Microscopic Abrams-Strogatz model of language competition

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Abstract: The differential equations of Abrams and Strogatz for the competition between two languages are compared with agent-based Monte Carlo simulations for fully connected networks as well as for lattices in one, two and three dimensions, with up to 10^9 agents.

Keywords: Monte Carlo, language competition

Many computer studies of the competition between different languages, triggered by Abrams and Strogatz [1], have appeared mostly in physics journals using differential equations (mean field approximation [2, 3, 4, 5]) or agent-based simulations for many [6, 7, 8, 9] or few [10, 11] languages. A longer review is given in [12], a shorter one in [13]. We check in this note to what extent the results of the mean field approximation are confirmed by agent-based simulations with many individuals. We do not talk here about the learning of languages [14, 15].

The Abrams-Strogatz differential equation for the competition of a language Y with higher social status $1 - s$ against another language X with lower social status s is

$$\mathrm{d}x/\mathrm{d}t = (1-x)x^a s - x(1-x)^a(1-s) \tag{1}$$

where $a \simeq 1.3$ [1] and $0 < s \leq 1/2$. Here x is the fraction in the population speaking language X with lower social status s while the fraction $1-x$ speaks language Y. As initial condition we consider the situation in which both languages have the same number of speakers, $x(t=0) = 1/2$. Figure 1 shows exponential decay for $a = 1.31$ as well as for the simpler linear case $a = 1$. For $s = 1/2$ the symmetric situation $x = 1/2$ is a stationary solution

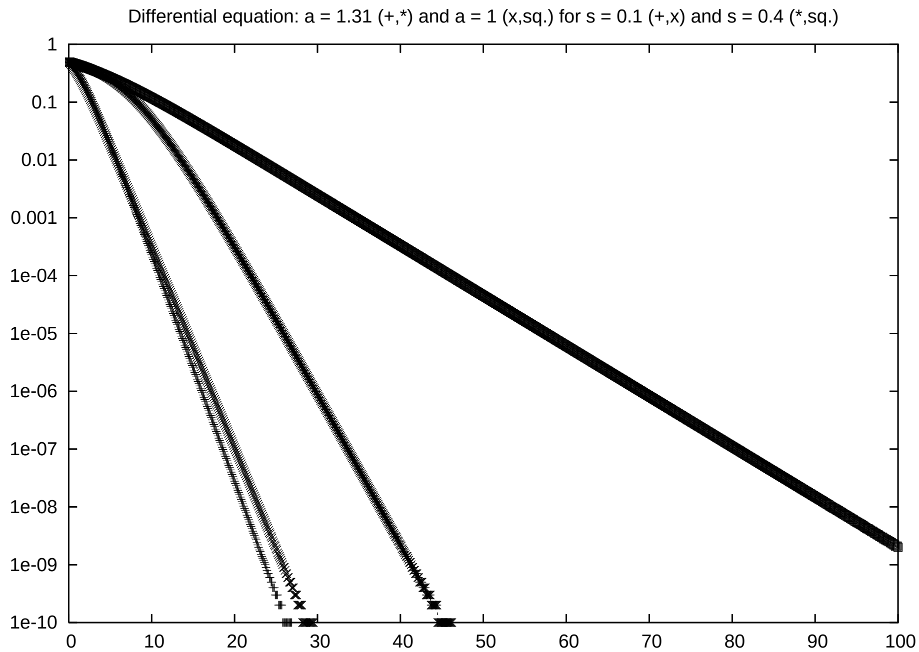

Figure 1: Fraction of X speakers from Abrams-Strogatz differential equation with $a = 1.31$ and $a = 1$, at status $s = 0.1$ (left) and $s = 0.4$ (right). For $a = 1.31$ the decay is faster than for $a = 1$.

which is stable for $a < 1$ and unstable for $a > 1$. From now on we use $a = 1$. This simplification makes the resulting differential equation

$$\mathrm{d}x/\mathrm{d}t = (2s - 1)(1 - x)x \tag{2}$$

for $s \neq 1/2$ similar to the logistic equation which was applied to languages before, as reviewed by [16]. For $s = 1/2$ any value of x is a marginally stable stationary solution.

