Asymmetric negotiation in structured language games

Han-Xin Yang,[1] Wen-Xu Wang,[2] and Bing-Hong Wang[1]
[1]*Department of Modern Physics, University of Science and Technology of China, Hefei 230026, China*
[2]*Department of Electronic Engineering, City University of Hong Kong, Hong Kong SAR, People's Republic of China*
(Received 5 October 2007; revised manuscript received 17 December 2007; published 21 February 2008)

We propose an asymmetric negotiation strategy to investigate the influence of high-degree agents on the agreement dynamics in a structured language game, the naming game. We introduce a model parameter, which governs the frequency of high-degree agents acting as speakers in communication. It is found that there exists an optimal value of the parameter that induces the fastest convergence to a global consensus on naming an object for both scale-free and small-world naming games. This phenomenon indicates that, although a strong influence of high-degree agents favors consensus achievement, very strong influences inhibit the convergence process, making it even slower than in the absence of influence of high-degree agents. Investigation of the total memory used by agents implies that there is some trade-off between the convergence speed and the required total memory. Other quantities, including the evolution of the number of different names and the relationship between agents' memories and their degrees, are also studied. The results are helpful for better understanding of the dynamics of the naming game with asymmetric negotiation strategy.

DOI: 10.1103/PhysRevE.77.027103 PACS number(s): 89.75.−k, 05.45.Ra, 05.45.Xt

The combination of complex network and social dynamical behavior has received much attention in the past few years, spurred by the rapid development of complex network theory [1–4]. Social and natural systems can be described by complex networks with individuals occupying nodes and interaction among individuals represented by edges. Much empirical evidence has indicated that the interaction networks of real systems are neither regular nor purely random, but show small-world and scale-free topological properties [1,2]. It is found that these properties play significant roles in the various dynamical processes taking place on complex networks, compared to regular and random networks [3,4]. Hence, it is necessary to model social dynamics on more realistic networks instead of regular, random, or well-mixed structures.

Language dynamics, as an important issue in social dynamics, has been extensively studied with focus on language evolution [5–8] and competition [9–14]. Recently, a language game, the naming game, related to opinion formation but with many differences, has been proposed to study the evolution of language or communication behaviors among a population of individuals [15–23]. In the naming game, agents mutually communicate about invented names for an unknown object to reach a final consensus on naming the object. A minimal version of the naming game was introduced in Ref. [17], in which a successful negotiation between two neighboring agents leads to the preservation of the name in the negotiation and the cancellation of the other names in memory. Although this model is simple, it can reproduce the agreement dynamics and the global consensus. In the minimal naming game, there are three possible negotiation strategies, called directed, reverse, and neutral strategies [19]. In the reverse strategy, agents occupying hubs are preferentially selected as speakers, who tell the name to others. It is found that in this case the evolutionary time for achieving the global consensus is much shorter than in the other two cases [19]. This phenomenon indicates that hubs play positive roles in reaching the global consensus if they have high probabilities to act as speakers. However, there exists one natural question: if hubs have much more chance to act as speakers, can the global consensus always be achieved faster?

In this paper, we propose a modified naming game, introducing an asymmetric negotiation strategy to study the hub effects on the agreement dynamics. We focus on the convergence time for reaching the final consensus of a population of individuals, which is of practical importance. Fast convergence to the same opinion or name can avoid difficulties in communication among social individuals and waste of resources in storing information in communication systems. We investigate the modified model on both small-world and scale-free networks. By tuning a free parameter, the probability of high-degree agents acting as speakers is controlled. We found that, although the effect of high-degree agents with high speaking probability can enhance the convergence efficiency, a very strong effect on the contrary delays the achievement of the final consensus; this is reflected in an optimal value of the parameter in the middle range of the parameter space. By studying the total memory, we found that some trade-off between the total memory and the convergence speed is required, i.e., to reach global consensus faster, more total memory is used by agents.

We first construct scale-free and small-world networks by using the Barabási-Albert (BA) [24] and Newman-Watts (NW) models [25]. Then we describe the modified naming game model with an asymmetric negotiation strategy. Each node of a network is occupied by an agent. The number of agents is equal to the network size. N agents observe a single object and communicate its name to neighboring agents. Each agent is endowed with a memory to restore a number of different names or opinions. Initially, agents' memories are empty and each agent i is assigned a weight k_i^{α}, where k is the degree of agent i and α is a tunable parameter. At each time step, a pair of connected nodes are randomly selected to communicate. The probability p_i of choosing one of them i as the speaker is proportional to i's weight:

