# Formation of Languages; Equality, Hierarchy and Teachers

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**Abstract:** A quantitative method is suggested, where meanings of words, and grammatic rules about these, of a vocabulary are represented by real numbers. People meet randomly, and average their vocabularies if they are equal; otherwise they either copy from higher hierarchy or stay idle. Presence of teachers broadcasting the same (but arbitrarily chosen) vocabulary leads the language formations to converge more quickly.

**Introduction:** Within the emerging physics literature on languages [**1-12**], birth of a language may be observed as a scarcely studied issue. In our opinion, the subject is important for researches on language competition, since quickly developed languages may have more chance to survive and to spread. In the present contribution, effect of inequality on the speed of originating a language is studied, where some social agents (hierarchical people, teachers) play the role of nucleation centers for clustering of words, meanings, and grammatic rules, etc. We present a quantitative model, where each subentry of a vocabulary is represented by a real number, and so are the words. Model is given in the following section; applications and results are displayed in next one. Last section is devoted for discussion and conclusion.

**Model:** We have a society composed of $N$ adults. Each person $k$ has a vocabulary of $M$ words ($w_{ki}$, $k \le N$, $i \le M$). For a word there exist many related items as meanings, rules for plural forms, adverb forms, tenses, prefixes, suffixes, etc. In real life, many words have a diversity of such peculiarities, which are not all easy to learn and to remember; since their meanings may be close to each other, as “dictionary” and “vocabulary”. Pronunciations may be similar too; as “a head”, and “ahead”. Also; as, “night”, “knight”, and “knife”. Such variations are symbolized by five different subentries ($j$) at most. So we take $1 \le j \le j_{max}$ for every word $w$ and for each $j$ we assign a representative real number $r$. Therefore our words are sets of up to five real numbers:

$$w_{ki} = \{r_{kij}\}. \tag{1}$$

The maximum number $j_{max}$ of subentries $r$ is also determined randomly between 1 and 5, independently for any word $w$. Clearly, $r_{kij} = 0$ ($w_{ki} = \{0,0,0,0,0\}$) corresponds to an unknown meaning (word) in the vocabulary of the adult $k$.

Initially there is no consensus about a common vocabulary, but the consensus may be set through several processes described in the following subsections, and the values for initial $r_{kij}$'s in Eq. (2) must be changed into time dependent ones, i.e. $r_{kij}(t)$.

Evolution of the language spoken by any adult may be described by

$$L_k(t) = \sum_{i=1}^{M} \sum_{j=1}^{j_{max}} r_{kij}(t), \tag{2}$$

wherex, $L_k$ varies from person to person, especially at the beginning of the formation period, and this fluctuation fades down with time since $L_k \to L$, if convergence occurs.

Eq. (2) may be summed over the members ($k$) of the society to consider all the vocabularies present at time $t$:

$$D(t) = \sum_{k}^{N} L_k(t). \tag{3}$$

As we observed, $D(t) - D(t-1)$ is a significant quantity within the present formalism, and we represent it by $V(t)$:

$$V(t) = D(t) - D(t-1). \tag{4}$$

As $t \to \infty$, $D(t)$ is expected to converge to its limit $D(t \to \infty)$, and $L_k(t)$ to some $L$, and $V(t)$ to zero. Then, the language $L = L_k(t \to \infty)$ may be evaluated as established. Minor fluctuations within $D(t)$ about $D(t \to \infty)$, and these within $V(t)$ about zero may be attributed to misuses due to lack of individual memories to remember all the relevant meanings, and rules, etc.

*Initiation*: We assign random real numbers for initial values of $r_{kij}$, with $0 \le r_{kij} < 1$, where $k \le N, i \le M$, and $j \le j_{max}(k,i)$.

*Evolution*: Once the initial vocabularies are set, we assume that two members ($k$, and $k$') meet randomly at a time $t$.