This differential equation is a mean-field approximation, ignoring the fate of individuals and the resulting fluctuations. We now put in N individuals which in the fully connected model feel the influence of all individuals, while on the d-dimensional lattice they feel only the influence of their $2d$ nearest neighbors. The probability p to switch from language Y to language X, and the probability q for the inverse switch, are

$$p = x^a s, \quad q = (1 - x)^a (1 - s) \quad . \tag{3}$$

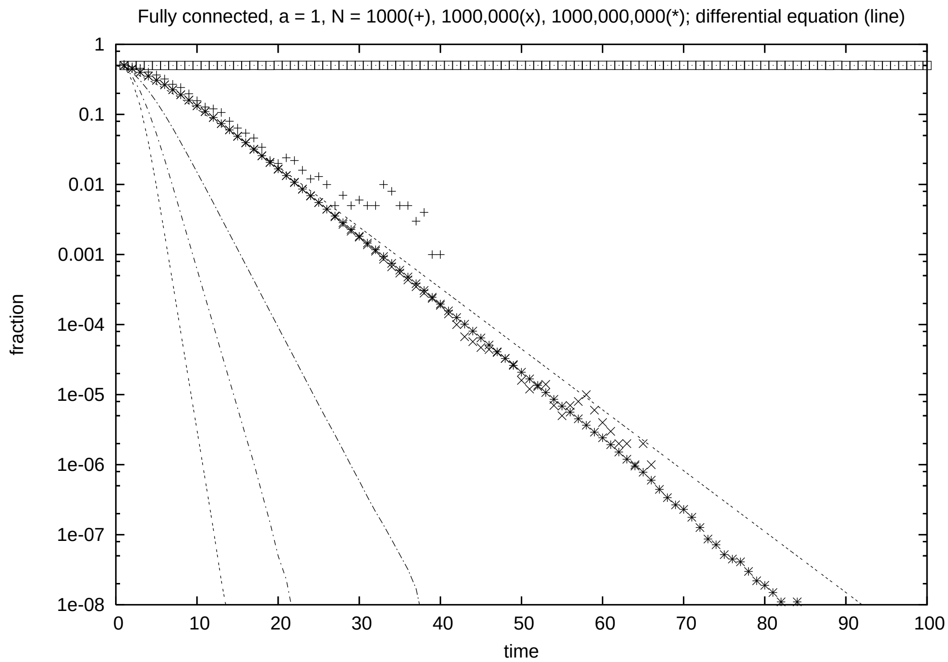

Figure 2: Fully connected model with 10^3, 10^6, 10^9 agents at $s = 0.4$ compared with differential equation (rightmost line) at $s = 0.4$. The three left lines correspond to $s = 0.1,\ 0.2,\ 0.3$ from left to right for $N = 10^9$. The thick horizontal line corresponds to $s = 0.5$ and $N = 10^6$ and changes away from 1/2 only for much longer times. Figs. 2 and 3 use one sample only and thus indicate self-averaging: The fluctuations decrease for increasing population.

On a lattice, this x is replaced by the fraction of X speakers in the neighborhood of $2d$ sites. We use regular updating for most of the results shown in this paper. Initially each person selects randomly one of the two languages with equal probability: $x(t = 0) = 0.5$. In the symmetric situation $s = 1/2$ with $a = 1$ that we will consider, our later lattice model becomes similar to the voter model [17].

Fig.2 shows our results for the fully connected case and Fig.3 for the square lattice with four neighbours; the results are quite similar to each other and to the original differential equation. A major difference with the differential equation (1) is seen in the symmetric case $s = 1/2$ when the two

