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A motor theory of how consciousness within language evolution led to mathematical cognition: the origin of mathematics in the brain

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Abstract

Invariance associated with Pribram's (1971, 1991) motor images-of-achievement (imaged consequences of movement) is proposed to provide the fundamental neurophysiological basis for mathematical cognition [Pribram, K. (1971) *Languages of the brain.* Englewood Cliffs, NJ: Prentice-Hall. Pribram, K. (1991) *Brain and perception*: *holonomy and structure in figural processing*. Hillsdale, NJ: Lawrence Erlbaum.] A three-part theory is outlined. First, linguistic representations of self-consciousness were instantiated through the evolutionary process of distinguishing one's own vocalizations from those of others. It is proposed that consciousness was imparted to these linguistic representations from an already corticalized neuromatrix described by Melzack (1992) [Melzack, R. (1992, April) Phantom limbs. *Scientific American*, *266*, 120–126.] Second, language evolved from motoric images-of-achievement associated with vocalization arising in the pre-Rolandic and inferior parietal cortex. Third, abstractive language processing that decomposes higher-order motor engrams into invariant image-schemas provides the basis for the awareness of pattern that constitutes mathematical cognition. It is concluded that mathematical cognition obtained an evolutionary connection with the physical world by way of the brain's somatic systems. © 1999 Elsevier Science Ltd. All rights reserved.

Keywords: Consciousness; Image-schemas; Language evolution; Mathematical cognition; Motor theory of language; Phantom limbs; Self-consciousnes

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1. Introduction

Elsewhere, I have described how I believe Mandler's (1988, 1992a, 1996) image-schemas comprise genotype-driven[1] *state variables* that estimate the states of complex perceptual-cognitive representations (Vandervert, 1997). Within this view, I proposed that image-schematic state variables are fed forward[2] from newly identified perceptual-cognitive functions of the cerebellum to other areas of the brain (Paulin, 1993, 1997; Schmahmann, 1997). It was argued that the fed-forward image-schematic information functioned to (a) estimate the probable future states of objects and events in perceptual-cognitive representations of moving systems for lower animals, and, continuing in the subsequent evolution of language, (b) constitute the basis of linguistic and mathematical representations of probable future states. The state-estimating connection with language and mathematics was described as an extension of Mandler's (1992a) suggestion that image-schemas provide, "a conceptual basis for the acquisition of the relational aspects of language" (p. 273).

1.1. Purpose

In this article, I provide a more detailed account of the theory, only hinted at in Vandervert (1997), that the awareness of *patterns*[3] that constitute mathematical cognition are derived from *invariant* image-schematic state-estimates embedded in language. This account will focus on the description of complex *systems* of image-schemas that I believe comprise yoked processing of linguistic and mathematical cognition in the brain. It will be argued that these complex systems of image-schemas are the motoric "images-of-achievement" (imaged consequences of movement) that were (a) described by Pribram (1971, 1991) in great detail in terms of brain anatomy and physiology, and (b) proposed by Pribram (1971) to be the evolutionary basis of language. (See also Pribram's discussion of the involvement of the motor cortices in "focusing" processes in the brain (pp. 211–212, this issue). In this article I examine such motor-focusing in the context of language processing.)

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[1] The genotype-driven process of neural development during ontogeny includes both experience-dependent and experience-expectant modification. Experience-expectant development involves the overproduction of neural synapses, followed by their selective pruning and selective preservation. See Black and Greenough (1986), and Greenough and Black (1992).

[2] The following technical meaning of "feedforward" is used here: A response in anticipation of a discrepancy between a future actual state and the reference state.

[3] In this article the idea of mathematics follows that described by Steen (1988):

> Mathematics is the science of patterns. The mathematician seeks patterns in number, in space, in science, in computers, and in imagination. Mathematical theories explain the relations among patterns; functions and maps, operators and morphisms bind one type of pattern to another to yield lasting mathematical structures. Applications of mathematics use these patterns to "explain" and predict natural phenomena that fit the patterns (p. 616).

For an additional "science of patterns" notion of mathematics, see Devlin (1994).

2. The general theoretical backdrop for a motor theory of mathematical cognition

A main thesis of this article is that mathematical cognition stands at the end of a long line of events in the interrelated evolution of visual-motor imagery, awareness of the body, movement, and consciousness *in* language processing. Accordingly, ideas from developmental cognitive psychology, language evolution, and neuroscience are involved. To prepare for the theoretical neuroscience discussion related to mathematical cognition, it is necessary to provide a brief overview of how ideas from developmental cognitive psychology, language evolution, and neuroscience research are related to it.

This preliminary sketch will consist of the following two lines of research and theory. First, to provide elemental perceptual-cognitive bedrock, Mandler's (1992b) image-schemas will be described. (Later in this article these image-schemas will be related to the patterns cognized as mathematical cognition.) Second, I will propose brain substrates from which consciousness arose. This discussion will involve a synthesis of the following:

1. Jerison's (1988, 1991) conception of why self-consciousness emerged within language evolution.
2. Arbib and Rizzolatti's (1996) idea of how premotor "mirror neurons" provided the gestural nidus for language
3. Melzack's (1992) notion of a phantom limb neuromatrix in the brain, that I have proposed provides the substrate for elemental consciousness (Vandervert, 1995).

Following this preliminary overview I will move to the theoretical neuroscience discussion that centers on how Pribram (1971, 1991) images-of-achievement are related to language evolution and mathematical cognition.

