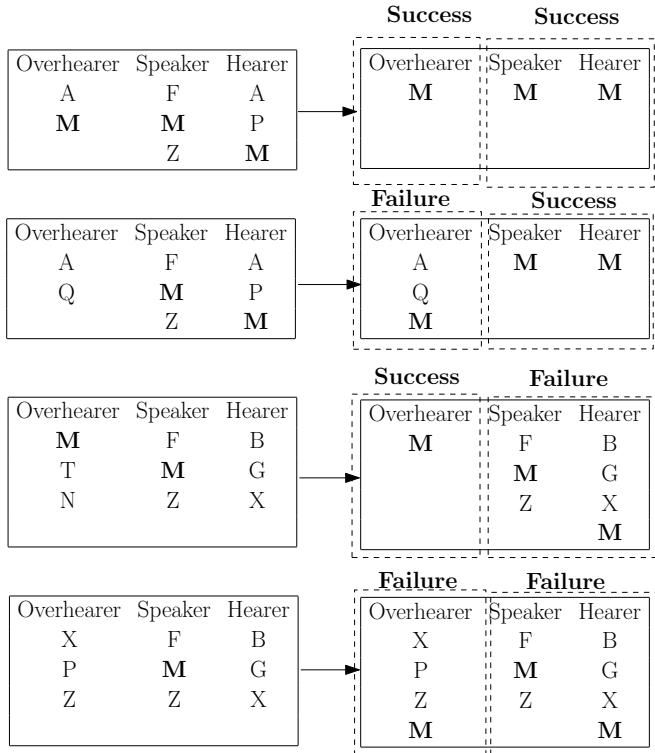


FIG. 1: Naming game interaction rules in presence of overearer. Each agent is described by her inventory, i.e., the repertoire of known names or words. The speaker selects randomly one of her names, or invents a new name if her inventory is empty (i.e., at the beginning of the game) and transmits it to the hearer. If the hearer does not know the selected name, she simply adds it to her inventory, and the interaction is a failure. If, on the other hand, the hearer recognizes the name, the interaction is a success, and both agents delete from their inventories all their names but the winning one. In both the situations, the overearners overhear the transmitted name and in case the name is known, they delete all names from their inventories except the transmitted one otherwise they perform a failure update. The above figure represents the names symbolically in English alphabet and boldface signifies the name that the speaker transmits to the hearer.

their names, but the winning one.

- If the hearer does not know the uttered name, the game is a failure, and the hearer inserts the name in her inventory.
- Each overearer overhears the word uttered by the speaker; if the word is in her inventory, she removes all the words from her inventory except this word (i.e., treats the event as a success) else she adds this word in her inventory (i.e., treats the event as a failure).

Fig. 1 shows a hypothetical example illustrating the inventory update rules of the different agents in the model of NG with overearners.

III. RESULTS AND DISCUSSIONS

The basic quantities to be measured in the NG are the total number of words $N_w(t)$, defined as the sum of the inventory sizes of all the agents at the given time instance t , and the number of different words $N_d(t)$ present in the system at time t , telling us how many synonyms are present in the system at that time instance. The dynamics proceeds as illustrated in fig. 2(a) and 2(b). At the beginning both $N_w(t)$ and $N_d(t)$ grow linearly as the agents invent new words. As invention ceases, $N_d(t)$ reaches a plateau, i.e. a maximum number of distinct words. On the other hand, $N_w(t)$ keeps growing till it reaches a maximum at time t_{max} . The total number of words then decreases and the system reaches the convergence state at time t_{conv} . At convergence all the agents share the same unique word, so that $N_w(t_{conv}) = N$ and $N_d(t_{conv}) = 1$. It is observed that all the global quantities in the basic naming game [1] follow a power-law scaling as a function of the population size N . In particular, $t_{max} \sim N^\alpha$, $t_{conv} \sim N^\beta$, $N_w^{max} \sim N^\gamma$ where $\alpha \approx \beta \approx \gamma \approx 1.5$ for the original naming game ($\delta = 1$) on a fully connected graph topology.

We now focus on analytically estimating the scaling of (i) N_w^{max} , (ii) t_{max} and (iii) t_{conv} with N in the presence of N^δ overearners.