In the simplest case of no inequality in status (*equality*), they average [**13-14**] subentries ($r_{kij}$) in their vocabularies, and share the new ones:

$$r_{kij}(t) = (r_{kij}(t-1) + r_{k'ij}(t-1))/2 = r_{k'ij}(t), \tag{5}$$

and the language spoken by each adult ($k$) becomes:

$$L_k(t) = L_{k'}(t) = (L_k(t-1) + L_{k'}(t-1))/2. \tag{6}$$

As interaction tours (time $t$) advance, $\mathrm{r}_{kij}(t \to \infty) = 1/2$ independent of the subindices. We have $D(t) = D(0)$ and $V(t) = V(0) = 0$, for all $t$, since $L_k(t) + L_{k'}(t) = L_k(t-1) + L_{k'}(t-1)$, due to Eq. (6).

We incorporate *inequality* into the society, by assigning some rank to adults in terms of real numbers (greater than or equal to zero, and less than one) determined randomly. Yet, any two adults will be considered as equivalent if their ranks are close to each other by a given $\Delta$, and each member will average her vocabulary with the other, Eq. (5) and (6). Otherwise, the one with lower rank (obeying) will copy down the vocabulary of the other (commander) and take it as her new vocabulary, till another possible meet with any adult occurs. In this case convergence (formation) of the language may be speeded up under certain conditions, as studied within the following section.

Furthermore, we may assume more stringent inequality: Some hierarchy (all, with rank of unity) broadcast the same (yet arbitrarily selected) vocabulary to the society, from the beginning on. We call them teachers. They will not change their common vocabulary and due to their ultimate rank, they will not average their vocabularies with anyone. Some other hierarchical people (within a given limit of $\Delta$) may average their vocabularies after they discuss with teachers. And the rest copies down from all those who have higher ranks by $\Delta$.

**Applications and Results:** In this section we will first consider uniqueness within society. Later, by assigning to each individual a random real number (rank; greater than or equal to zero, and less than one) we will establish hierarchy. And finally, we will incorporate some teachers with ultimate rank of unity into society.

We handled *equality* within adults by assuming an averaging process for the words ($w_{ki}$ of Eqs. (1), (2), and (5)), and the meanings ($r_{kij}$ of Eq. (1)) [**13-14**]. Evolution of $r_{kij}(t)$, for a randomly selected $j$ is displayed in Figure 1, where adults ($N = 500$) are all equal and only arbitrarily chosen hundred adults are displayed. Each adult had her own initial randomly selected meanings ($r_{kij}(0)$) as used by herself and suggested to the society. Whenever any two of the adults randomly meet, they obey Eq. (5); each interacts equally with the other and averages her vocabulary. $D(t) = D(0)$

and $V(t) = V(0) = 0$, for all $t$, since $D(t)$ of Eq. (3) does not change during interactions $L_k(t) + L_{k'}(t) = L_k(t-1) + L_{k'}(t-1)$, Eq. (6)). Corresponding probability density function (PDF) for $r_{kij}(t)$ (with $N = 500$, $M = 100$ and $j \leq j_{max}$) is a delta function, i.e., $\text{PDF}(V) = \delta(0)$ (inset, Fig. 1.).

We incorporate *inequality* into the society, by assigning some rank to adults in terms of real numbers (greater than or equal to zero, and less than one) determined randomly. Yet, any two adults will be considered as equivalent if their ranks are close to each other by a given $\Delta$. Under the present condition, each member will average her vocabulary with the other, Eq. (3). Otherwise, the one with lower rank will copy down the vocabulary of the other and take it as her new vocabulary, till another possible meet with any adult occurs.

