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Rediscovering the Co-occurrence Principles of Vowel Inventories: A Complex Network Approach

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In this work, we attempt to capture patterns of co-occurrence across vowel systems and at the same time figure out the nature of the force leading to the emergence of such patterns. For this purpose we define a weighted network where the vowels are the nodes and an edge between two nodes (read vowels) signify their co-occurrence likelihood over the vowel inventories. Through this network we identify communities of vowels, which essentially reflect their patterns of co-occurrence across languages. We observe that in the assortative vowel communities the constituent nodes (read vowels) are largely uncorrelated in terms of their features and show that they are formed based on the principle of maximal perceptual contrast. However, in the rest of the communities, strong correlations are reflected among the constituent vowels with respect to their features indicating that it is the principle of feature economy that binds them together. We validate the above observations by proposing a quantitative measure of perceptual contrast as well as feature economy and subsequently comparing the results obtained due to these quantifications with those where we assume that the vowel inventories had evolved just by chance.

Keywords: Vowels; complex network; community structure; feature entropy.

1. Introduction

Linguistic research has documented a wide range of regularities across the sound systems of the world's languages [2, 5, 12, 13, 17, 18]. Functional phonologists argue that such regularities are the consequences of certain general principles like *maximal* *perceptual contrast*[a] [12], *ease of articulation*[b] [2, 14], and *ease of learnability*[c] [2]. In the study of vowel systems the optimizing principle, which has a long tradition [9, 25] in linguistics, is maximal perceptual contrast. A number of numerical studies based on this principle have been reported in literature [12, 13, 21]. Of late, there have been some attempts to explain the vowel systems through multi agent simulations [2] and genetic algorithms [10]; all of these experiments also use the principle of perceptual contrast for optimization purposes.

An exception to the above trend is a school of linguists [3, 6] who argue that perceptual contrast-based theories fail to account for certain fundamental aspects such as the patterns of co-occurrence of vowels based on similar acoustic/articulatory *features*[d] observed across the vowel inventories. Instead, they posit that the observed patterns, especially found in larger size inventories [3], can be explained only through the principle of *feature economy* [7, 16]. According to this principle, languages tend to maximize the combinatorial possibilities of a few distinctive features to generate a large number of sounds.

The aforementioned ideas can be possibly linked together through the example illustrated by Figure 1. As shown in the figure, the initial plane P constitutes of a set of three very frequently occurring vowels /*i*/, /*a*/ and /*u*/, which usually make up the smaller inventories and do not have any single feature in common. Thus, smaller inventories are quite likely to have vowels that exhibit a large extent of contrast in their constituent features. However, in bigger inventories, members from the higher planes (P' and P'') are also present and they in turn exhibit feature economy. For instance, in the plane P' comprising of the set of vowels /ĩ/, /ã/, /ũ/, we find a nasal modification applied equally on all the three members of the set. This is actually indicative of an economic behavior that the larger inventories show while choosing a new feature in order to reduce the learnability effort of the speakers. The third plane P'' reinforces this idea by showing that the larger the size of the inventories the greater is the urge for this economy in the choice of new features. Another interesting facet of the figure are the relations that exist across the planes (indicated by the broken lines). All these relations are representative of a common

[a]Maximal perceptual contrast, is desirable between the phonemes of a language for proper perception of each individual phoneme in a noisy environment

[b]Ease of articulation requires that the sound systems of all languages are formed of certain universal (and highly frequent) sounds.

[c]Ease of learnability is required so that a speaker can learn the sounds of a language with minimum effort.

[d]In linguistics, features are the elements, which distinguish one phoneme from another. The features that describe the vowles can be broadly categorized into three different classes namely the *height*, the *backness* and the *roundedness*. Height refers to the vertical position of the tongue relative to either the roof of the mouth or the aperture of the jaw. Backness refers to the horizontal tongue position during the articulation of a vowel relative to the back of the mouth. Roundedness refers to whether the lips are rounded or not during the articulation of a vowel. There are however still more possible features of vowel quality, such as the velum position (e.g., nasality), type of vocal fold vibration (i.e., phonation), and tongue root position (i.e., secondary place of articulation).