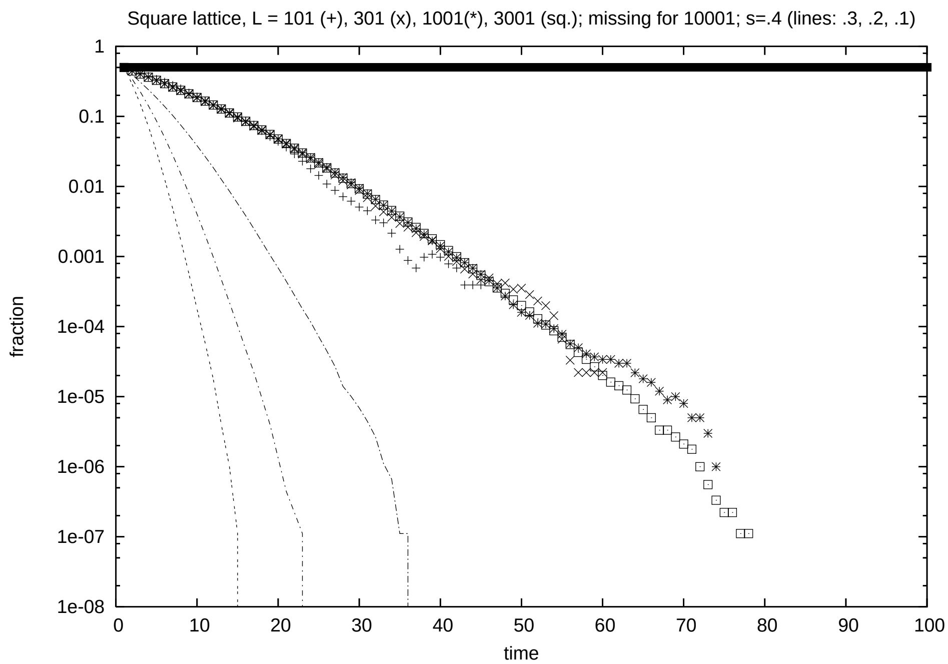

Figure 3: $L \times L$ square lattice with $L = 101$ to 3001 at $s = 0.4$. The three left lines correspond to $s = 0.1,\ 0.2,\ 0.3$ from left to right for $L = 3001$. The thick horizontal line corresponds to $s = 0.5$.

languages are completely equivalent. Then the differential equation has x staying at 1/2 for all times, while random fluctuation for finite population destabilize this situation and let one of the two languages win over the other, with x going to zero or unity.

This latter case can be described in a unified way by looking at the number of lattice neighbours speaking a language different from the center site. It corresponds to an energy in the Ising magnet and measures microscopic interfaces. Initially this number equals d on average, and then it decays to zero, first possibly as a power law, and then exponentially after a time which increases with increasing lattice size, Fig.4. The first decay describes a coarsening phenomenon, while the exponential decay is triggered by finite size fluctuations. In one dimension the initial decay follows a power law, $t^{-1/2}$, while in three dimensions an initial plateau is reached. This is followed by an exponential decay in $d = 1, 3$ as in two dimensions, Fig.5. Figure 6

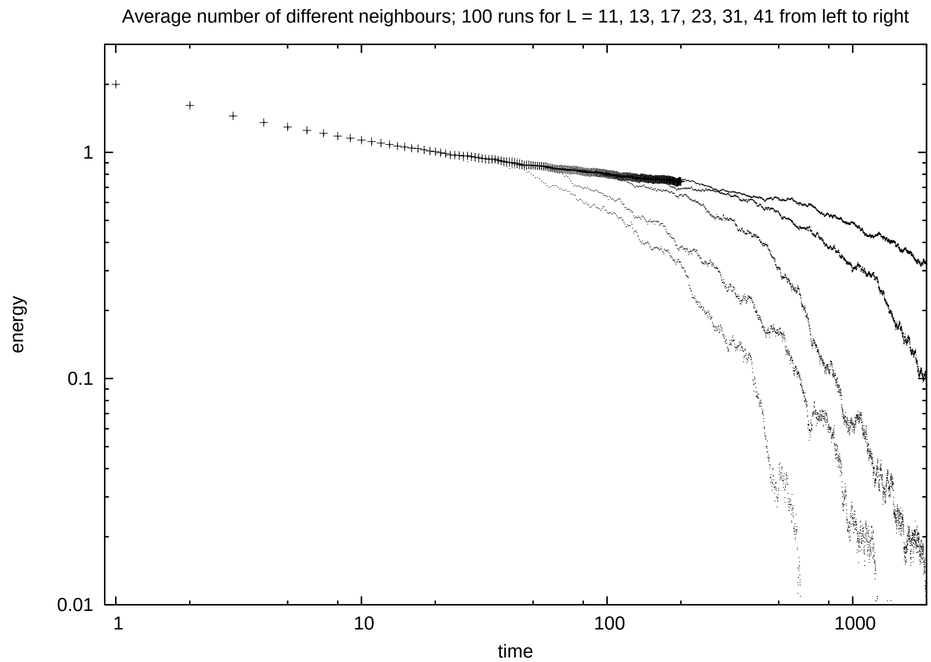

Figure 4: Decay of unstable symmetric solution $x = 1/2$ for $s = 1/2$ for square lattices of various sizes; the larger is the lattice the longer do we have to wait. A semilogarithmic plot, not shown, indicates a simple exponential decay. Figs.4-6 average over 100 samples.