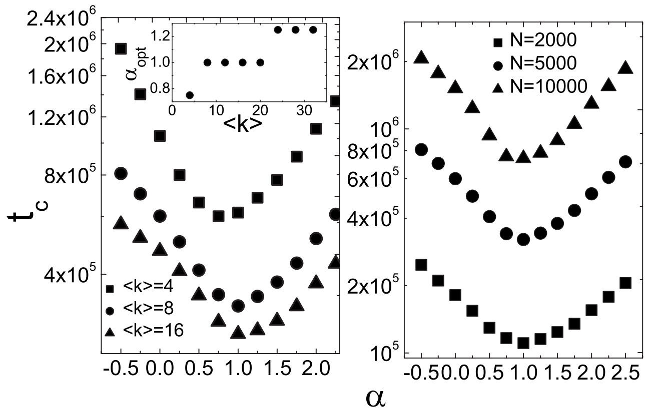

FIG. 1. Convergence time t_c vs α on BA networks. Left panel: different $\langle k\rangle$ with network size $N=5000$. The inset shows the optimal values of α, α_{opt}, as a function of the average degree $\langle k\rangle$. Right panel: different network size N with fixed average degree $\langle k\rangle=8$. Each data point is obtained by averaging over 1000 runs on each of ten different network realizations. In the BA model, there are m_0 nodes initially. At each time step, a new node with m edges is preferentially attached to the existing network. The average degree $\langle k\rangle=2m$.

$$p_i = \frac{k_i^\alpha}{k_i^\alpha + k_j^\alpha}. \tag{1}$$

Once one agent is selected as the speaker, the other plays as the hearer. If the speaker's memory is empty, it invents a new name; otherwise, it randomly selects one of the names stored in its memory. Then the speaker transmits the invented or selected name to the hearer. If the hearer finds the same name existing in its memory, the negotiation is successful and both agents delete all other names but preserve the agreed name. If the hearer does not know the name, it adds the name into its memory. By repeating the above process, the system evolves.

The asymmetric negotiation refers to the strategy of choosing speakers according to their weights. If $\alpha>0$, higher-degree agents have more chances to act as speakers; if $\alpha<0$, lower-degree agents have more chances to be speakers. In the case of $\alpha=0$, the modified model reduces to the neutral naming game in which two selected neighboring agents have the same probability to be the speaker [19]. Hence, by tuning the value of α, one can investigate the dynamics of the language game with different influence strength of high-degree agents.

In the following we present simulation results on the convergence time t_c for reaching the global consensus on both BA scale-free and NW small-world networks. Figure 1 shows t_c as a function of the parameter α for different average degrees $\langle k\rangle$ of BA networks. One can see that there exists an optimal value of α for all studied $\langle k\rangle$, resulting in the fastest convergence. The inset of the left panel shows the optimal value α_{opt} vs $\langle k\rangle$. There is a slow increase as $\langle k\rangle$ increases. The optimal values are near 1, which means that, when the influence of agents is proportional to their degrees, the system can achieve the final consensus most quickly. The

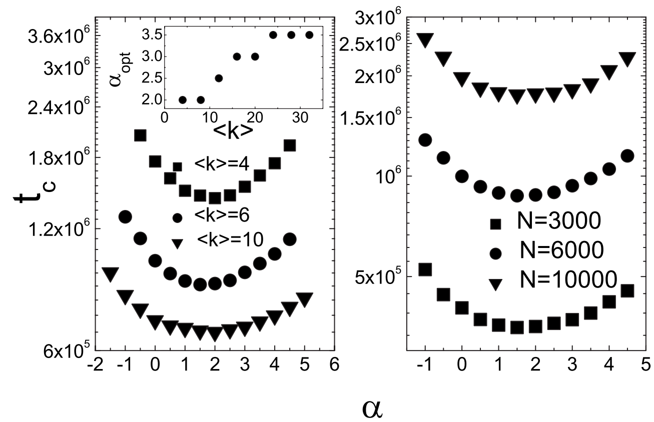

FIG. 2. Convergence time t_c as a function of α on NW networks. Left panel: different average degree $\langle k\rangle$ with $N=6000$. The inset shows the optimal values of α, α_{opt}, as a function of the average degree $\langle k\rangle$. Right panel: different network size N with fixed average degree $\langle k\rangle=6$. Other parameters are the same as in Fig. 1. The NW model is a modified version of the Watts-Strogatz small-world network model [26]. The NW network is constructed by randomly adding edges to a regular ring network. $\langle k\rangle$ of the NW network is controlled by the probability of randomly adding edges.