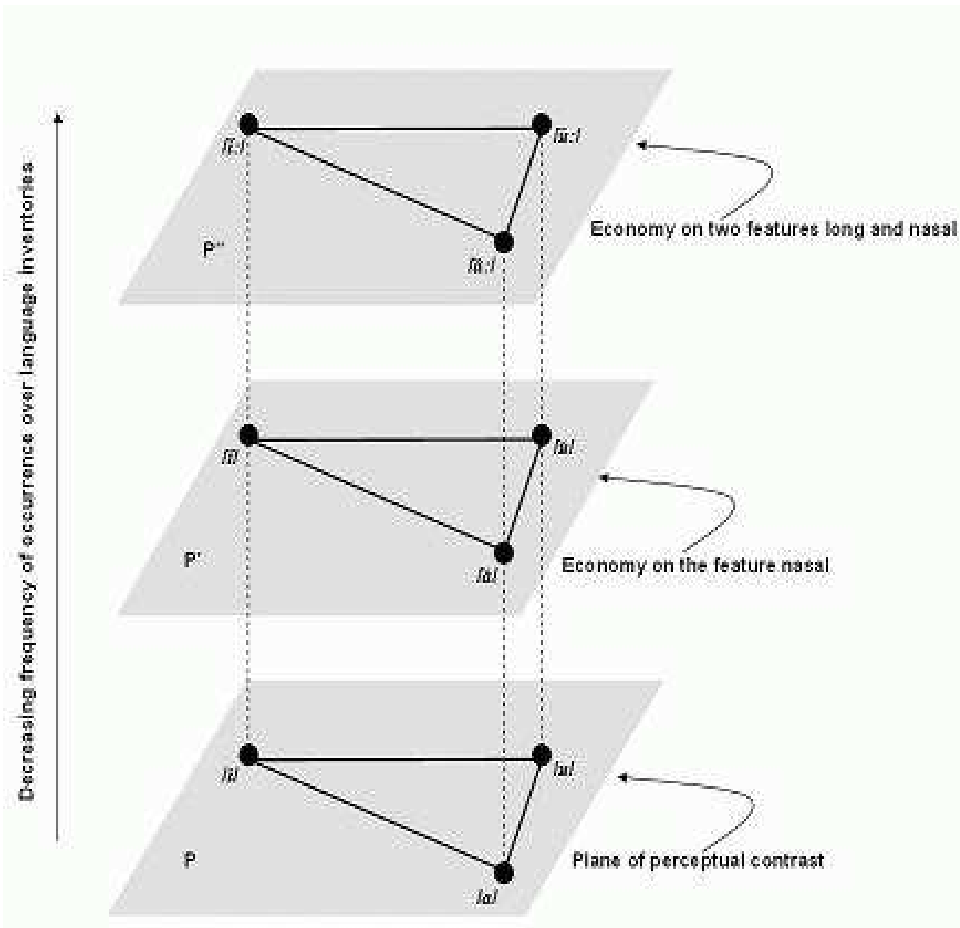


Fig. 1. The organizational principles of the vowels (in decreasing frequency of occurrence) indicated through different hypothetical planes.

linguistic concept of *robustness* [6] in which one frequently occurring vowel (say $/i/$) implies the presence of the other (and not vice versa) less frequently occurring vowel (say $/\tilde{\mathrm{i}}/$) in a language inventory. These cross-planar relations are also indicative of feature economy since all the features present in the frequent vowel (e.g., $/i/$) are also shared by the less frequent one (e.g., $/\tilde{\mathrm{i}}/$). In summary, while the basis of organization of the vowel inventories is perceptual contrast as indicated by the plane P in Figure 1, economic modifications of the perceptually distinct vowels takes place with the increase in the inventory size (as indicated by the planes P' and P'' in Figure 1).

In this work we attempt to corroborate the above conjecture by automatically capturing the patterns of co-occurrence that are prevalent *in* and *across* the planes illustrated in Figure 1. We also present a quantitative measure of the driving forces that lead to the emergence of such patterns and show that the real inventories are significantly better in terms of this measure than expected. In order to do so, we define the "**Vo**wel-Vowel **Net**work" or **VoNet**, which is a weighted network where the vowels are the nodes and an edge between two nodes (read vowels) signify their co-occurrence likelihood over the vowel inventories. We conduct community structure analysis of different versions of VoNet in order to capture the patterns of co-occurrence in and across the planes P, P' and P'' shown in Figure 1. The plane P consists of the communities, which are formed of those vowels that have a very high frequency of occurrence (usually *assortative* [19] in nature). We observe that the constituent nodes (read vowels) of these assortative vowel communities are largely uncorrelated in terms of their features and quantitatively show that they indeed exhibit a higher than expected level of perceptual contrast. On the other hand, the communities obtained from VoNet, in which the links between the assortative nodes are absent, corresponds to the co-occurrence patterns of the planes P' and P''. In these communities, strong correlations are reflected among the constituent vowels with respect to their features and they indeed display a significantly better feature economy than it could have been by random chance. Moreover, the co-occurrences across the planes can be captured by the community analysis of VoNet where only the connections between the assortative and the non-assortative nodes, with the non-assortative node co-occurring very frequently with the assortative one, are retained while the rest of the connections are filtered out. We also show that these communities again exhibit a significantly higher feature economy than feasible by chance.

This article is organized as follows: Section 2 describes the experimental setup in order to explore the co-occurrence principles of the vowel inventories. In this section we formally define VoNet, outline its construction procedure, present a community-finding algorithm, and also present a quantitative definition for maximal perceptual contrast as well as feature economy. In section 3 we report the experiments performed to obtain the community structures, which are representative of the co-occurrence patterns in and across the planes discussed above. We also report results where we measure the driving forces that lead to the emergence of such patterns and show that the real inventories are substantially better in terms of this measure than those where the inventories are assumed to have evolved by chance. Finally, we conclude in section 4 by summarizing our contributions, pointing out some of the implications of the current work and indicating the possible future directions.

2. Experimental Setup

In this section we systematically develop the experimental setup in order to investigate the co-occurrence principles of the vowel inventories. For this purpose, we formally define VoNet, outline its construction procedure, describe a community-finding algorithm to decompose VoNet to obtain the community structures, and

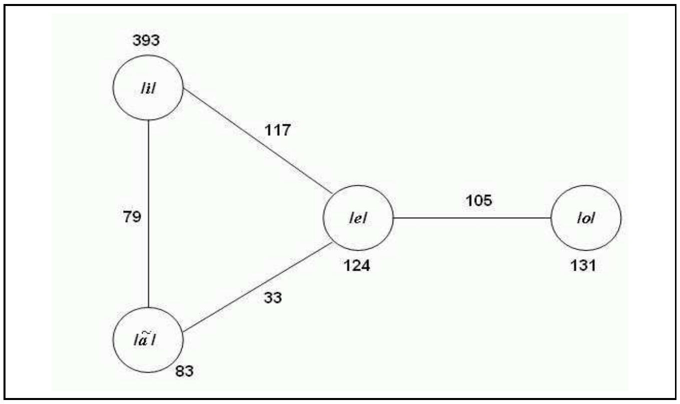


Fig. 2. A partial illustration of the nodes and edges in VoNet. The labels of the nodes denote the vowels represented in IPA (International Phonetic Alphabet). The numerical values against the edges and nodes represent their corresponding weights. For example /*i*/ occurs in 393 languages; /*e*/ occurs in 124 languages while they co-occur in 117 languages.

define the metrics required in order to explore the co-occurrence principles of the observed communities.