For small $\Delta$, almost everybody (except the top of hierarchy with rank $1 - \Delta$) may copy from others, and almost everybody (except the bottom of hierarchy with rank $\Delta$) may be copied by others. Within this content, the averaging process between equals is ignored within the society ($N$). On the other hand if $0.5 < \Delta$, only the top of hierarchy with rank $1 - \Delta$ will be copied by the bottom of that with rank $\Delta$, and more than half of the society will average. Clearly, averaging process will dominate as $0.5 \ll \Delta \to 1.0$; therefore this regime implies more freedom and more discussion. $\Delta = 1.0$ case corresponds to equality of all the adults.

Evolutions of $r_{kij}(t)$ with various $N$, and $M$ as designated in the figure captions and $j \leq j_{max}$, and $D(t)$, and $V(t)$ are displayed, in Figures 2a, and b, and c, respectively, where $\Delta = 0.2$ for all. One may remark that, discussing and averaging mechanism between (close) equals (by $\Delta$), or copying from the vocabulary of some higher rank people causes the language to converge, yet convergence is very small for $\Delta \sim 0$, and speeds up as $\Delta \to 1$. For $\Delta \sim 0$, all the society speaks ultimately the language of the one with highest rank which is very close to unity.

*Teachers*: Figure 3a displays evolution of $r_{kij}(t)$, with randomly selected number of meanings ($j \leq j_{max}$), where the meanings of words belong to language of arbitrarily chosen hundred adults out of $N = 5000$ adults. Please note the horizontal limiting line representing the language broadcasted by teachers. The greater the distance from this line is, the greater is the needed effort to learn the language. Figure 3b displays $D(t)$, and Figure 3c displays $V(t)$, with $\Delta = 0.2$ and $\tau = 0.2$ in all, where $\tau$ designates the number of teachers per population of the society ($N$). Please note the rapid convergence in $D(t)$ and $V(t)$.

In Figure 3c there exist three behaviors in $V(t)$: For $t \sim 0$ region we have comparably big fluctuations; for $t \to \infty$ we have very small fluctuations, both about zero. And in between we have exponential decay. Initial fluctuations originate from randomness, and the number of equilibrated ones may be increased by increasing the number of tours (and also, precision of real numbers in the utilized software). So, the characteristic regime is the intermediate one and exponential decay implies that the envelope function for $D(t)$ (which passes through local maxima and minima) is also an exponentially decaying one. (We had observed similar exponential decays within our computations on opinion dynamics. [**16**]) The pronounced threefold behavior is reflected in PDF's in Figure 3c; where, the horizontal axis is for $V^2$, and the perpendicular one is logarithmic.

Small-speed regimes in PDF's of Figure 3d correspond to $t \to \infty$ region in $V(t)$, which may be ignored totally. Please note that PDF($V$) (and PDF($V^2$)) goes to $\delta$, as $\Delta$ approaches unity and high speed wing tips in PDF's are coming from $t \sim 0$ region in $V(t)$, where randomness is dominant. Teachers shape the intermediate region, and due to them we have the exponential convergence in $D(t)$. And one new language emerges, which is spoken by the majority of adults, and will be learned by children.

**Discussion and Conclusion:** Clearly, increasing the number of teachers (and $\tau$) increases rates of exponential decays in $V(t)$ and $D(t)$: There will be more chance to check personal vocabularies, and number of ordinary adults will be lowered. Big differences between the real numbers associated to entries of the broadcasted common vocabulary and those to initial settings may be considered as a kind of measure for difficulty to learn the relevant language, since more interaction tours will be needed for averaging before the personal vocabularies approach the broadcasted one. If the equilibrium level of $D(t)$ is far from the initial one, then the emerging language may be considered as a tough one to learn. (We run the case, with $0.9 \leq r_{kij} < 1.0$, and $0.0 \leq r_{kij} < 0.1$ (Eq. 3) for the teachers' vocabulary many times and verified the last remark in all.)