2.1. *Elemental perceptual-cognitive building blocks*: *image-schemas as state variables which represent physical world dynamics*

Mandler (1988, 1992a,b) proposed that image-schematic *conceptual primitives* are derived from perceptual analytic processes occurring in infants. She believes that these image-schemas provide the basis for conceptual and language development. The following abstract from Mandler's major theoretical paper provides a brief overview of what image-schemas are, and how she believes they operate to form the conceptual structure of early space–time representations:

> The theory proposes that perceptual analysis redescribes perceptual information into meanings that form the basis of an accessible conceptual system. These early meanings are represented in the form of image-schemas that abstract certain aspects of the spatial structure of objects and their movements in space. Image-schemas allow infants to form concepts such as animate and inanimate objects, agents, and containers. It is proposed that this form of representation serves a number of functions, including providing a vehicle for simple inferential

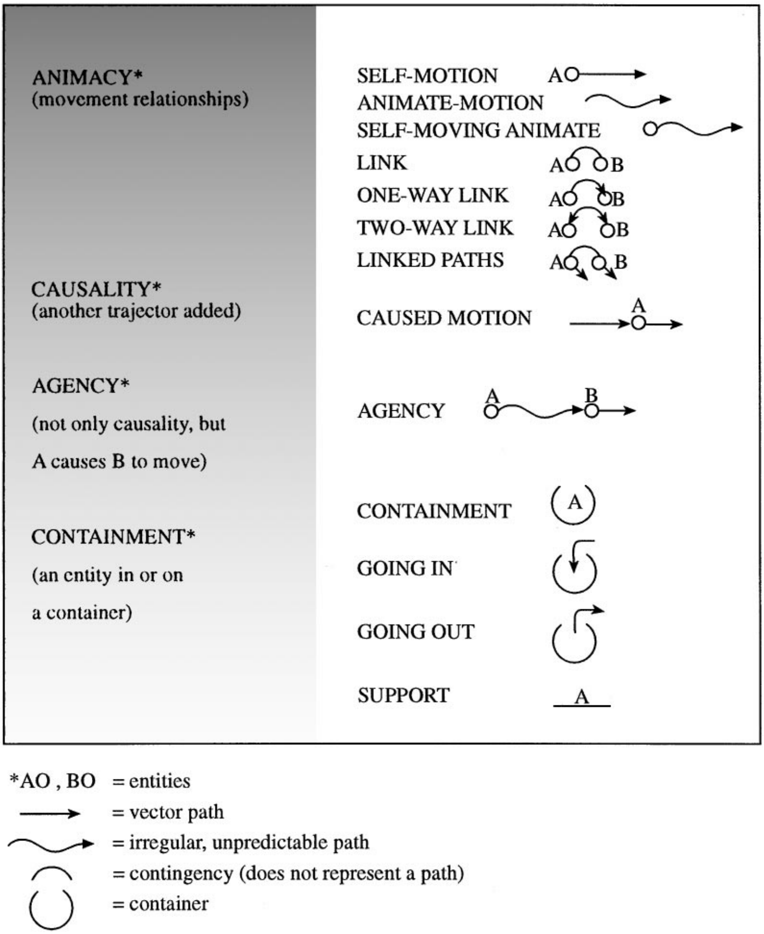

Fig. 1. Pictorial representations of Mandler's image-schemas (conceptual primitives). These image-schemas are building blocks of cognition.

> and analogical thought, enabling the imitation of actions of others, and providing a conceptual basis for the acquisition of the relational aspects of language (1992a, p. 273).

Fig. 1 shows Mandler's (1992b) depictions of image-schematic conceptual primitives or *meanings* from which she believes understandings and, later, language develop in the child. The depiction of the image-schemas is not meant to imply that they represent actual visual-motor images in the brain. The depictions are abstractions gleaned from careful observation of the perceptual-motor activity of children.

In my theoretical framework (Vandervert, 1997), the image-schemas depicted in Fig. 1 are *elemental state variables* though which the brain (a) constructs coordinate system information about probable future states of the physical world in cognitive representations, and (b) within the reference frame of (a), coordinates future movements. Later in this article, I will propose how *invariant* categories of image-schemas, as state variables in such representations, participate in mathematical cognition.

In order to appreciate their full potential for the representation and coordination of movement, the image-schemas portrayed in Fig. 1 should be thought of as they would appear in animation and as continuously combining and recombining. Mandler (1992b) described these animated, combinatory properties as follows:

> Image-schemas can be defined as dynamic analogue representations of spatial relations and movements in space. They are dynamic in that they can represent continuous change in location, such as an object moving along a path.... Because image-schemas are analogue in nature, they have parts. One can focus on the path itself, its beginning, or its ending. In this sense, image-schemas embed. BEGINNING OF PATH can be embedded in PATH; each can be considered an image-schema in its own right. Similarly, perception of contingent motion is recoded into the contingent notion of coupled paths or LINK. I propose that image-schemas such as PATH (with focus on BEGINNING OF PATH) and LINK constitute the meanings involved when a concept such as animacy is formed (p. 591).

In summary of this section, Mandler's image-schemas were examined for two reasons. First, within Mandler's theory, image-schemas provide a fundamental connection between motor processes in the brain and cognition in language development. Second, the motor characterizations of image-schemas are movement vectors (see legend, Fig. 1) that later in this article will tie directly into larger images-of-achievement systems of motor functions in the brain, to the "feel" of consciousness, and, finally, to that which is *cognized* in mathematical cognition. I now turn to a synthesis of the ideas and research of Jerison (1988, 1991), Arbib and Rizzolatti (1996), and Melzack (1992) on how the evolution of communication and language are related to our awareness of our own representational states-our own systems of image-schemas.

3. The evolutionary basis of self-consciousness: distinguishing representations communicated by others from one's own reference frame

In this section, I outline my view of the evolution of language that set the stage for mathematical cognition. Within this view, the evolution of language is believed to have occurred slowly during the Pleistocene-era evolution of hominids, beginning with a *combination* of gesturing and pre-language vocalization. For the theory and evidence on gesturing, see, for example, Arbib and Rizzolatti (1996), Armstrong, Stokoe and Wilcox (1995), Corballis (1998, 1999), Hewes (1973), Kimura (1993), and for pre-language vocalization, see, for example, Corballis (1998, 1999), Jerison (1988, 1991), and Liberman and Mattingly (1985). The theoretical conceptions of the evolutionary scenario that follow below will be seen to fit hand-in-glove with parallel evolutionary selection that led to the involvement of image-schemas in concept and language formation.

Jerison (1988, 1991) proposed that language evolved through the selective advantage accrued through hominid *vocal-auditory* mapping of large Pleistocene-era prey–predator ranges. (Jerison did not address the possibility of the involvement of gesturing in language evolution.) In performing the cognitive functions of range mapping, the vocal-auditory system would be a hominid analogue of, for example, wolves' olfactory mapping of territorial ranges by urination. Gestural and vocalized image-schematic information (rather than patterns of urine deposits) would constitute the working framework for dynamic perceptual-cognitive representations of ranges shared by vocal-auditory hominids. One can imagine such gestures and vocalizations as undergirded by the image-schematic information portrayed in Fig. 1. In these perceptual-cognitive fields, gestural and vocal versions of image-schematic information would translate into the communication of the movement of one's self, other hominids, animals, and features in the territory, such as refuges. See Vandervert (1997) for more detail on how image-schemas can be related to Jerison's hominid range-mapping.