2.1. *Definition and Construction of VoNet*

Definition of VoNet: We define VoNet as a network of vowels, represented as G = $\langle$ V_V, E $\rangle$ where V_V is the set of nodes labeled by the vowels and E is the set of edges occurring in VoNet. There is an edge $e \in$ E between two nodes, if and only if there exists one or more language(s) where the nodes (read vowels) co-occur. The weight of the edge e (also *edge-weight*) is the number of languages in which the vowels connected by e co-occur. The weight of a node u (also *node-weight*) is the number of languages in which the vowel represented by u occurs. In other words, if a vowel v_i represented by the node u occurs in the inventory of n languages then the node-weight of u is assigned the value n. Also if the vowel v_j is represented by the node v and there are w languages in which vowels v_i and v_j occur together then the weight of the edge connecting u and v is assigned the value v. Figure 2 illustrates this structure by reproducing some of the nodes and edges of VoNet.

Construction of VoNet: Many typological studies [5, 8, 11, 14, 17, 18] of segmental inventories have been carried out in past on the UCLA Phonological Segment Inventory Database (UPSID) [15]. Currently UPSID records the sound inventories of 451 languages covering all the major language families of the world. In this work we have therefore used UPSID comprising of these 451 languages and 180 vowels found across them, for constructing VoNet. Consequently, the set V_V comprises of 180 elements (nodes) and the set E comprises of 3135 elements (edges). Figure 3 presents a partial illustration of VoNet as constructed from UPSID.

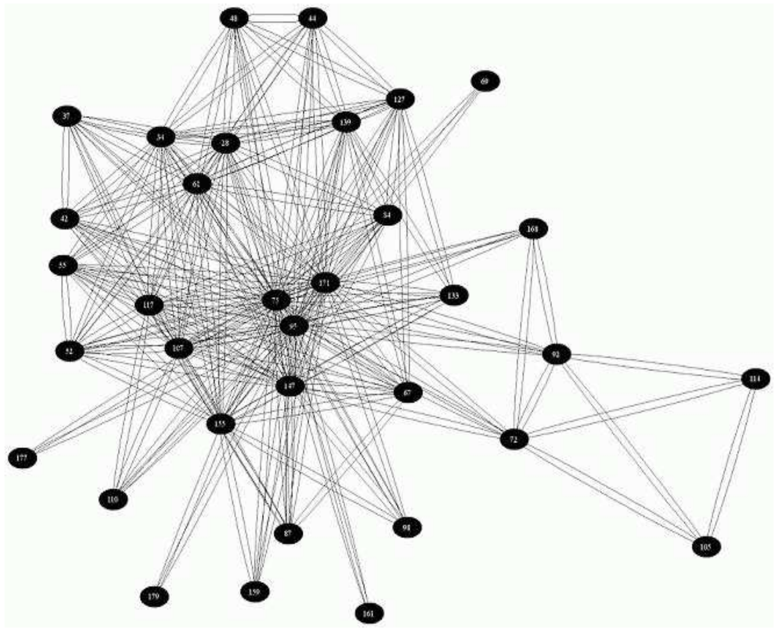


Fig. 3. A partial illustration of VoNet. All edges in this figure have an edge-weight greater than or equal to 15. The number on each node corresponds to a particular vowel. For instance, node number 72 corresponds to /ĩ/.

2.2. *Finding Community Structures*

We attempt to identify the communities appearing in VoNet by the extended Radicchi et al. [20] algorithm for weighted networks as introduced by us in an earlier article [17]. The basic idea is that if the weights on the edges forming a triangle (loops of length three) are comparable then the group of vowels represented by this triangle highly occur together rendering a pattern of co-occurrence while if these weights are not comparable then there is no such pattern. In order to capture this property we define a strength metric S for each of the edges of VoNet as follows. Let the weight of the edge (u,v), where u, $v \in \mathrm{V}_C$, be denoted by w_{uv}. We define S as,

$$S = \frac{w_{uv}}{\sqrt{\sum_{i \in V_C - \{u,v\}} (w_{ui} - w_{vi})^2}} \tag{1}$$

if $\sqrt{\sum_{i \in V_C - \{u,v\}} (w_{ui} - w_{vi})^2} > 0$ else $S = \infty$. The denominator in this expression essentially tries to capture whether or not the weights on the edges forming triangles are comparable (the higher the value of S the more comparable the weights are). The network can be then decomposed into clusters or communities by removing edges that have S less than a specified threshold (say η).

At this point it is worthwhile to clarify the significance of a vowel community. A community of vowels actually refers to a set of vowels which occur together in the language inventories very frequently. In other words, there is a higher than expected probability of finding a vowel v in an inventory which already hosts the other members of the community to which v belongs. For instance, if /i/, /a/ and /u/ form a vowel community and if /i/ and /a/ are present in any inventory then there is a very high chance that the third member /u/ is also present in the inventory.

2.3. *Definition of the Metrics*

Once the communities are obtained through the algorithm discussed earlier the next important task is to analyze them so as to capture the binding force that keeps them together. For this purpose, we need to have a quantitative measure for perceptual contrast as well as feature economy. In order to establish that the above forces really play a role in the emergence of the communities, we also need to compare and show that the communities are much better in terms of this measure than it would have been if the vowel inventories had evolved by chance. In the rest of this section we detail out the metric for quantification as well as the metric for comparison.