We run also the case, where each teacher broadcasted (keeping her ultimate rank) a different vocabulary, rather than a common one. This case corresponds to a richer language. And we obtained still, but rather slower, exponential decays. In our opinion, this result agrees well with the reality that those languages involving more words and grammatic rules are harder to learn than those with less words and rules. In any case, presence of nuclei speeds up clustering of words and rules; and the relevant language emerges quickly. So, when a group of people immigrate to a new society, and if they gain rank (power) they may broadcast their language to the present society, which may be considered as one of the possible mechanism to spread languages besides colonization, conquest, etc. As a final remark it may be stated that we varied the number of words (upper limit within the sum of Eq. (2)) and the number of adults ($N$) within the society from 10, 100 to 1000, 5000, all respectively and obtained similar results. As the numbers decreased, fluctuations increased; yet, the envelope of $D(t)$ always came out as exponential.

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**FIGURES**

**Figure 1** Evolution of $\sum_i^M r_{kij}(t)$, for three adults, where $j$ is arbitrary with $j \leq j_{max}$, and $M = 100$, for $N = 500$. Inset shows PDF for time rate of change of $r_{kij}(t)$.

**Figure 2a** Evolution of $\sum_i^M r_{kij}(t)$, for three adults, where $j$ is arbitrary and $M = 100$, for $N = 1000$, where $\Delta = 0.2$.

**Figure 2b** Evolution of $D(t)$ with $M = 300$**,** $N = 5000$, for $\Delta = 0.2$.

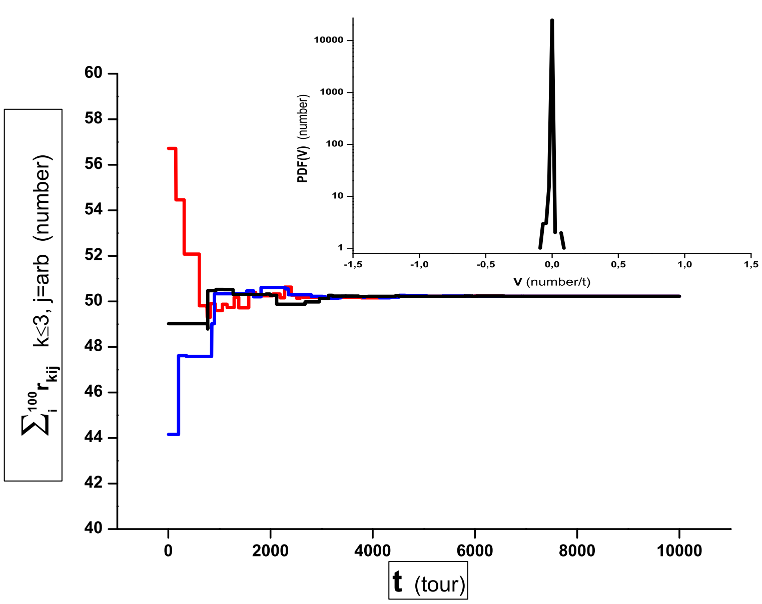
**Figure 2c** Evolution of V(t) with $M = 300$**,** $N = 5000$, for $\Delta = 0.2$.

**Figure 3a** Evolution of $\sum_i^M r_{kij}(t)$, for three adults, where $j$ is arbitrary and $M = 300$, for $N = 1000$, with $\Delta = 0.2$, $\tau = 0.2$.