fact that the optimal value of α is above 0 also indicates that the presence of a hub favors the convergence efficiency, compared to the neutral strategy ($\alpha=0$). This phenomenon is consistent with previously reported results that the reverse strategy leads to faster convergence than the neutral strategy, because high-degree agents more frequently act as speakers when the reverse strategy is adopted [19]. On the other hand, when α continuously increases from 1, t_c becomes longer. For very large α, for example $\alpha>2$, t_c is even larger than when the neutral strategy is used, which demonstrates that a too strong influence of high-degree agents plays a negative role in achieving global consensus. The right panel of Fig. 1 shows t_c depending on α for different network sizes. One can see that larger network sizes result in longer convergence times but the optimal value of α remains fixed. Hence, α_{opt} is only slightly correlated with the average degree $\langle k\rangle$ of BA networks and independent of the network size.

Figure 2 shows t_c versus α on NW small-world networks for different $\langle k\rangle$. There also exists an optimal value of α for the small-world network, but the optimal value is higher than that for the scale-free networks. The inset of the left panel shows the optimal value α_{opt} in dependence on $\langle k\rangle$. α_{opt} is an increasing function of $\langle k\rangle$ and is not less than 2. This phenomenon can be explained by considering the difference in degree distribution between scale-free and small-world networks. The degree distribution of scale-free networks is highly heterogeneous, with a majority of small-degree nodes, and a few nodes possessing very high degrees. In contrast, small-world networks follow a Poisson distribution and have no very high-degree nodes. Hence, for the same value of α, the weight of high-degree nodes in scale-free networks is obviously larger than that in small-world networks. In this perspective, the best value $\alpha=1$ for the scale-free network does not provide enough influence for high-degree agents in the case of small-world networks. Therefore, α_{opt} is not less

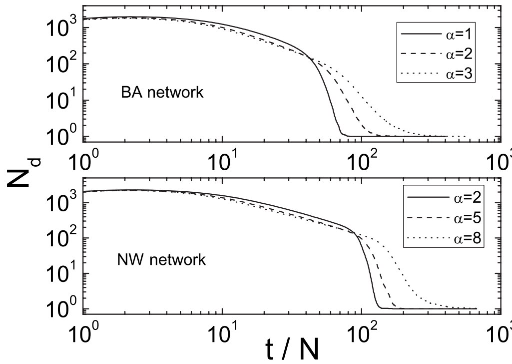

FIG. 3. Evolution of the number of different names versus rescaled time t/N for different values of α with fixed average degree $\langle k\rangle=8$ on the BA (top panel) and the NW (bottom panel) network. Network size is 5000.

than 2 instead of 1. The investigation of t_c for different network sizes shows similar network-size-independent behavior as in the scale-free network.

High-degree agents have more neighbors to communicate with, so if they more frequently act as the speaker, the names spoken by them more easily diffuse among agents and consequently benefit the success rate in negotiation. A higher success rate leads to faster name deletion in the memories of both speakers and hearers, so that the convergence of global consensus is promoted. That is why the reverse strategy is more efficient than the neutral strategy in achieving consensus. As shown in Fig. 3, the decrease in the number of different names N_d in the early stage for higher values of α is faster than for lower values. On the other hand, the agreement dynamics of the naming game is enhanced by the formation of some big name clusters, as shown in Ref. [18]; within each cluster, agents share a common name. Through the competition of these name clusters, one big cluster invades the others and finally dominates the system with a global consensus. A very strong influence of high-degree agents inhibits the invasion and merging of clusters, which is reflected by the final stage in Fig. 3. When a few different names remain in the system, it needs a longer time to reach final agreement for larger α. Thus there should exist an optimal value of α in the middle range, resulting in the fastest convergence.

Furthermore, we investigate the maximum total memory of agents $N_w^{\max}$ depending on the parameter α, as shown in Fig. 4. Here we adopt a normalized $N_w^{\max}$ obtained by dividing by the maximum value of $N_w^{\max}$ for each $\langle k\rangle$. The normalized $N_w^{\max}$ shows a nonmonotonic behavior with a peak in the middle range for both scale-free and small-world networks. Comparing Fig. 3 with Fig. 4, one can see that the peak value of α is lower than the fastest convergence value of α, which indicates that larger maximum total memory may not induce faster convergence. However, we note that the total memory needed to achieve the fastest convergence is considerably larger than that for much longer convergence time, as shown in Fig. 4. This result implies that some trade-off between the total memory used by agents and the convergence time is required, and, to converge faster, more memory is used by agents.