shows that the average of $|x(t) - 1/2|$ increases in two dimensions roughly as the square-root of time until it saturates at $1/2$, indicating random walk behavior. (Note that first averaging over x and then taking the absolute value $| < x > -1/2|$ would not give appropriate results since $< x >$ would always be $1/2$ apart from fluctuations.)

In all the simulations described above, we went through the population regularly, like a typewriter on a square lattice, and for full connectivity kept the probabilities constant within each iteration. Using random updating is more realistic but takes more time. The long-time results are similar, and the power-law decay holds for $t < 10^2$ with exponents 0.5 for $d = 1$ (Fig. 5), and 0.1 (compatible with $1/\ln t$) for $d = 2$. For $d = 3$ a plateau is also reached. For the simpler regular updating we checked when the fraction x, initially $1/2$, leaves the interval from 0.4 to 0.6 on its way to zero or one, Fig.7a. For the random updating we checked when the energy reaches a small fraction of its initial value, taken as $2/L$, 0.04 and 0.6 for $d = 1, 2, 3$, Fig.7. Both figure parts are quite similar, with scaling laws for the characteristic time which are compatible with the ones obtained for a voter model [17]: $\tau \simeq N^2$ in $d = 1$, $\tau \simeq N \ln N$ in $d = 2$, and $\tau \simeq N$ in $d = 3$, where $N = L^d$.

We conclude that agent-based simulations differ appreciably from the results from the mean-field approach for the symmetric case $s = 1/2$: While Eqs.(1,2) then predict x to stay at $x = 1/2$, our simulations in Fig.4 and later show that after a decay everybody speaks the same language. In a fully connected network and in $d = 3$ the decay is triggered by a finite size fluctuation, while in $d = 1, 2$ the intrinsic dynamics of the system causes an initial ordering phenomena in which spatial domains of speakers of the same language grow in size.

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References

[1] D.M. Abrams and S.H. Strogatz, Nature 424 (2003) 900.

[2] M. Patriarca and T. Leppänen, Physica A 338 (2004) 296.

[3] W.S.Y. Wang and J.W. Minett, Trans. Philological Soc.103 (2005) 121 and unpublished.

[4] J.Mira and A. Paredes, Europhys. Lett. 69 (2005) 1031.

[5] J.P. Pinasco and L. Romanelli, Physica A 361 (2006) 355.

[6] C. Schulze and D. Stauffer, Int. J. Mod. Phys. C 16 (2005) 781; Physics of Life Reviews 2 (2005) 89;

[7] T. Tesileanu and H. Meyer-Ortmanns, Int. J. Mod. Phys. C 17, No. 3, 2006, in press.

[8] D. Stauffer, C. Schulze, F.W.S. Lima, S. Wichmann and S. Solomon, e-print physics/0601160 at arXiv.org.

[9] V.M. de Oliveira, M.A.F. Gomes and I.R. Tsang, Physica A 361 (2006) 361; V.M. de Oliveira, P.R.A. Campos, M.A.F. Gomes and I.R. Tsang, e-print physics/0510249 at arXiv.org for Physica A.

[10] K. Kosmidis, J.M. Halley and P. Argyrakis, Physica A, 353 (2005) 595; K.Kosmidis, A. Kalampokis and P.Argyrakis, physics/0510019 in arXiv.org to be published in Physica A.

[11] V. Schwämmle, Int. J. Mod. Phys. C 16 (2005) 1519; *ibidem* 17, No. 3, 2006, in press.

[12] D. Stauffer, S. Moss de Oliveira, P.M.C. de Oliveira, J.S. Sa Martins, *Biology, Sociology, Geology by Computational Physicists*, Elsevier, Amsterdam 2006 in press.