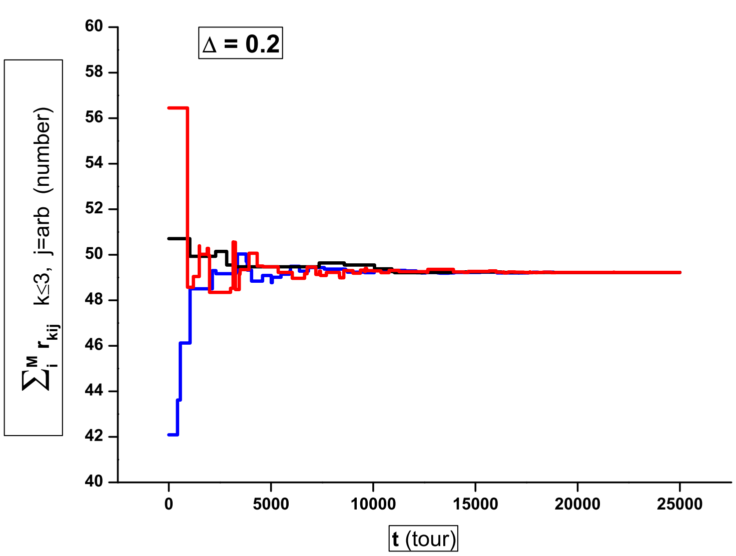
**Figure 3b** $D(t)$ with $\Delta = 0.2$ and $\tau = 0.2$. Please notify the rapid convergence.

**Figure 3c** $V(t)$ with $\Delta = 0.2$ and $\tau = 0.2$. Please notify the rapid convergence in $V(t)$. Perpendicular axis for $V(t)$ is logarithmic. The inset shows PDF for the given $V(t)$.

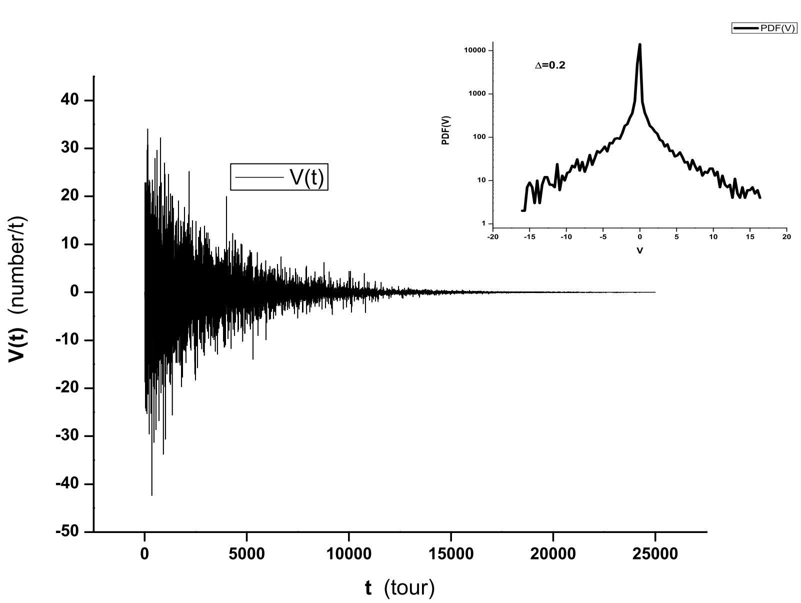
**Figure 3d** PDF($V$) for $\Delta = 0.2$, and various $\tau$, where the horizontal axis is $V^2$, and the perpendicular one is logarithmic.



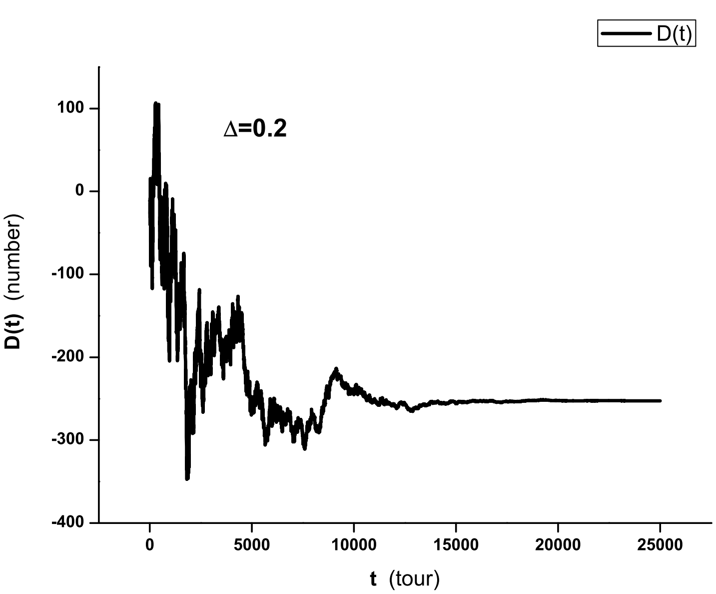
$\sum_i^{100} r_{kij}$ k≤3, j=arb (number)
t (tour)
PDF(V) (number)
V (number/t)