2.3.1. *Metric for Quantification*

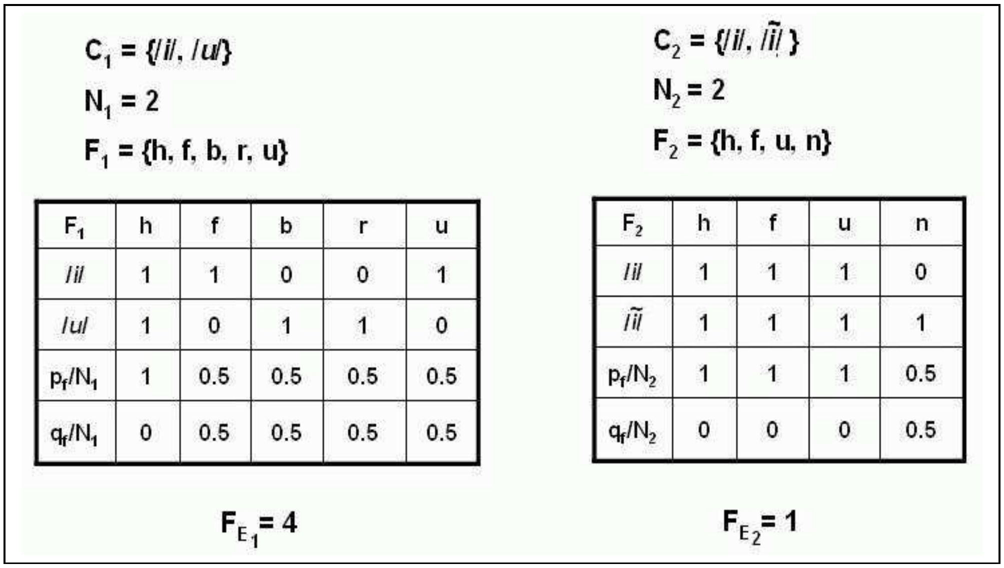
For a community C of size N let there be p_f vowels, which have a particular feature f (where f is assumed to be boolean in nature)[e] in common and q_f other vowels, which lack the feature f. Thus, the probability that a particular vowel chosen uniformly at random from C has the feature f is $\frac{p_f}{N}$ and the probability that the vowel lacks the feature f is $\frac{q_f}{N}$ (=1$-\frac{p_f}{N}$). If F be the set of all features present in the vowels in C then *feature entropy* F_E can be defined as

$$F_E = \sum_{f \in F}(-\frac{p_f}{N}\log\frac{p_f}{N} - \frac{q_f}{N}\log\frac{q_f}{N}) \quad (2)$$

F_E is essentially the measure of the number of bits that are required to communicate the information about the entire community C through a channel.

Capturing Perceptual Contrast: If C comprises of a set of perceptually distinct vowels, then larger number of bits should be required to communicate the information about C over the transmission channel since in this case the set of features that constitute the vowels are more in number. Therefore, the higher the perceptual contrast the higher is the feature entropy. The idea is illustrated through the example in Figure 4. In the figure, F_E exhibited by the community C_1 is higher than that of the community C_2, since the set of vowels in C_1 are perceptually more

[e]There are 28 such boolean features that are found across the vowel systems recorded in UPSID.



$C_1 = \{/i/, /u/\}$
$N_1 = 2$
$F_1 = \{h, f, b, r, u\}$

F_1	h	f	b	r	u
/i/	1	1	0	0	1
/u/	1	0	1	1	0
p_f/N_1	1	0.5	0.5	0.5	0.5
q_f/N_1	0	0.5	0.5	0.5	0.5

$F_{E_1} = 4$

$C_2 = \{/i/, /\tilde{i}/\}$
$N_2 = 2$
$F_2 = \{h, f, u, n\}$

F_2	h	f	u	n
/i/	1	1	1	0
/ĩ/	1	1	1	1
p_f/N_2	1	1	1	0.5
q_f/N_2	0	0	0	0.5

$F_{E_2} = 1$

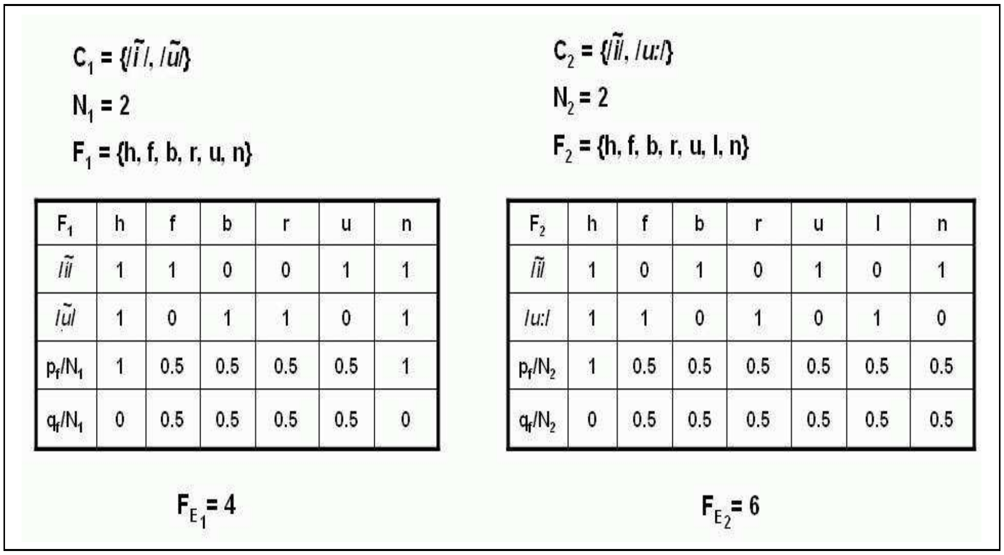
Fig. 4. F_E for the two different communities C_1 and C_2. The letters **h**, **f**, **b**, **r**, **u**, **l**, and **n** stand for the features high, front, back, rounded, unrounded, and nasalized respectively.

distinct than those in C_2.

Capturing Feature Economy: To have more information conveyed using a fewer number of bits, maximization of the combinatorial possibilities of the features used by the constituent vowels in the community C is needed, which is precisely the prediction made by the principle of feature economy. Therefore the lower the feature entropy the higher is the feature economy. In fact, it is due to this reason that in Figure 5, F_E exhibited by the community C_1 is lower than that of the community C_2, since in C_1 the combinatorial possibilities of the features is better utilized by the vowels than in C_2.