3.1. An evolutionary scenario of the selection of self-consciousness

Jerison (1988) described how vocal-auditory communication among hominids led to the evolution of an entirely new aspect of the unique cognitive world of hominids–the evolutionary emergence of self-consciousness:

> The construction [of our representation of reality] is clearly based on the sensory and motor systems of the brain. In our personal lives the construction works as our knowledge of the external world, which we know as a truly real world ....
>
> Consider now what would happen if among the sensory elements in the construction there is one that is also an element of communication. Communication with that element would effectively *share the reality itself* that the individual experiences. *That* is the peculiar feature of human language. When we communicate with it we share realities. This is so odd a thought that it takes a bit of accommodation to accept it. Yet there is plenty of evidence that it is true. The simplest is in the effectiveness of the written word as a source of imagery about real events, the ease with which we enter the lives of others when they are described with language, the reality of the world we enter and live in as we read a realistic novel.
>
> Our ordinary communications also have this character .... In our ordinary communications, we routinely expect to share images, and we use this to avoid direct commands. We may say: "There is a car coming", to warn a friend to be attentive and avoid an accident. Our statement invites the friend to share our experience of the moment and to act on that experience.
>
> When we communicate with verbal language, according to this argument, we share consciousness with one another, because we share a constructed reality. Language, like vision and hearing, contributes to the construction. This

> argument implies a biological and evolutionary explanation for self-consciousness as a human trait, at least in terms of its biological function. Self-consciousness *would have to arise to distinguish the reality generated by one's own information (sensory, linguistic, etc.) from the reality generated by verbal information from another individual* [*italics added*]. It would lead to problems, and sometimes does, when we cannot distinguish the really real world; that is, the world our brain normally builds, from worlds it can build using only the evidence of language, whether our own or someone else's (pp. 7–8).

Jerison's scenario provides a valuable global-level insight into how the uniquely human vocal-auditory cognitive world, including an *awareness* of self (self-consciousness), might have emerged. The account Jerison proposes is instructive — as far as it goes. But what already corticalized neural mechanisms would have provided the basis for the concomitant selections (the differentiation of representations *and* the emergence of self-consciousness) that he describes? To answer this question, I propose a synthesis of Jerison's scenario and the newer discoveries of (a) so-called "mirror neurons" in premotor cortex of the monkey — the monkey homologue of Broca's speech area, and (b) a neuromatrix of consciousness attributes from research on the experience of phantom limbs.

3.2. Mirror neurons of the premotor cortex: a micro-level nidus for the selection of the distinction of one's own reality from those of others

Premotor mirror neurons in the monkey cortex discharge when the monkey *observes* a grasping action by others and also when it *executes* the action itself (Rizzolatti, Fadiga, Gallese & Fogassi, 1996). In the mirroring of observation/execution, mirror neurons clearly *link* the actions of others to action taking place in the monkey's own analogous representations. Arbib and Rizzolatti (1996) theorize that since mirror neurons are found in the monkey homologue of Broca's area and are also present in Broca's area in humans, their mirrored observation/execution functions likely provided the selection basis for motor receiver/sender functions of gesturing that subsequently led to speech.

However, if the mirror neuron provided the nidus for sharing realities through communication in the manner suggested by Arbib and Rizzolatti, then by the same argument it is equally plausible that it provided the simultaneous nidus for Jerison's *differentiation* of the receiver's reality from those of senders. This conclusion is based upon the simple fact that the monkey *doesn't* perform grasping movements when observing grasping in others. Thus, the mirror neuron reminds us that, even here at this primitive, micro-level of function, the execution (or intentional) side of motor functions is already corticalized to be distinguished from what the monkey sees others doing. Therefore, I propose that the mirror neuron not only may provide the germinal basis in motor functions for receiving/sending in gesturing and speech, as Arbib and Rizzolatti propose, but also that the mirror neuron provides the simultaneous *motoric* basis for the distinction of the monkey's reality from those of others–although in the monkey this distinction is not yet "conscious" in the human sense.

3.3. Phantom limbs and the larger neuromatrix of the self-consciousness of one's own body reference frame

If, as Jerison proposes, the differentiation between one's own reality and the vocal incoming realities of others leads to self-*consciousness*, then exactly *from where* would this process *acquire* or "win" this consciousness?

Elsewhere (Vandervert, 1995), I proposed that consciousness originates in a mostly hardwired neuromatrix in the brain. This neuromatrix was postulated by Melzack (1992) to explain the experience of phantom limbs in amputees and in those with congenital limb deficiencies. The experience of phantom limbs is a fully *conscious* experience (in the same manner that one might be conscious of their own bodies or their own thoughts) of the phantom presence of the missing limb. The person experiencing a phantom limb may, for example, try to stand on a phantom foot or pick up a cup with a phantom hand (see Melzack, 1992, p. 120; Melzack, Israel, Lacroix & Schultz, 1997, Table 2A & 2B).

The consciousness *attributes* of the three brain circuits of Melzack's neuro-matrix illustrate how the phantom limb experience is nearly indistinguishable from the experience of our everyday active "stream of consciousness". Melzack (1992) described these circuits and their phenomenal attributes as follows:

> In essence, I postulate that the brain contains a neuromatrix, or network of neurons, that, in addition to responding to sensory simulation, continuously generates a characteristic patterns of impulses indicating that the body is intact and unequivocally one's own. If such a matrix operated in the absence of sensory inputs from the periphery of the body, it would create the impression of having a limb even when that limb is removed.
>
> To produce all of the qualities I have described for phantoms, the matrix would have to be quite extensive, including at least three major neural circuits in the brain. One of them, of course, is the classical sensory pathway passing through the thalamus to the somatosensory cortex [giving rise to phantom experiences of the limb being physically extended in space as an 'entity' that can be moved and mentally manipulated, and having sensory experiences-pressure, warmth, cold, wet, itches].
>
> A second system must consist of the pathways leading through the reticular formation of the brain stem to the limbic system, which is critical for emotion and motivation. I include this circuit in part because I an other have noted that paraplegics who suffer a complete spinal break high in the upper body continue to experience themselves as still being in their old body, and they describe the feelings in the denervated areas with the same kinds of affective terms as they did before they were injured, such as 'painful', 'pleasurable' or 'exhausting'.
>
> A final system consists of cortical regions important to the recognition of the self and to the evaluation of sensory signals. A major part of this system is in the parietal lobe, which in studies of brain-damage patients has been shown to be essential to the sense of self.