A. Scaling of N_w^{max}

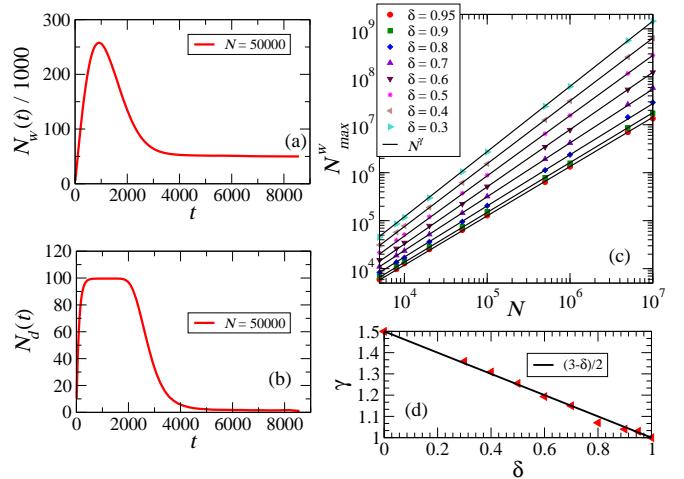


FIG. 2: Evolution of (a) the total number of words $N_w(t)$, (b) the number of different words present in the system, with time t when the number of overearners is ηN ($\delta = 1$ as in the original NG) with $\eta = 0.05$. Data refer to a population of $N = 50000$ agents. (c) Scaling of N_w^{max} with N for different values of δ . (d) The figure expresses the relation of γ vs δ . Each point in the above curves represents the average value obtained over 100 simulation runs.

In the original NG the maximum number of distinct words scales as N with an average value of $N/2$. This

is because at each time step only two agents can update their inventories, inventing in particular a new word if their inventories are empty. When N^δ overhearers are present the fraction of agents who can invent new words is reduced by a factor N^δ . In this way the number of unique words in the system when the total number of words is close to the maximum is $\propto N/N^\delta = N^{1-\delta}$. Further, let us assume that each agent has on average cN^a words in her inventory when the total number of words is close to the maximum. As in the original NG, $N_w^{max} \sim N^\gamma$ so that $\gamma = a + 1$ holds here also. In the following, we shall attempt to find a relation between γ and δ . We can write the evolution equation of $N_w(t)$ as

$$\frac{dN_w(t)}{dt} \propto \left(1 - \frac{cN^a}{N^{1-\delta}}\right) N^\delta - \frac{cN^a}{N^{1-\delta}} cN^a N^\delta \quad (1)$$

where the first term is related to unsuccessful games (increase in N_w is proportional to N^δ times the probability of a single failure) and the second term is for successful games (decrease in N_w is proportional to $cN^a N^\delta$ times the probability of a single success). At maximum, $\frac{dN_w(t_{max})}{dt} = 0$ and therefore in the limit $N \rightarrow \infty$ the only relation possible is $a = \frac{1-\delta}{2}$ which implies $\gamma = \frac{3-\delta}{2}$. When $\delta = 0$, $\gamma = 1.5$ we recover the original NG behavior. In general, as one varies δ in the interval $[0, 1]$, N_w^{max} varies as N^γ where $\gamma \in [1, 1.5]$. The scaling of N_w^{max} with δ for different values of N is shown in fig. 2(c). In other words, for all values of N , N_w^{max} monotonically decreases as δ increases and in the limit $\delta \rightarrow 1$ we have $N_w^{max} \rightarrow N$. This behaviour of γ vs δ is confirmed by the simulation results shown in fig. 2(d).

B. Scaling of t_{max}

We have to analyze the behaviour of the success rate in the beginning of the process in order to estimate the scaling relations for t_{max} . At early stages, most successful interactions involve agents which have already met in previous games. Thus, the probability of success is proportional to the ratio between the number of couples that have interacted before time t , which is $\propto tN^\delta(N^\delta - 1)/2$ and the total number of possible pairs is $N(N - 1)/2$. Thus, in the early stages, success rate $S(t) \propto \frac{tN^{2\delta}}{N^2} = tN^{2(\delta-1)}$. Note that if we put $\delta = 0$, we immediately recover $S(t) \propto t/N^2$ which is the case for the original NG. If $\delta \rightarrow 1$, we have $S(t) \propto t$, while if $\delta = \frac{1}{2}$ we have $S(t) \propto t/N$. Both these observations are validated by fig. 3(a) and 3(b) respectively for different values of N . With this information about $S(t)$ we can now easily estimate the value of t_{max} by once again writing the evolution equation:

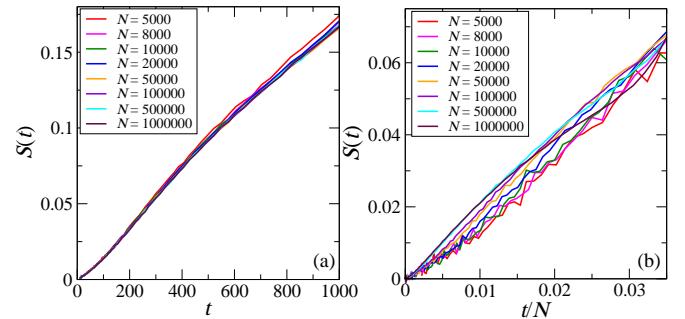


FIG. 3: Success rate at the onset of the dynamics (a) Success rate $S(t) \propto t$ when no. of overhearers = ηN where $\eta = 0.05$. (b) $S(t) \propto t/N$ when $\delta = \frac{1}{2}$. All the curves have been generated averaging over 100 simulation runs.

$$\frac{dN_w(t)}{dt} \propto \left(1 - tN^{2(\delta-1)}\right) N^\delta - tN^{2(\delta-1)} cN^{(1-\delta)/2} N^\delta \quad (2)$$

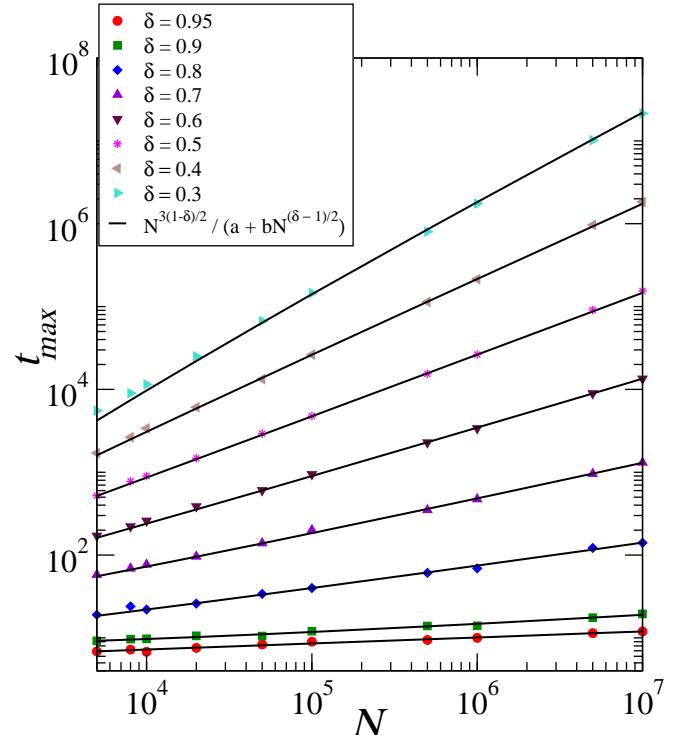


FIG. 4: Scaling of t_{max} with population size N . As one varies δ , t_{max} scales as $\frac{N^{\frac{3(1-\delta)}{2}}}{a+bN^{(\delta-1)/2}}$ where a and b are some constants. Each data point of all the above curves represents averaged value taken over 100 simulation runs. The bold lines show the fit from the analytical results.

If we now impose $\frac{dN_w(t_{max})}{dt} = 0$, then in the limit $N \rightarrow \infty$ we have $t_{max} \propto \frac{N^{\frac{3(1-\delta)}{2}}}{a+bN^{-(1-\delta)/2}}$ where the de-

nominator is precisely a correction term with a and b as constants. Once again, for $\delta = 0$ we have $t_{max} \propto N^{3/2}$ thus recovering the original NG property. On the other hand, in the limit $\delta \rightarrow 1$, t_{max} approaches $O(1)$. The results of the scaling of t_{max} with N for different values of δ are shown in fig 4.

C. Scaling of t_{conv}

The exponent for the convergence time, β , deserves a more intricate discussion, and we can only attempt to provide a naïve argument here. We concentrate on the scaling of the interval of time separating the peak of $N_w(t)$ and the convergence, i.e., $t_{diff} = (t_{conv} - t_{max})$, since we already have an argument for t_{max} . t_{diff} is the time span required by the system to get rid of all the words but the one which survives in the final state.