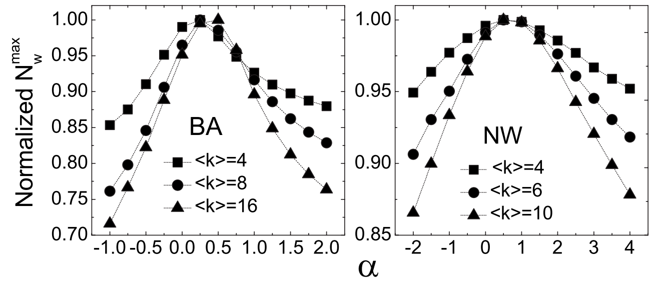

FIG. 4. Normalized maximum total memory used by agents $N_w^{\max}$ as a function of α for different $\langle k\rangle$ on BA (left panel) and NW (right panel) networks. Other parameters are the same as in Fig. 1.

Another interesting point concerns the relationship between agents' degrees and their maximum total memories. As shown in Fig. 5, for negative values of α, the maximum memory used by an agent with degree k is proportional to $\sqrt{k}$, which is consistent with previous reported results [19]. In this case, high-degree agents act more frequently as hearers, and hence receive more names transmitted from other parts of the network. For positive values of α, the agents' maximum memories and their degrees displays a negative correlation, i.e., higher-degree agents use less memory to record names, which is very different from the case of negative values of α and previously reported results [19]. The observation can be explained by noting the fact that, for positive values of α, higher-degree agents act as speakers more frequently, thus receiving fewer names transmitted from others and using less memory during the communication. We note also that the fact that high-degree agents use less memory may favor the achievement of final agreement, as in the case of the optimal value $\alpha=1$ displayed in Fig. 5. From the practical point of view, if we consider the naming game as a model for communication among computer agents, it is more natural to assign larger memories to agents with many more connections. Hence, the use of positive values of α seems not easy to implement, in particular for scale-free networks, because in these networks a large number of agents with small degrees need more memory capacity compared to the original minimal naming game. However, if the total memory used by agents is considered as the cost for reaching global consensus, the cost for the fastest convergence is not higher than that of the symmetric naming game ($\alpha=0$), as displayed in Fig. 4. This means that the cost of the high-

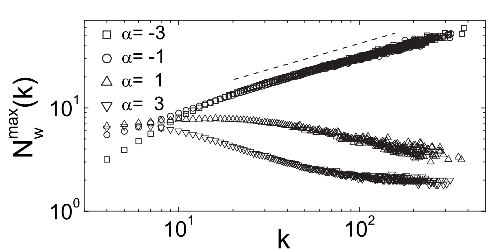

FIG. 5. Maximum memory used by an agent, $N_w^{\max}$, as a function of its degree for different values of α. The dashed line is proportional to $\sqrt{k}$. The average degree $\langle k\rangle$ is 8 and network size is 20 000. Each data point is obtained by averaging over 100 runs of each of ten network realizations.

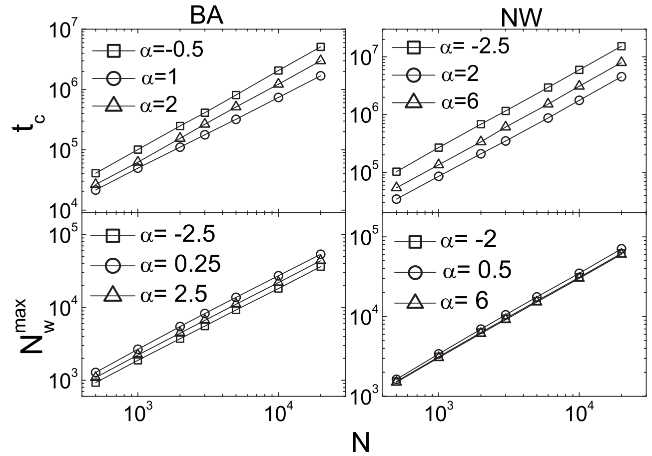

FIG. 6. Convergence time t_c and maximum total memory $N_w^{\max}$ used by agents as functions of the network size N for different values of α in BA and NW networks. t_c scales as N^{β} in both BA and NW networks. For the BA networks, β from top to bottom is 1.30, 1.28, and 1.18. For the NW networks, β is about 1.35. For both BA and NW networks, $N_w^{\max}$ scales linearly with the network size.

degree agents is considerably reduced and that of the small-degree agents is slightly increased. The total cost increment is still less than the total cost reduction. From this point of view, the asymmetric negotiation strategy is still recommended since it can lead to faster convergence with less cost.