> Indeed, patients who have suffered a lesion of the parietal lobe in one hemisphere have been known to push one of their own legs out of a hospital bed because they were convinced it belonged to a stranger. Such behavior shows that the damaged area normally imparts a signal that says, 'This is my body; it is a part of myself'. (see also Melzack, Israel, Lacroix & Schultz, 1997, p. 123) for a detailed account, based on 46 people, of the attributes of phantom experiences)

The phantoms generated through the brain circuits Melzack describes have all of the attributes including the "feel" of everyday consciousness and self-consciousness. The fact that the phantoms occur not only in child and adult amputees, but in people congenitally limb-deficient is, I believe, convincing evidence that this continuously generating neuromatrix circuitry is largely hardwired and is the body reference frame's foundational neurophysiological source from which a consciousness leading to Jerison's self-consciousness in language was selected.

But how and why would the consciousness attributes generated by the neuromatrix be imparted to language? That is, how and why does the neuromatrix become involved in the selection of language that differentiates one's own representations from those of others? The answer, although requiring elaboration, is quite simple. As will be seen in the motor theory of language to be described below, gesturing and speech were selectively evolved from the interrelated functions of the much the same brain systems that produce the consciousness attributes of Melzack's neuromatrix. Thus I argue that, in the evolution of language, the consciousness attributes of this neuromatrix were selectively extended from their already intricate involvement in *movement* processing (see Melzack, Israel, Lacroix & Schultz, 1997) to the symbolic form of *actions*-the motor activity associated with gesturing and speech. Following this idea, the evolution of language would *not* have created consciousness out of the air, so to speak, but would have begun with the primitive form of consciousness that was already corticalized in neuromatrix circuitry and that was coming to be involved in the control of movement–movement associated with gesturing and vocalization.

In sum of this section, language constitutes an evolutionary extension of *actions* executed by evolving hominids coupled with the neuromatrix of consciousness attributes that subserves the recognition to whom those actions belong. This means that the reference frame for linguistic representations of one's reality is the body reference frame encompassed by the overlapping motor and neuromatrix areas of the brain. Thus, the self-consciousness that Jerison's says must emerge with language is more accurately a linguistic consciousness of actions of the body reference frame. I now turn to a more detailed account of the neurophysiology behind actions that led to speech.

4. A neurophysiological model of the emergence of self-consciousness from the body reference frame: establishing invariance

Within the overall forgoing position we can describe Jerison's proposed self-consciousness as a naturally selected consciousness of one's own body reference frame (one's "reality") as distinguished from those of others. Such a vocal-auditory self-consciousness would require the selection of a *separable* system of actions that would establish an *invariance* of one's *own* actions in relation to those communicated by others. This self-consciousness of one's own reference frame would serve as an *internal comparator*[4] (standard body reference) in relation to the realities communicated by others.

4.1. The invariant comparator reference frame: the corporeal self

I will outline the operations of the internal comparator in terms of brain functions involved in Pribram's (1971, 1991) model of the *corporeal self*. Pribram (1971) placed the functions of the corporeal self at the heart of his central-motor theory of language evolution. The neuroanatomy and neurophysiology of Pribram's corporeal self substantially overlay that of Melzack's bodily neuromatrix.

Pribram described the corporeal self as a joint visual-somatosensory action space reference frame generated in the pre-Rolandic (motor), and inferior parietal cortex (see Fig. 2). The corporeal self-centers on proximal (axial parts of the body) representation and, as a reference frame, constitutes the highest level of action control in a top-down sequence of innervation: "The corporeal self… [is] defined as that which remains *invariant* [italics added] across all targets, that is, all achievements" [imaged consequence of movement] (Pribram, 1991, p. 155). In Melzack's neuromatrix this invariance manifests as the continuously generated pattern of impulses that indicates "that the body is intact and unequivocally one's own" (1992, p. 123). The idea of *invariance across all targets* is of critical importance. It will be proposed below that the notion of corporeal-self invariance, acting through the evolution of language, is the ultimate source of categories of invariance in the *patterns* inherent in mathematical cognition.

The corporeal self-mediates functions of the *left* parietal lobe providing command functions for the operations of the limbs, hands, and eyes within the immediate extra-personal space (Mountcastle, Lynch, Sakata & Acuna, 1975). These corporeal-self command functions lead either to the carrying out of imaged bodily achievements, or to simply erecting imagined achievements that may be communicated but not actually carried out.

4.2. Sources of invariance within the corporeal self: images-of-achievement

The corporeal self consists of collections of *images-of-achievement* that are generated within its action space. Images-of-achievement are *anticipatory* "images of load" in motor representation, that is, sort of dynamic internal mirror images of the field of

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[4] Comparator functions involve a standard reference of relatively fixed ranges of biological processes, and are ubiquitous in the regulation of interdependent systems in the brain. Everything from, for example, the regulation of blood levels of water, oxygen and so forth, to body temperature, to sleep cycles, and auditory, motor, and visual mappings involves some type of comparator functions in the brain.