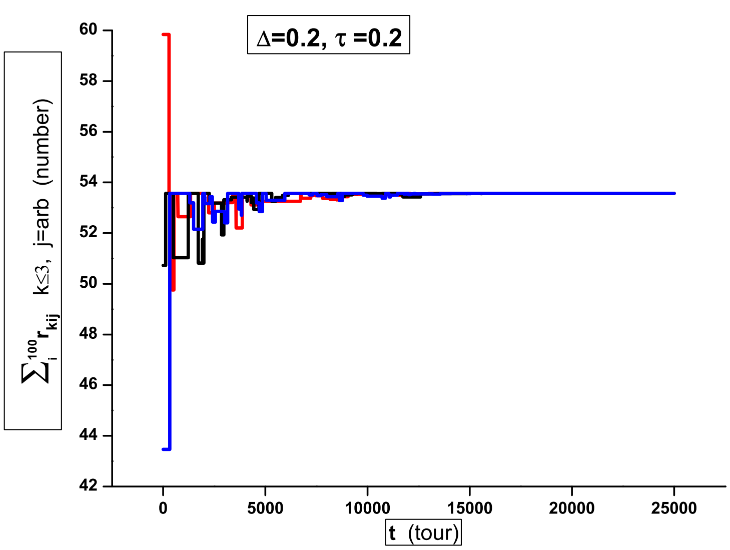
60
58
56
54
52
50
48
46
44
42
40
$\Delta$ = 0.2
$\sum_i^M r_{kij}$  k≤3,  j=arb  (number)
0
5000
10000
15000
20000
25000
t (tour)



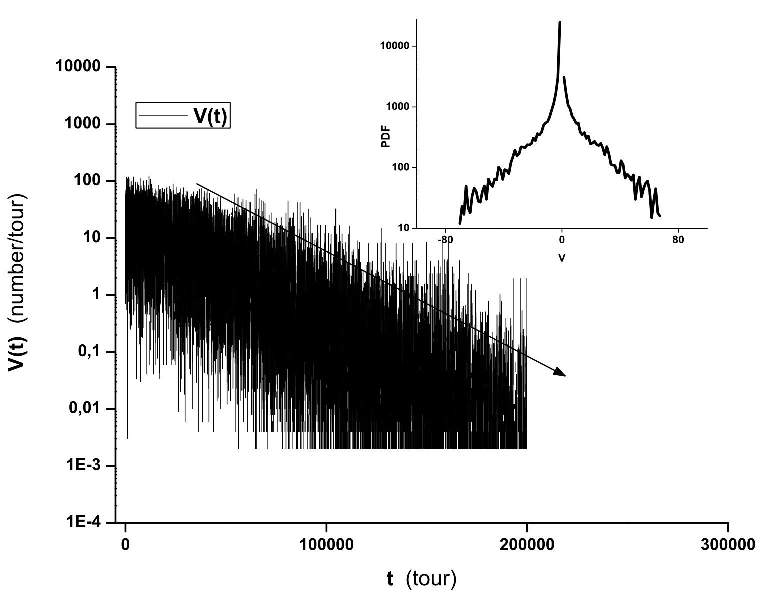
PDF(V)
Δ=0.2
PDF(V)
V
V(t)
V(t) (number/t)
t (tour)