2.3.2. *Metric for Comparison*

For the purpose of the comparison as discussed earlier, we construct a random version of VoNet, namely VoNet$_{rand}$. Let the frequency of occurrence for each vowel v in UPSID be denoted by f_v. Let there be 451 bins each corresponding to a language in UPSID. f_v bins are then chosen uniformly at random and the vowel v is packed into these bins. Thus the vowel inventories of the 451 languages corresponding to the bins are generated. In such randomly constructed inventories the effect of none of the forces (perceptual contrast or feature economy) should be prevalent as there is no strict co-occurrence principle that plays a role in the inventory construction. Therefore these inventories should show a feature entropy no better than expected



$C_1 = \{/\tilde{i}/, /\tilde{u}/\}$
$N_1 = 2$
$F_1 = \{h, f, b, r, u, n\}$

F_1	h	f	b	r	u	n
/ĩ/	1	1	0	0	1	1
/ũ/	1	0	1	1	0	1
p_f/N_1	1	0.5	0.5	0.5	0.5	1
q_f/N_1	0	0.5	0.5	0.5	0.5	0

$F_{E_1} = 4$

$C_2 = \{/\tilde{i}/, /u{:}/\}$
$N_2 = 2$
$F_2 = \{h, f, b, r, u, l, n\}$

F_2	h	f	b	r	u	l	n
/ĩ/	1	0	1	0	1	0	1
/u:/	1	1	0	1	0	1	0
p_f/N_2	1	0.5	0.5	0.5	0.5	0.5	0.5
q_f/N_2	0	0.5	0.5	0.5	0.5	0.5	0.5

$F_{E_2} = 6$

Fig. 5. F_E for the two different communities C_1 and C_2. The letters **h**, **f**, **b**, **r**, **u**, **l**, and **n** stand for the features high, front, back, rounded, unrounded, long, and nasalized respectively.

by random chance and hence can act as a baseline for all our experiments reported in the following section. VoNet$_{rand}$ can be then constructed from these new vowel inventories similarly as VoNet. The method for the construction is summarized in Algorithm 1.

Algorithm 1. Algorithm to construct VoNet$_{rand}$

for *each vowel* v
{
  for i = *1 to* f_v
  {
    Choose one of the 451 bins, corresponding to the languages in UPSID, uniformly at random;
    Pack the vowel v into the bin so chosen if it has not been already packed into this bin earlier;
  }
}
Construct VoNet$_{rand}$, similarly as VoNet, from the new vowel inventories (each bin corresponds to a new inventory);

3. Experiments and Results

In this section we describe the experiments performed and the results obtained from the analysis of VoNet. In order to find the co-occurrence patterns in and across the

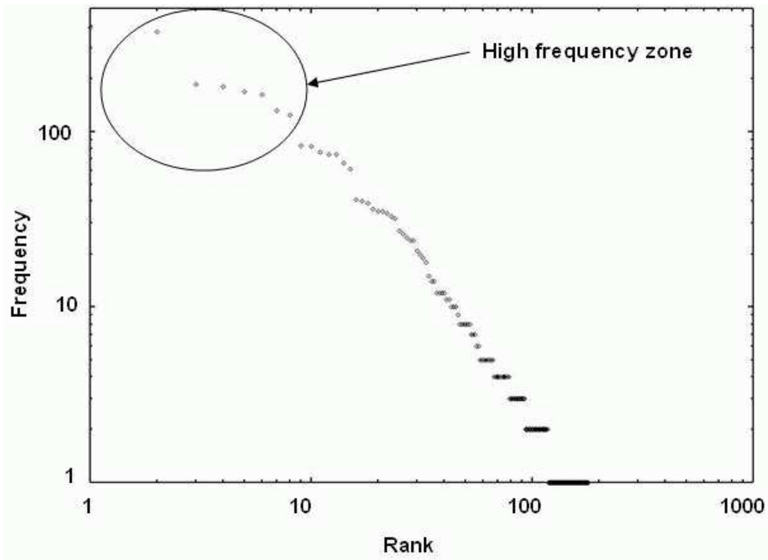


Fig. 6. The frequency (y-axis) versus rank (x-axis) curve in log-log scale illustrating the distribution of the occurrence of the vowels over the language inventories of UPSID.

planes of Figure 1 we define three versions of VoNet namely VoNet$_{assort}$, VoNet$_{rest}$ and VoNet$_{rest'}$. The construction procedure for each of these versions are presented below.

Construction of VoNet$_{assort}$: VoNet$_{assort}$ comprises of the assortative[f] nodes having node-weights above 120 (i.e, vowels occurring in more than 120 languages in UPSID), along with only the edges inter-connecting these nodes. The rest of the nodes (having node-weight less than 120) and edges are removed from the network. We make a choice of this node-weight for classifying the assortative nodes from the non-assortative ones by observing the distribution of the occurrence frequency of the vowels illustrated in Figure 6. The curve shows the frequency of a vowel (y-axis) versus the rank of the vowel according to this frequency (x-axis) in log-log scale. The high frequency zone (marked by a circle in the figure) can be easily distinguished from the low-frequency one since there is distinct gap featuring between the two in the curve.

Figure 7 illustrates how VoNet$_{assort}$ is constructed from VoNet. Presently, the number of nodes in VoNet$_{assort}$ is 9 and the number of edges is 36.

Construction of VoNet$_{rest}$: VoNet$_{rest}$ comprises of all the nodes as that of VoNet. It also has all the edges of VoNet except for those edges that inter-connect

[f]The term "assortative node" here refers to the nodes having a very high node-weight.