Finally, we investigate the scaling behavior of the convergence time and the maximum total memory with the network size. As shown in Fig. 6, for BA networks, t_c scales as N^{β} with β slightly depending on the value of α. For NW networks, t_c scales as $N^{1.35}$, independent of the value of α. The scaling parameter β of the asymmetric negotiation naming game is slightly smaller than in the original minimal naming game, where the scaling parameter is 1.4 [19], while the maximum total memory $N_w^{\max}$ still scales linearly with the network size, which is the same as in the original minimal naming game [19].

In conclusion, we have investigated a modified naming game with asymmetric negotiation strategy on both scale-free and small-world networks. The most interesting result is that there exists an optimal value of the parameter α that leads to the fastest convergence. This result demonstrates that a proper influence of high-degree agents in negotiation best benefits the achievement of final consensus, and high-degree agents can play both positive and negative roles in the agreement dynamics of the naming game. We have qualitatively explained the results for the convergence time in terms of the evolution of the total number of different names. We have also investigated the dependence of the total maximum memory used by agents on the parameter α and found a peak in the middle range of the parameter space. The relationship between the maximum memory used by an agent and its degree shows different behavior compared to previously reported results in the naming game, while the convergence time and the total maximum memory show similar scaling behavior. It may be interesting to explore asymmetric negotiation on networks with degree correlation in future work.

This work is funded by the National Basic Research Program of China (973 Program Grant No. 2006CB705500) and by the NNSFC under Grants No. 60744003, No. 10635040, No. 10532060, and No. 10472116.

[1] R. Albert and A.-L. Barabási, Rev. Mod. Phys. **74**, 47 (2002).
[2] S. N. Dorogovtsev and J. F. F. Mendes, Adv. Phys. **51**, 1079 (2002).
[3] M. E. J. Newman, SIAM Rev. **45**, 167 (2003).
[4] S. Boccaletti *et al.*, Phys. Rep. **424**, 175 (2006).
[5] M. Nowak and D. Krakauer, Proc. Natl. Acad. Sci. U.S.A. **96**, 8028 (1999).
[6] V. M. de Oliveira, M. A. F. Gomes, and I. R. Tsang, Physica A **361**, 361 (2006).
[7] C. Schulze and D. Stauffer, Int. J. Mod. Phys. C **16**, 781 (2005).
[8] C. Schulze, D. Stauffer, and S. Wichmann, Comput. Phys. Commun. **3**, 271 (2008).
[9] D. Abrams and S. H. Strogatz, Nature (London) **424**, 900 (2003).
[10] M. Patriarca and T. Leppännen, Physica A **338**, 296 (2004).
[11] K. Kosmidis, J. M. Halley, and P. Argyrakis, Physica A **353**, 595 (2005).
[12] V. Schwämmle, Int. J. Mod. Phys. C **16**, 1519 (2005).
[13] P. M. C. de Oliveira *et al.*, Physica A **376**, 609 (2007).
[14] A. Kalampokis and P. Argyrakis, Physica A **379**, 665 (2007).
[15] T. Briscoe, *Linguistic Evolution through Language Acquisition: Formal and Computational Models* (Cambridge University Press, Cambridge, U.K., 1999).
[16] *Approach to the Evolution of Human Language*, edited by J. Hurford, C. Knight, and M. Studdert-Kennedy (Cambridge University Press, Cambridge, U.K., 1999).
[17] A. Baronchelli, M. Felici, E. Caglioti, V. Loreto, and L. Steels, J. Stat. Mech.: Theory Exp. (2006), P06014.
[18] A. Baronchelli, L. Dall'Asta, A. Barrat, and V. Loreto, Phys. Rev. E **73**, 015102(R) (2006).
[19] L. Dall'Asta, A. Baronchelli, A. Barrat, and V. Loreto, Phys. Rev. E **74**, 036105 (2006).
[20] L. Dall'Asta, A. Baronchelli, A. Barrat, and V. Loreto, Europhys. Lett. **73**, 969 (2006).
[21] T. Lenaerts, B. Jansen, K. Tuyls, and B. D. Vylder, J. Theor. Biol. **235**, 566 (2005).
[22] C.-L. Tang, B. Y. Lin, W. X. Wang, M. B. Hu, and B. H. Wang, Phys. Rev. E **75**, 027101 (2007).
[23] A. Baronchelli, L. Dall'Asta, A. Barrat, and V. Loreto, Phys. Rev. E **76**, 051102 (2007).
[24] A.-L. Barabási and R. Albert, Science **286**, 509 (1999).
[25] M. E. J. Newman and D. J. Watts, Phys. Rev. E **60**, 7332 (1999).
[26] D. J. Watts and S. H. Strogaz, Nature (London) **393**, 440 (1998).