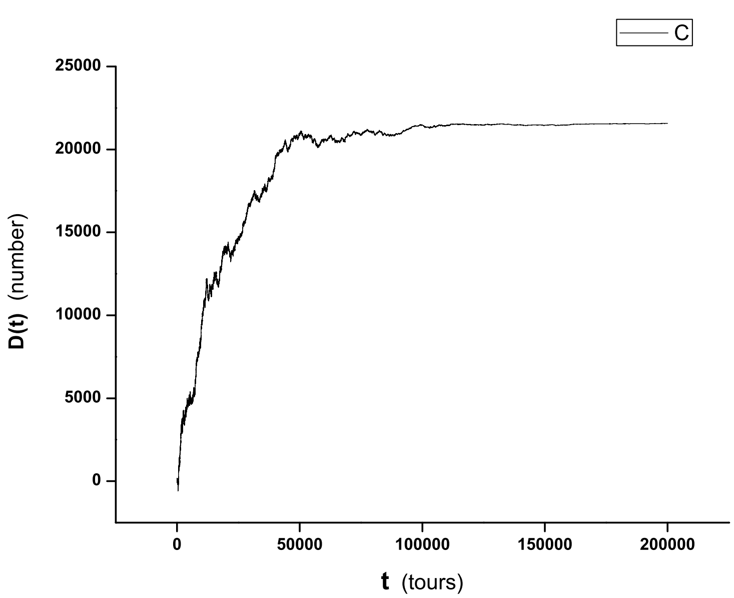
D(t)
Δ=0.2
100
0
-100
-200
-300
-400
0
5000
10000
15000
20000
25000
D(t) (number)
t (tour)

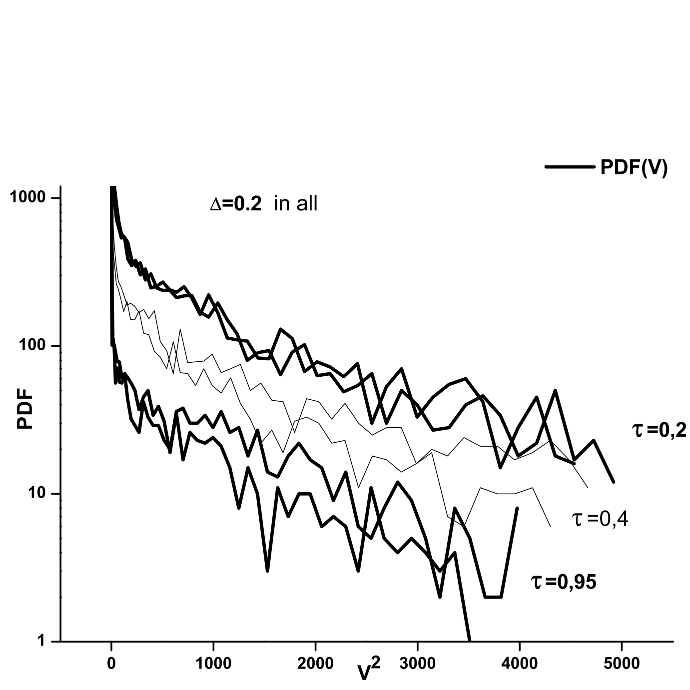


Δ=0.2, τ =0.2
$\sum_i^{100} r_{kij}$ k≤3, j=arb (number)
t (tour)
60
58
56
54
52
50
48
46
44
42
0
5000
10000
15000
20000
25000



10000
1000
100
10
1
0,1
0,01
1E-3
1E-4
V(t)
V(t) (number/tour)
0
100000
200000
300000
t (tour)
PDF
10000
1000
100
10
-80
0
80
V


C
25000
20000
15000
10000
5000
0
D(t) (number)
0
50000
100000
150000
200000
t (tours)



PDF(V)
Δ=0.2 in all
1000
100
10
1
PDF
τ =0,2
τ =0,4
τ =0,95
0
1000
2000
3000
4000
5000
V2