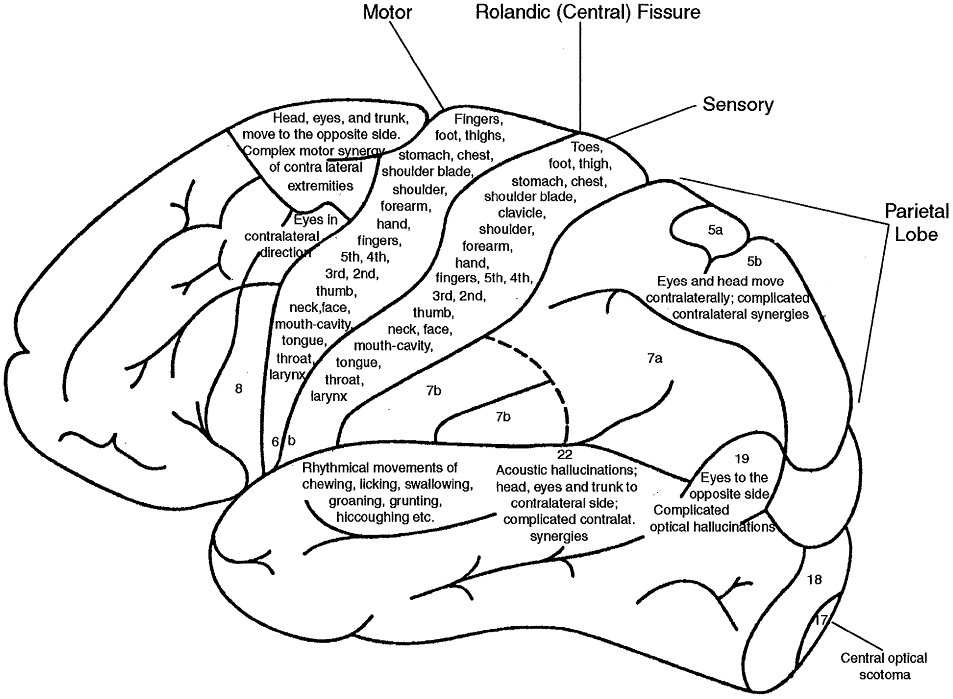

Fig. 2. Pre-Rolandic and parietal areas of the cerebral cortex.

external forces.[5] An image-of-achievement is thus the anticipatory imaged consequence of movement (assemblages of anticipatory image-schema states, see Fig. 1). Images-of-achievement are higher-order, *invariant* "images" of intended achievement that precede all *actions*.[6]

The fact that gestures and vocalizations are actions led Pribram (1971) to propose his central-motor theory of the origins of human language:

> Motor mechanisms of the brain may well be responsible [for language]-especially that part of the sensory motor cortex where the representations of

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[5] The differentiation between "force" and "load" is largely irrelevant to the thesis of this article. The interested reader may consult Pribram's (1991) discussion of this issue, "A Vector Space: Force Defined In Terms of Load", pp. 145–148.

[6] Images-of-achievement are essentially *invariant* across movement involved in various actions, such as, riding a bicycle, writing, and so on. For example, one might pedal and steer a toy bicycle with the finger movements, or write large, sweeping cursive with arm movements rather than the normal finger movements. At a perhaps more cognitively involved level of *language* generation, one may choose to execute the imaged "structure in the mind" by saying whatever it is, writing it, or by using bodily gestures as in the game of charades. In all of the above examples, from the bicycle to language, the "structure of the imaged consequence of movement in the mind" of the person/speaker remains essentially invariant, while the particular choice of muscle systems and movements vary.

> Images-of-Achievement of the vocal apparatus are engendered, since this cortex overlaps so extensively the apparatus in which auditory images are formed. The increase in the size of the posterior superior temporal cortex (and adjacent angular gyrus) in man can also be attributed as readily to an enhancement of their subcortical motor connections as to any augmentation of associative processes (p. 369).

Thus images-of-achievement directly or indirectly characterize much, perhaps most, of the structure of information ensuing from the body reference frame that is transmitted in Jerison's vocal-auditory sharing of realities.

4.3. Images-of-achievement: a brief introspective account

The idea of an image-of-achievement is difficult to bring to mind. It is helpful in this regard to close one's eyes and imagine, for example, walking across the room to open a window. Within the series of mental events that ensues in this imagery, many people report that some sort of mental outline of action seems to be taking place, but it is imagery that is quite difficult to satisfactorily describe. However, a unique photographic technique that isolates a dynamic outline of bodily movement provides an intriguing visual analogy, and it is only an analogy, to the world of motor imagery. Fig. 3(a) is a high-speed photograph of a person walking that is in some ways similar the eyes-closed imagery alluded to above.

For example, because images-of-achievement are anticipations of future states of action, they may be likened to the unfolding sequence of movement in portion (a) shown in Fig. 3. Further, there actually may be substance behind the analogy, in that a small portion of the movement sequence in the rapidly photographed movements can be used to predict the future states of the action (Bernstein, 1967, pp. 23, 24; Pribram, 1991, pp. 136, 137). That is, both the image-of-achievement and the dynamic sequence of the walking person in Fig. 3 contain faster-than-real-time anticipatory information. See Pribram (p. 212, this issue).

4.4. Images-of-achievement as comparator "set points" within the corporeal self

Pribram (1991) described how the *invariance* of images-of-achievement served as setpoints in the everyday flow of top-down movement and action:

> [action systems within the corporeal self] embody controls that act much like those embodied in thermostats in which tests match *input* [italics added] against a setpoint.... The set-point (which in regard to action systems is composed of sets of Images-of-Achievement) is the temporally projected target of the operation (p. 123).

Within these thermostat-like feedback loops, an input *match* with the images-of-achievement results in a transfer of function toward the execution of the imaged action. A *mismatch* produces a destabilization that transfers the control of the

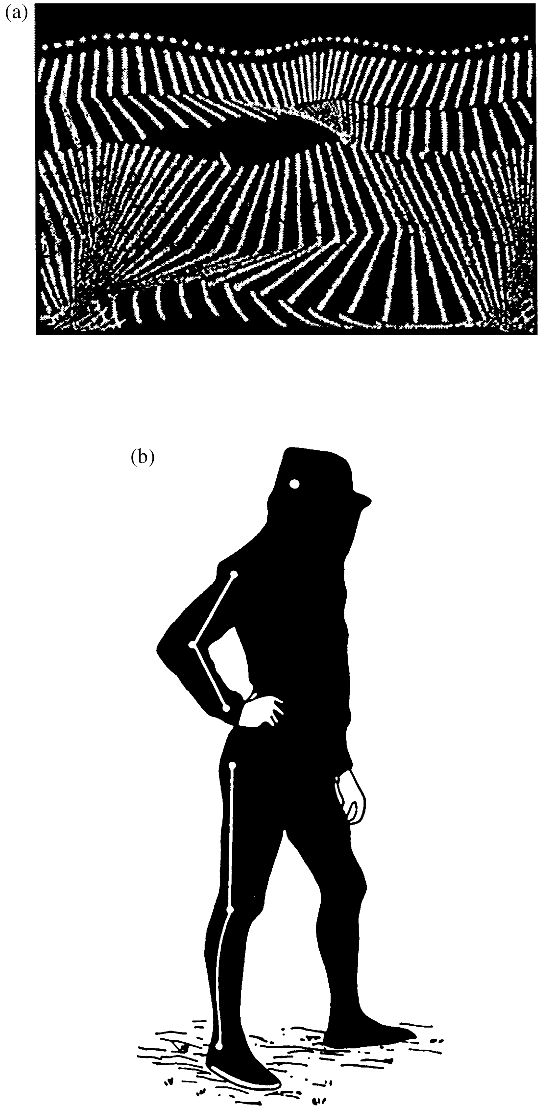

Fig. 3. Load sequence in walking movement. The upper chronophotographic image (about 20 exposures per second) is of the man in the black costume with white tape. Movement is from left to right (notice the foot movement along the bottom). Adapted Marey (1901).