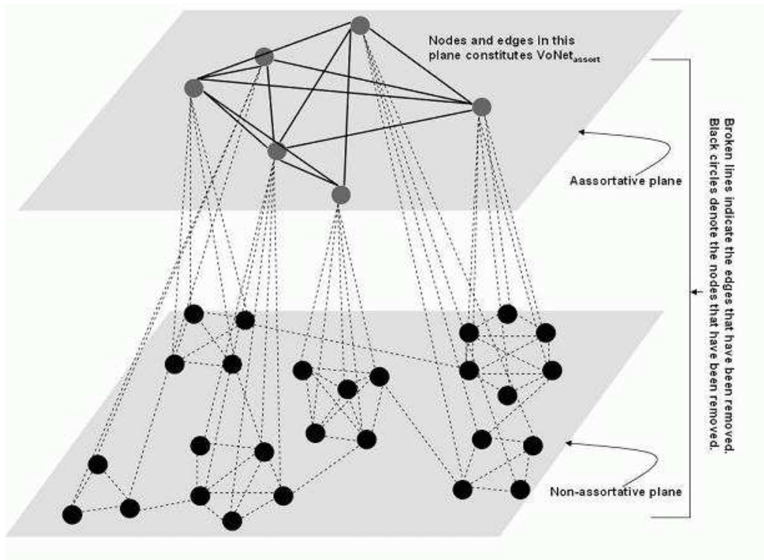


Fig. 7. The construction procedure of VoNet$_{assort}$ from VoNet.

the assortative nodes. Figure 8 shows how VoNet$_{rest}$ can be constructed from VoNet. The number of nodes and edges in VoNet$_{rest}$ are 180 and 1293[g] respectively.

Construction of VoNet$_{rest'}$: VoNet$_{rest'}$ again comprises of all the nodes as that of VoNet. It consists of only the edges that connect an assortative node with a non-assortative one if the non-assortative node co-occurs more than ninety five percent of times with the assortative nodes. The basic idea behind such a construction is to capture the co-occurrence patterns based on robustness [6] (discussed earlier in the introductory section) that actually defines the cross-planar relationships in Figure 1. Figure 9 shows how VoNet$_{rest'}$ can be constructed from VoNet. The number of nodes in VoNet$_{rest'}$ is 180 while the number of edges is 114[h].

We separately apply the community-finding algorithm (discussed earlier) on each of VoNet$_{assort}$, VoNet$_{rest}$ and VoNet$_{rest'}$ in order to obtain the respective vowel communities. We can obtain different sets of communities by varying the threshold η. A few assortative vowel communities (obtained from VoNet$_{assort}$) are noted in Table 1. Some of the communities obtained from VoNet$_{rest}$ are presented in Table 2. We also note some of the communities obtained from VoNet$_{rest'}$ in Table 3.

[g]We have neglected nodes with node-weight less than 3 since these nodes correspond to vowels that occur in less than 3 languages in UPSID and the communities they form are therefore statistically insignificant.

[h]The network does not get disconnected due to this construction since, there is always a small fraction of edges that run between assortative and low node-weight non-assortative nodes of otherwise disjoint groups.

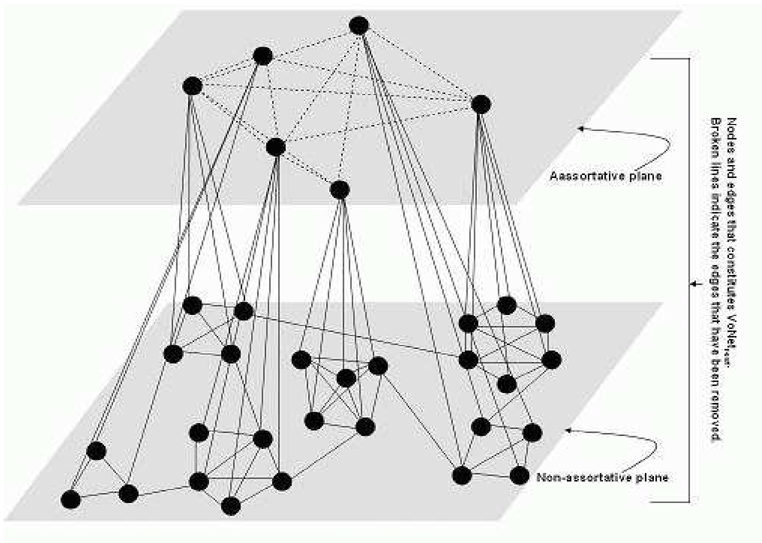


Fig. 8. The construction procedure of VoNet$_{rest}$ from VoNet.

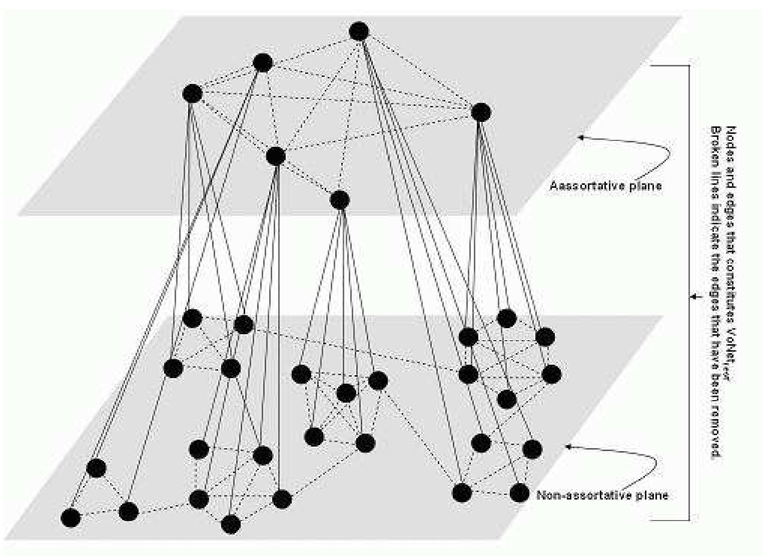


Fig. 9. The construction procedure of VoNet$_{rest'}$ from VoNet.