If we adopt the mean field assumption that at $t = t_{max}$ each agent has on average $N_w^{max}/N \sim N^{\frac{1-\delta}{2}}$ words, we see that, by definition, in the interval t_{diff} , each agent must have won at least once. This is a necessary condition to have convergence, and it is interesting to investigate the timescale over which this happens. Assuming that \bar{N} is the number of agents who did not yet have a successful interaction at time t , we have:

$$\bar{N} = N(1 - p_s p_w)^t \quad (3)$$

where p_s is the probability of choosing a specific agent and $p_w = S(t)$ is the probability of a success. In this case, $p_s = \frac{1}{N^{1-\delta}}$ and $p_w = tN^{2(\delta-1)}$. In order to estimate t_{diff} , we require the number of agents who have not yet had a successful interaction to be finite just before the convergence, i.e., $\bar{N} \sim O(1)$ and we consider $p_w(t_{max}) = t_{max}N^{2(\delta-1)} = \frac{N^{-(1-\delta)/2}}{a+bN^{-(1-\delta)/2}}$. In this way one gets:

$$t_{diff} \propto N^{\frac{3(1-\delta)}{2}} (a + bN^{-(1-\delta)/2}) \log N \quad (4)$$

The above scaling relation of t_{diff} is well confirmed by the simulation results in fig 5.

Thus, when $\delta = 0$, and we ignore the correction, we recover the original NG case: $t_{conv} \propto N^{3/2} \log N$. On the other hand, in the limit $\delta \rightarrow 1$, we have $t_{conv} \rightarrow \log N$.

IV. RELATED WORK

Most previous studies in semiotic dynamics has focused on populations of agents in which all pairwise interactions are allowed, i.e., the agents are placed on the vertices of a fully connected graph. In statistical mechanics, this topological structure is commonly referred to as mean-field topology. In the original work on the minimal naming game model [1], Baronchelli et al. studied, numerically and analytically, the behavior of the mean-field model, providing theoretical arguments in order to explain the main properties of the global behavior of the population.

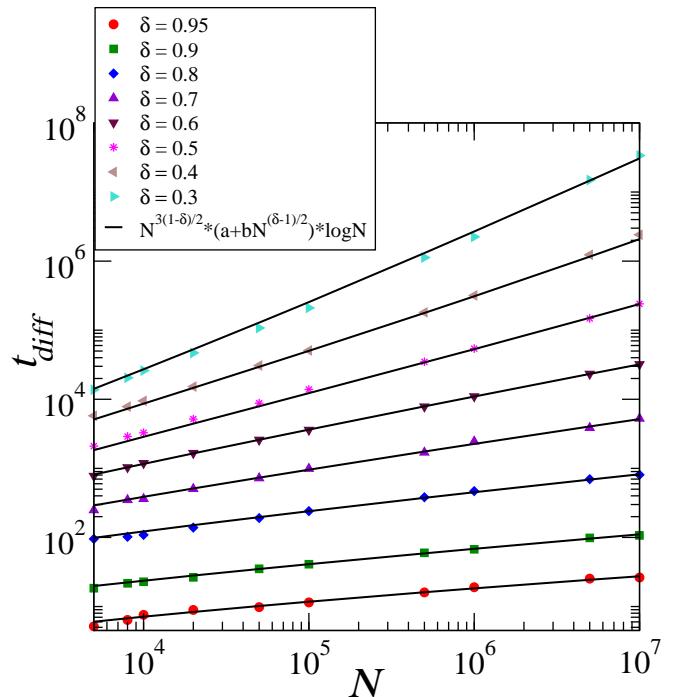


FIG. 5: Scaling of t_{diff} with the population size N . As δ varies, this interval time t_{diff} scales as $N^{\frac{3(1-\delta)}{2}} (a + bN^{(\delta-1)/2}) \log N$ where a and b are some constants. Each point represents the average value obtained from 100 simulation runs. The bold lines show the fit from the analytical results.