execution of the targeted action to lower levels of operation that aim to restore stability. Inputs are tested against the set-point until stability is finally achieved in the imaged action itself. Thus, images-of-achievement serve as *comparator* elements or set-points that guide motor systems toward action — they comprise what is *invariant* in the action system feedback loop. This feedback loop with its image-of-achievement set-points will now be placed within an overall reference frame for speaker differentiation (identification) and Jerison's scenario of the emergence of self-consciousness.

4.5. Speaker differentiation (*identification*)*: how we become conscious of ourselves and of invariance*

To illustrate how image-of-achievement set points would have selectively operated (both in phylogeny and ontogeny) to accomplish the differentiation of realities and the simultaneous instantiation of a linguistic consciousness of the actions of the body reference frame, we can suppose that the comparator input-testing functions to be involved in some type of *speaker identification* system. Computerized speaker identification systems have long been available and now are extremely sophisticated (e.g., Furui, 1996). Fig. 4 is a diagram of such a speaker identification system, which has been substantially modified to include the general organization of mechanisms in Pribram's central-motor theory of language evolution.

In Fig. 4, a feature extraction function is used to determine a MATCH or MISMATCH between the hearer's own voice and incoming speech waves from other speakers. A MATCH means that the hearer is talking or talking to itself, and the speaker is identified *as* itself. This match feeds back through comparator functions that (a) stabilizes lifelong calibrations through differing environments, but (b) selectively modifies settings over the hearer's developmental history. I will return to the significance of these fed back vocalizations for self-consciousness in a moment.

A MISMATCH momentarily destabilizes the speaker identification process to subcategories of corporeal action space in memory, as tests for *similarity* are conducted on the basis of alternative speaker models that are stored in memory. When a MATCH is found among the alternatives, the speaker model is identified, and the identified action space is retrieved for an updated and ongoing construction of that speaker's images-of-achievement "reality".

4.6. Linguistic consciousness of one's own images-of achievement: how the self is spoken into existence

In the process of identifying incoming vocalizations shown in Fig. 4, the hearer, when speaking, conjointly separates its own vocalizations. The hearer-speaker's own vocalizations complete a continual negative feedback loop (steady-state loop) which engenders a comparator "reference model" of the hearer's own images-of-achievement representations.

With each iteration through the negative feedback loop the speaker's own vocal *input* is matched again (reinstated) against a set point consisting of the speaker's own image-of-achievement. This processing constitutes an internal linguistic *act* that constructs the collection of images-of-achievement that constitutes one's own corporeal-self action space — that is, that constitutes one's own corporeal reality as separate from those of others. Moreover, the countless selective iterations of particular vocal executions of images-of-achievement (see footnote 6) through the negative feedback