Tables 1 , 2 and 3 indicate that the communities in VoNet$_{assort}$ are formed based on the principle of perceptual contrast whereas the formation of the communities in VoNet$_{rest}$ as well as VoNet$_{rest'}$ is largely governed by feature economy. We dedicate the rest of this section mainly to verify the above argument. For this reason we

Table 1. Assortative vowel communities. The contrastive features separated by slashes (/) are shown within parentheses. Comma-separated entries represent the features that are in use from the three respective classes namely the height, the backness, and the roundedness.

Community	Features in Contrast
/i/, /a/, /u/	(low/high), (front/central/back), (unrounded/rounded)
/e/, /o/	(higher-mid/mid), (front/back), (unrounded/rounded)

Table 2. Some of the vowel communities obtained from VoNet$_{rest}$.

Community	Features in Common
/ĩ/, /ã/, /ũ/	nasalized
/ĩː/, /ãː/, /ũː/	long, nasalized
/iː/, /uː/, /aː/, /oː/, /eː/	long

Table 3. Some of the vowel communities obtained from VoNet$_{rest'}$. Comma-separated entries represent the features that are in use from the three respective classes namely the height, the backness, and the roundedness.

Community	Features in Common
/i/, /ĩ/	high, front, unrounded
/a/, /ã/	low, central, unrounded
/u/, /ũ/	high, back, rounded

present a detailed study of the co-occurrence principles of the communities obtained from VoNet$_{assort}$, VoNet$_{rest}$, and VoNet$_{rest'}$. In each case we compare the results with those of VoNet$_{rand}$ obtained from Algorithm 1.

3.1. *Co-occurrence Principles of the Communities of VoNet$_{assort}$*

We apply the community-finding algorithm (discussed earlier) on VoNet$_{rand}$ in order to obtain the assortative communities similarly as outlined for VoNet. Figure 10 illustrates, for all the communities obtained from the clustering of VoNet$_{assort}$ and its random version, the average feature entropy exhibited by the communities of a particular size[i] (y-axis) versus the community size (x-axis).

A closer inspection of Figure 10 immediately reveals that the feature entropy exhibited by the communities of VoNet$_{assort}$ is higher as compared to the random version of the same. The two curves finally intersect due to the formation of a single giant component, which is similar for the real and the random edition of VoNet$_{assort}$. Nevertheless, the data points that appear on these curves are fairly

[i]Let there be n communities of a particular size k picked up at various thresholds. The average feature entropy of the communities of size k is therefore $\frac{1}{n}\sum_{i=1}^{n} F_{E_i}$ where F_{E_i} signifies the feature entropy of the i^{th} community.

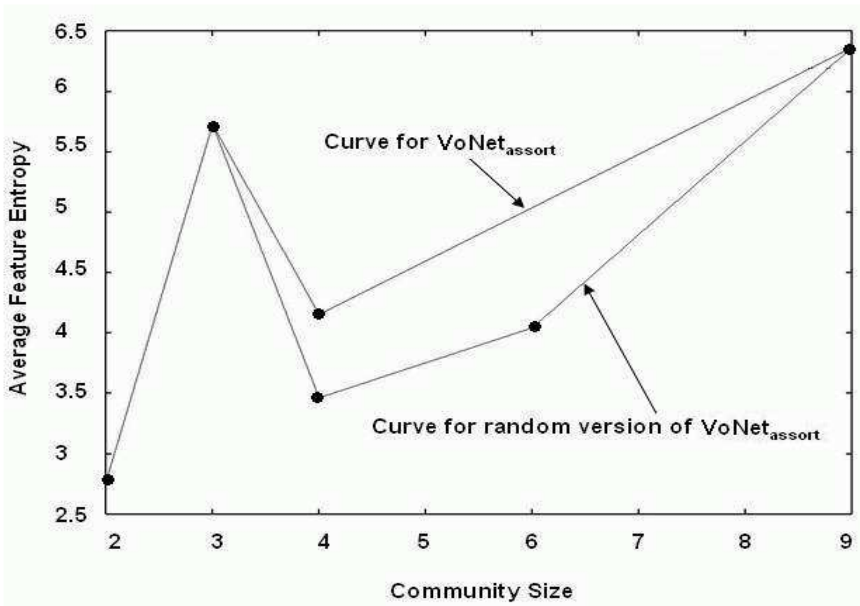


Fig. 10. Curves showing the average feature entropy of the communities of a particular size versus the community size for VoNet$_{assort}$ as well as its random counterpart.

less in number and hence Figure 10 alone is not sufficient enough to establish that the communities in VoNet$_{assort}$ are formed based on the principle of perceptual contrast. Another possible way to investigate the problem would be to look into the co-occurrence principles of the smaller vowel inventories (of size ≤ 4) since they mostly comprise of the members belonging to the assortative vowel communities. Table 4 for instance, shows the number of occurrences of the members of the community formed by /i/, /a/, and /u/, as compared to the average occurrence of other vowels, in the inventories of size 3 and 4. The figures in the table points to the fact that the smaller inventories can be assumed to be good representatives of the assortative vowel communities. We therefore compare the average feature entropy of these inventories as a whole with their random counterparts (obtained from Algorithm 1). Figure 11 illustrates the result of this comparison. The curves depict the average feature entropy of the vowel inventories of a particular size (y-axis) versus the inventory size (x-axis). The two different plots compare the average feature entropy of the inventories obtained from UPSID with that of the randomly constructed ones. The figure clearly shows that the average feature entropy of the vowel inventories of UPSID is substantially higher for inventory size 3 and 4 than that of those constructed randomly.

The results presented in Figures 10 and 11 together confirms that the assortative vowel communities are formed based on the principle of maximal perceptual contrast.

Table 4. Frequency of occurrence of the members of the community /*i*/, /*a*/, and /*u*/, as compared to the frequency occurrence of other vowels, in smaller inventories. The last column indicates the average number of times that a vowel other than /*i*/, /*a*/, and /*u*/ occurs in the inventories of size 3 and 4.