[13] C. Schulze and D. Stauffer, Comput. Sci. Engin. 8 (2006) in press.

[14] M.A. Nowak, N.L. Komarova and P. Niyogi, Nature 417 (2002) 611.

[15] A. Baronchelli, M. Felici, E. Caglioti, V. Loreto, L. Steels, e-prints physics/0509075, 0511201 and 0512045 at arXiv.org.

[16] W.S.Y. Wang, J. Ke, J.W. Minett, in: Computational linguistics and beyond, eds. C.R. Huang and W. Lenders (Academica Sinica : Institute of Linguistics, Taipei, 2004); www.ee.cuhk.edu.hk/∼wsywang

[17] R. Holley and T.M. Liggett, Ann. Probab. 3 (1975) 643; K. Suchecki, V.M. Eguíluz and M. San Miguel, Phys. Rev. E 72 (2005) 0361362 and Europhys. Lett. 69 (2005) 228; M. San Miguel, V.M. Eguíluz, R. Toral and K. Klemm, Comp. Sci. Engin. 7 (Nov/Dec 2005) 67.

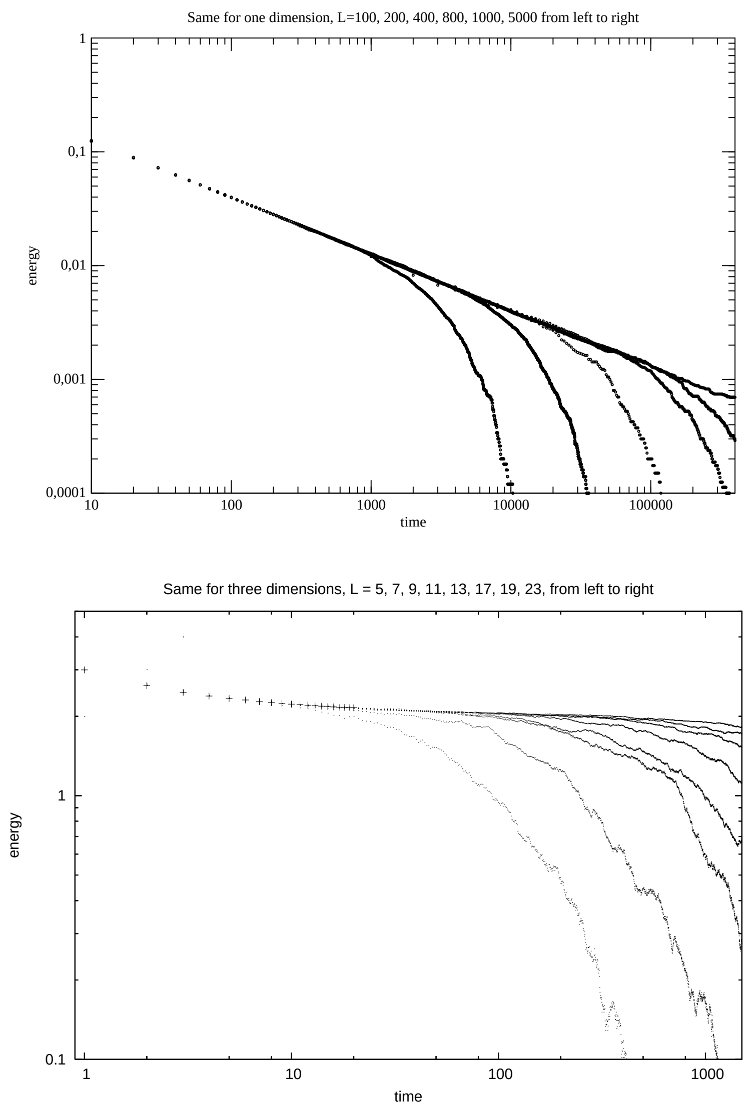

Figure 5: Same as Fig.4 but in one (top) or three (bottom) dimensions. Simulations shown for $d = 1$ are done with random updating