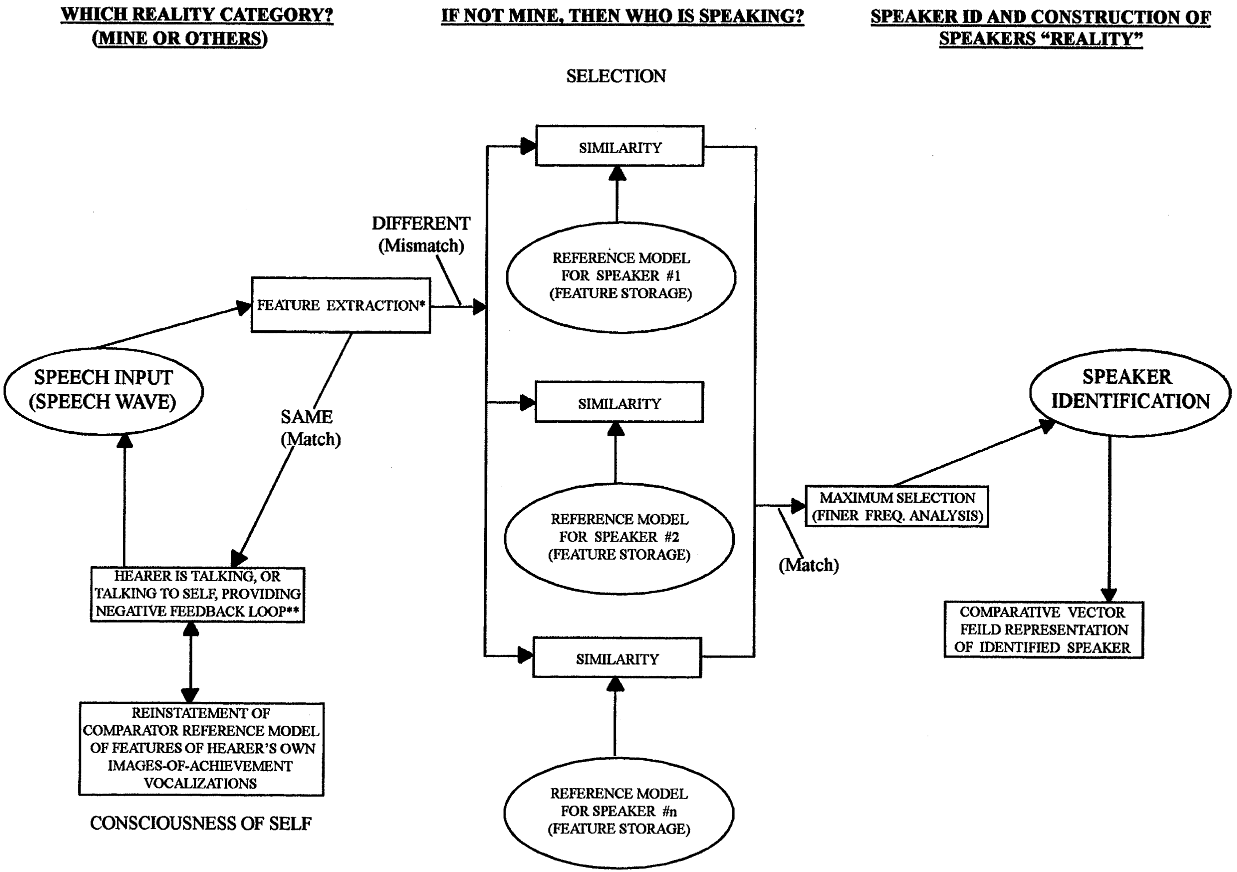

Fig. 4. Speaker identification system. *Feature extraction consists of speech wave frequency separation and feature analysis. **The *abstractive* return loop at the left reinstates image-of-achievement representations, thus giving rise to self-consciousness. Ideas for portions of this diagram were kindly provided by Professor Sadaoki Furui (see Furui, 1996).

loop constitutes an *abstractive* process. In phylogeny this abstractive process results in the progressive symbolic evolution of human language.

Thus, I propose that the cognition and memory of this fed back reinstatement of images-of-achievement of the corporeal self *as overlaid on the already corticalized consciousness attributes of Melzack's neuromatrix* "is" self-consciousness in language cognition.[7] Via the negative feedback loop engendered by one's own vocalizations, this consciousness of the corporeal self's images-of-achievement is literally spoken into existence and into an ongoing *stability*. That the corporeal self should develop a linguistic consciousness of itself in this manner would seem to be a selective necessity. This would be so, as Jerison intimated, because it is the corporeal self's collection of actions that is communicated in language, and one cannot be conscious of linguistic representations of other corporeal selves and not of one's own.

4.7. The adaptive value of talking to one's self

The forgoing view helps explain both how it is possible to talk to one's self, and what its adaptive role might have been. One is able to talk to one's self, because the self is composed of a *separate* field of load information that has all of the characteristics of other hearer-speaker fields. Directly connected with the separateness of this "self-field", the selective value of talking to one's self would be that it continually further instantiates, stabilizes, and hones its own structure. During Pleistocene-era hunting and foraging such instantiation, stabilization, and honing would have been powerfully adaptive both when the hearer was bombarded by a din of incoming speaker information (see Fig. 4), and when one was temporarily cut off from others and must orient itself within a vocal-auditory mapping of territory (see Jerison (1991, p. 85) for more on the latter adaptation). The above phylogenetic selective value of talking to one's self is no doubt equally important in the ontogeny of modern humans.

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[7] My view of self consciousness as a product of this reinstatement process provides an evolutionary basis for Rolls's (1995) suggestion that consciousness may be an outcome of "second-order" language processing:

> Arbitrary symbol manipulation-using important aspects of language processing, and used for planning, but not used for initiating all types of behavior — is what consciousness is about. Indeed consciousness may *be* the state when this type of processing is being performed. This is consistent with the points made...that the brain systems that are required for consciousness and language are either very similar or the same. According to this explanation, the feeling of anything is the state that is present when linguistic processing, involving thoughts of the second or higher order, is being performed (p. 1102).

While Rolls is talking about consciousness *itself* (i.e., of anything), and I am talking about consciousness of *self*, the proposed role of a comparator reinstatement or of second-order language in the instantiation of either type of consciousness is quite similar. The problem with Rolls' use of the term "second-order" language is that he does not suggest what process his term would refer to, how it "wins" consciousness, or how it might be understood within an account of the evolution of the role of language in cognitive representations of *any* type of consciousness.

4.8. The "feel" of self-consciousness

The cognitive attributes of this experienced self-consciousness are necessarily those shared among those using agreed upon symbolic representations of images-of-achievement. Since, as Mandler (1992) has argued, this language symbolism arises from the image-schemas, the framework for the experience of self-consciousness can be seen consisting of the image-schematic attributes she has observed emerging in the infant, namely, animacy, causation, agency, and containment (see Fig. 1).

These image-schematic attributes comprise the "feel" of our consciousness of ourselves (of the achieving corporeal self). Running through the stability of self-consciousness are these core experiential attributes. They are the attributes that through language give us symbols representative of a consciousness of animated life in the minds of others, and, *by these virtues breathe such life into our own consciousness of self.*

According to this view, the "feel" of self-consciousness "is" not the words in/of vocalization, nor is it even imagery of their everyday referents. Rather, the feel is the ongoing image-schematic context *afloat, so to speak, on the consciousness attributes of the hardwired neuromatrix circuits*. Thus, part and parcel of our linguistic actions, we feel a consciousness that is animated, causative, willful and intentional (agency), and contained; and, in fact, it is all of these.

5. From consciousness of self to consciousness of mathematical images-of-achievement (mathematical cognition)

In this section, I propose how the language-driven, *abstractive* processes going on in the continuous negative feedback language loop in Fig. 4 result in an additional new category of consciousness and cognition, namely, mathematical cognition.

5.1. The cognitive framework for mathematical cognition

Recall that images-of-achievement are *higher-order* images of action that can be executed in a variety of ways (see footnote 6). To provide the cognitive basis for mathematical cognition we must further understand (a) how images-of-achievement entail the ultimate abstract basis for mathematical cognition, and (b) how we come to *cognize* the elements of this abstract basis as mathematical cognition.