The model is extensively studied apart from fully connected network, in regular lattices [27, 28]; small world networks [28–31]; random geometric graphs [28, 32, 33]; and static [34–36], dynamic [37], and empirical [38] complex networks. The final state of the system is usually a complete consensus [39], but stable polarized states can be reached introducing a simple confidence/trust parameter [40]. NG as defined in [1] is also modified in several ways [28, 32, 38, 40–49] and it represents the fundamental stepping stone of more complex models in computational cognitive sciences [50–53]. In [27], effects of topological embedding on the naming game dynamics is reported and it has been shown that the convergence process requires a memory per agent scaling as N and lasts a time $N^{1+\frac{2}{d}}$ in dimension $d \leq 4$ (the upper critical dimension), while in mean field both memory and time scale as $N^{\frac{3}{2}}$. Thus, low dimensional lattices require more time to reach the consensus compared to mean-field but for a lower memory. In [34], for both the ER and BA network models, the convergence time t_{conv} scales as N^β , where $\beta \approx 1.4$. In [31], Barrat et al. show that for small-world networks the convergence towards consensus is reached on a timescale of order $N^{\beta_{SW}}$, with $\beta_{SW} \approx 1.4 \pm 0.1$, close to the mean-field case ($N^{\frac{3}{2}}$) and this is in strong contrast with the N^3 behavior of purely one-dimensional systems. In particular, time to converge scales as $p^{1.4 \pm 0.1}$, which is

consistent with the fact that for p of order $\frac{1}{N}$ one should recover an essentially one-dimensional behavior with convergence times of order N^3 . The small-world topology therefore allows to combine advantages from both finite dimensional lattices and mean-field networks.

There has been a long history in the area of rumor spreading which closely parallels the major concepts of the model investigated here. One of the benchmarks is the PUSH-PULL strategy introduced in [24] and then further extended and made incrementally more efficient in [25, 26]. The simple PUSH-PULL mechanism is as follows: at each round, a node that knows the rumor selects a random neighbor and forwards the rumor (PUSH), or if the node does not know the rumor selects a neighbor uniformly at random and asks for the information (PULL). This scheme informs all N agents in a fully connected network in time $\log_3 N + O(\ln \ln N)$ with probability at least $1 - O(N^{-\alpha})$ where $\alpha > 0$. In [26], Chierichetti et al. investigated the rumor spreading in a connected network of N nodes with conductance ϕ . The conductance of a graph, a name borrowed from electrical networks, is a quantity that measures how well information spreads in the graph, its maximum value $\phi = 1$ being reached for a fully connected graph. In [26] it has been shown that the PUSH-PULL strategy broadcasts a message within $O(\frac{\log^2 \phi^{-1}}{\phi} \log N)$ rounds with a high probability of $1 - o(1)$ which the authors claim to be a tight bound. Estimates regarding the amount of memory required per agent for the purpose of spreading have not been presented so far in the above literature. In general, since there is usually one rumor to be spread the memory estimate becomes trivial ($O(1)$). However, one can always envisage a situation where new rumors need to be constantly invented in the population.

In our work, we theoretically as well as by means of simulations show that opinion spreading in a fully connected network (i.e., conductance $\phi = 1$) of N nodes takes a $O(\log N)$ time to reach the global agreement with a maximum memory estimate of N as $\delta \rightarrow 1$ which is

comparable with the time requirement for the spreading of rumor in [24, 26]. It is important to stress that in our model the case of maximal conductance $\phi = 1$ is obtained on a fully connected graph only in the limit $\delta = 1$. Further, we point out that the model of NG in overhearing population can be recast for rumor spreading when constantly new rumors can be generated that should compete to spread in the whole population.

V. CONCLUSIONS AND FUTURE WORK

In this article, we have introduced the agreement dynamics of naming game to describe the convergence of population of agents on assigning a unique name to an object in the domain of multi-party communication. We have investigated the basic naming game model in an overhearing population and computed the scaling behaviour of the main global quantities: $N_{max}^w \propto N^\gamma$ where the exponent $\gamma = \frac{3-\delta}{2}$; $t_{max} \propto N^\alpha$ where roughly the exponent $\alpha = \frac{3(1-\delta)}{2}$ and $t_{conv} \propto N^\alpha \log N$. In particular, we achieve a very fast agreement in the population with significantly low memory requirement. Moreover, we have also suggested that this model with overhearers can find relevant application in rumour spreading.

There could be many interesting future directions. First of all, it will be interesting to explore the model in a scenario where agents make their success update probabilistically as studied in [40]. The role of agent topology can also be one of the future perspectives. Different complex topologies could be studied where agents are embedded on more realistic networks. Furthermore, while in this article we have concentrated only on the study of the scaling properties of the system, performing a detailed analysis of the microscopic aspects of the dynamics could be another interesting topic for future research. One might also extend the idea of overhearers to the more complex tasks like categorization [50–53].

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