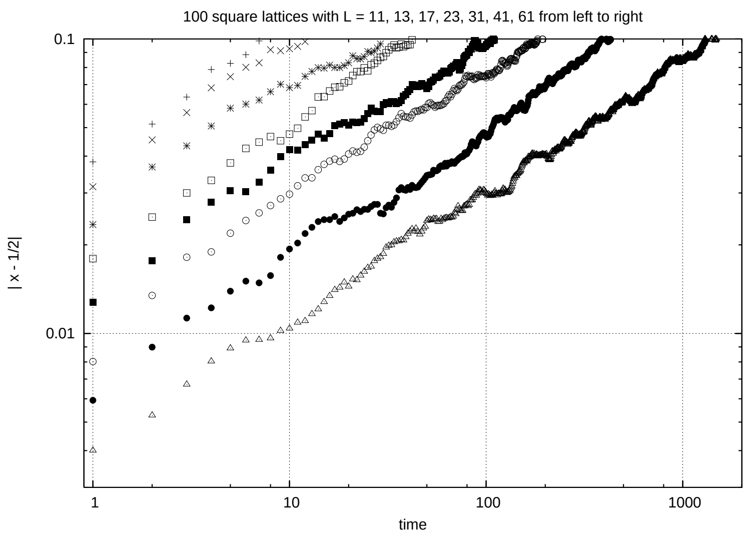

Figure 6: Average over absolute difference between $x(t)$ and $x(t = 0) = 1/2)$ for $d = 2$.

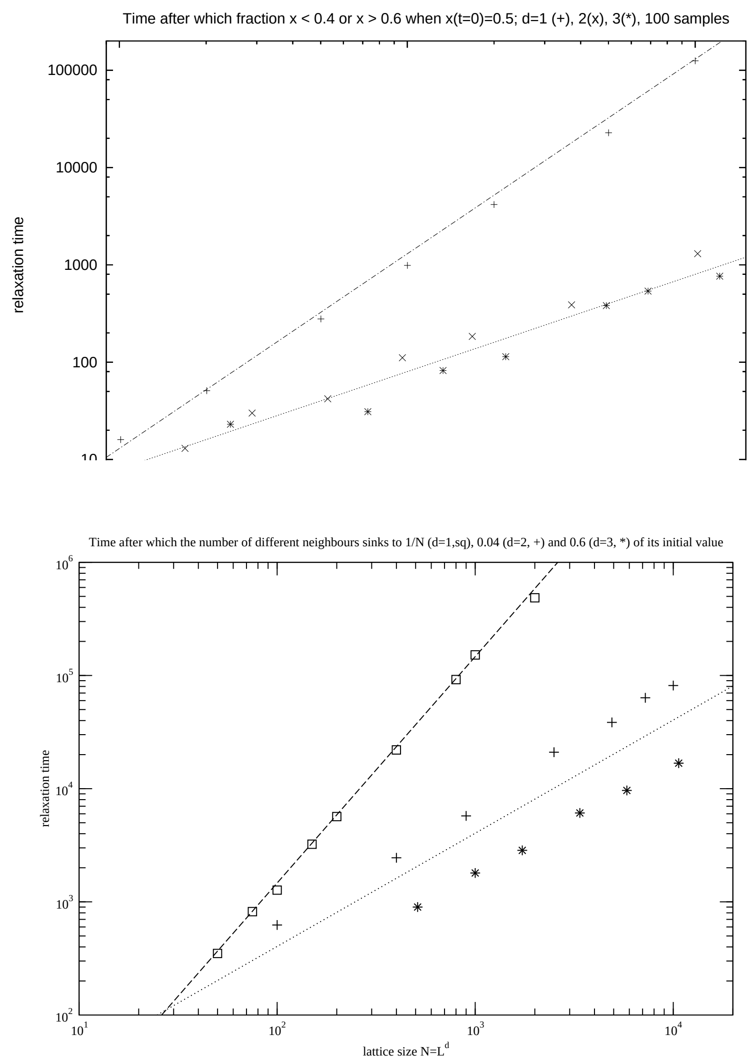

Figure 7: Time for the energy (= number of different lattice neighbours) to sink to some constant fraction of its initial value, versus population $N = L^d$, in one (+), two (x) and three (*) dimensions, from $x(t)$ averaged over 100 samples. Part a uses regular updating, part b the better random updating. The straight lines have slope 1 for $d = 2, 3$, and 2 for $d = 1$.