5.2. Images-of-achievement: the brain's ultimate congruence with the physical world

Bernstein (1967), who we mentioned in relation to Fig. 3, provided much of the basic conceptual framework for Pribram's images-of-achievement. Bernstein offered the following deeper-level discussion of the "higher-order" or more abstract aspects of what Pribram was to later call images-of-achievement:

> It is clear that each of the variations of a movement (for example, drawing a circle large or small, directly in front of one's self or to one side, on a horizontal

> piece of paper or on a vertical blackboard, etc.) demands a quite different muscular formula; and even more that this, involves a completely different set of muscles in the action. The almost equal facility and accuracy with which all these variations can be performed is evidence for the fact that they are ultimately determined by one and the same higher directional engram in relation to which dimensions and position play a secondary role.... We must conclude... that the higher engram, which may be called the engram of a topological class... is extremely geometrical, representing a very abstract motor image of space. This makes us suppose — for the time being merely as an hypothesis though it forces itself upon us very strongly — that the localizational areas of these higher-order motor engrams have also the same topological regulation as is found in external space or in the motor field [and that in any case the pattern is by no means that which maintains the joint-muscle apparatus]. In other words, there is considerable reason to suppose that in the higher motor centres of the brain (it is very probable that these are in the cortical hemispheres) the localizational pattern is none other than some form of projection of external space in the form present for the subject in the motor field (the relations between movements and external space, analogous to the concept of the visual field). This projection [the higher engram], from all that has been said above, must be congruent with external space, but only topologically and in no sense metrically.... The topological properties of the projection of space in the C.N.S. may prove to be very strange and unexpected; we must not expect to find in the cortex some sort of photograph of space, even an extremely deformed one. (1967, pp. 49–50)

The important conclusion that Bernstein provides is that the fundamental (the most abstract) *patterns of the external world* are encapsulated *in* the theoretical higher "engrams" — thus the workability of mathematics in the physical world. As described earlier in this article, Pribram has argued that environmental force dynamics (mirror images of external loads), rather that topological properties of space, are represented in these higher cortical engrams.

Following Pribram's agrument, I propose that the higher-order engrams of force *dynamical* load images in the brain are seen decomposed or dimensionalized in Mandler's image-schemas which are *dynamic* image elements (see Fig. 1). It is the *cognition* of these physical world-congruent image-schematic conceptual primitives that *is mathematical* cognition. Such mathematical cognition results from the abstracting iterations of the negative feedback loop in Fig. 4. That is, whereas the abstraction within and across images-of-achievement forms the basis of linguistic cognition, the "residual" and concomitant abstraction of images-of-achievement into image-schematic form provides the basis of mathematical cognition-mathematical cognition is a by-product of language processing.

Thus, arising from the corporeal-self action space areas of the brain illustrated in Fig. 2, the above invariant image-schematic cognitions are constantly composed as *mathematical cognition*. We can refer to these structural components of mathematical cognition as *mathematical images-of-achievement*. According to this view, mathematical cognition is hidden deep within the evolutionary structure of language processes, but it is *not* language per se. Nor is mathematical cognition to be taken as this or that culturally devised system of mathematics. That is, mathematical cognition is not number or geometry, it is consciousness of patterns that potentially can be systematized in numbers or geometry.[8]

6. Conclusions and discussion

In the evolutionary process of differentiating one's own reality from those of others, a linguistic consciousness of self-frame (the corporeal self) arises from the reinstatement of one's own image-of-achievement vocalizations. This linguistic consciousness of self is a consciousness of the corporeal-self action space of the brain's cerebral cortex. From a consciousness of invariances associated with load images of the corporeal self, mathematical cognition arises. Thus, as a "mirror image" of patterns of external load information, mathematical cognition is encoded in the brain to be congruent with the physical world.

Directly in this regard, Pribram (1991) insightfully pointed out the epistemological significance of the corporeal self:

> An organism … endowed [with only the distance senses] remains a passive spectator unable to act in any way within a universe so richly perceived. Furthermore, the distance senses provide no direct contact with that universe. Contact is made by the somatic systems (p. 121).

To paraphrase this idea in the context of the motor/language theory of mathematical cognition presented in this article, it is fundamentally through the corporeal-self's contact with physical world that mathematics obtains its workability in the physical world "out there". See also Pribram (pp. 211–212, this issue).

In closing, it is important to juxtapose the theory presented in this article with a leading research effort on mathematical cognition that takes a different approach.

The theory of mathematical cognition I have proposed coincides to a small extent with Dehaene's (1997) suggestion of *number processing* in the inferior parietal and

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[8] The behavior of retarded savants represents a telling example of the difference between mathematical cognition and the logic of particular mathematical systems. Savants who can, for example, calendar calculate far faster than normal people, at the same time often have no understanding of the meaning of the arithmetic operations they execute. Such savants might give the product of 6 times 3 as 8. In savants, the higher level engrams are likely being transformed into idiosyncratic mathematical cognition (e.g., calendar calculation) via the particular level of language facility they happen to have at their disposal.

Further, and not unrelated to the savant situation, a distinction between mathematical cognition and logical mathematical systems provides insight into the brain's capacities to provide "meta-mathematical" analyses of axiomatic systems as in Gödel's incompleteness theorem (see Nagel & Newman, 1958). Gödel's theorem may place limitations on the logic of computing systems and on the probable "logic" of the savant's calendar calculating, but as Nagel and Newman point in their closing chapter, not on the brain's adaptive capacity to mathematically cognize beyond the existing formal structure of mathematics.

verbal processing areas of the brain (see Fig. 2). However, while Dehaene's (1997) research has begun to localize mathematical functions to the inferior parietal cortex, his conclusions on a recurrent theme of his book shows that he has not begun to formulate the broader and deeper evolutionary picture of the brain mechanisms involved:

> Numbers do not have full latitude to invade any available neuronal networks of the child's brain. Only certain circuits are capable of contributing to calculation — either because they are part of our innate sense of numerical quantities, such as, perhaps some areas of the inferior parietal cortex, or because, though they were initially destined for some other use, their neural organization turns out to be sufficiently flexible and close to the desired function so that they can be "recycled" for number processing (p. 206).

Certainly, Dahaene is on the right track in ascribing a sense of numerical quantification to the inferior parietal cortex. However, in my view mathematical cognition is not about "number" or "quantity", but more essentially about *pattern* (see footnote 3). Further, the involvement of language in mathematical cognition that I have described goes far beyond any notion of "recycling" in ontogeny. In the view I have presented, mathematical cognition represents the "deepest" (most abstractive) aspect of language processing in the brain. Finally, Dahaene has not seemed to recognize the important distinction between mathematical *cognition* and culturally designed systems of mathematics, which I described earlier. This distinction is absolutely critical to many philosophical and theoretical questions in mathematics (see footnote 8). According to the central-motor theory presented in this article, the intertwining of image-schemas and language, and subsequently language processing and the cognition of invariant patterns in images-of-achievement (image-schemas) represents the fuller, longer time-scale evolutionary story of the development of mathematical cognition.

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