Inv. Size	No. of Invs.	Occ. /i/	Occ. /a/	Occ. /u/	Avg. Occ. other vowels
3	23	15	21	12	3
4	25	19	24	11	3

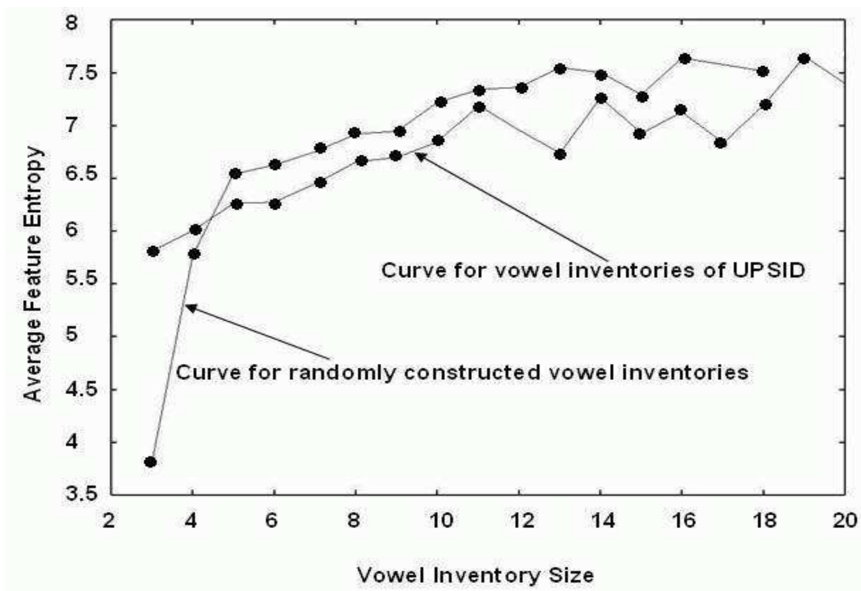


Fig. 11. Curves showing the average feature entropy of the vowel inventories of a particular size versus the inventory size. The two different plots compare the average feature entropy of the inventories obtained from UPSID with that of the randomly constructed ones.

3.2. *Co-occurrence Principles of the Communities of VoNet$_{rest}$*

In this section, we investigate whether or not the communities obtained from VoNet$_{rest}$ are better in terms of feature entropy than they would have been, if the vowel inventories had evolved just by chance. We construct the random edition of VoNet$_{rest}$ from VoNet$_{rand}$ and apply the community-finding algorithm on it so as to obtain the communities. Figure 12 illustrates, for all the communities obtained from the clustering of VoNet$_{rest}$ and its random version, the average feature entropy exhibited by the communities of a particular size (y-axis) versus the community size (x-axis). The curves in the figure makes it quite clear that the average feature entropy exhibited by the communities of VoNet$_{rest}$ are substantially lower than that of their random counterpart (especially for a community size ≤ 7). As the community size increases, the difference in the average feature entropy of

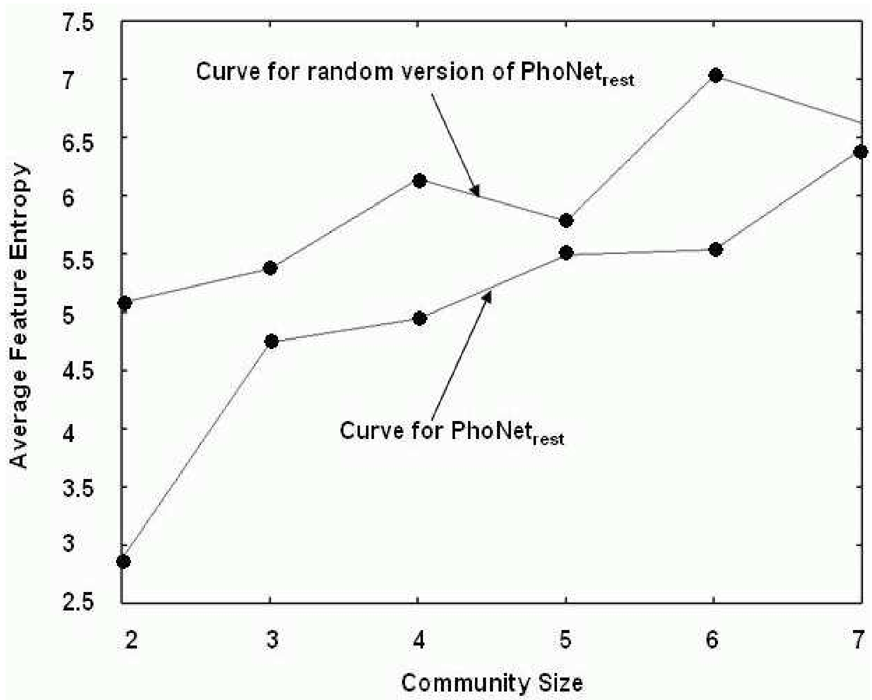


Fig. 12. Curves showing the average feature entropy of the communities of a particular size versus the community size for VoNet$_{rest}$ as well as its random counterpart.

the communities of VoNet$_{rest}$ and its random version gradually diminishes. This is mainly because of the formation of a single giant community, which is similar for the real and the random versions of VoNet$_{rest}$.

The above result indicate that the driving force behind the formation of the communities of VoNet$_{rest}$ is the principle of feature economy. It is important to mention here that the larger vowel inventories, which are usually comprised of the communities of VoNet$_{rest}$, also exhibit feature economy to a large extent. This is reflected through Figure 11 where all the real inventories of size ≥ 5 have a substantially lower average feature entropy than that of the randomly generated ones.

3.3. *Co-occurrence Principles of the Communities of VoNet$_{rest'}$*

In this section we compare the feature entropy of the communities obtained from VoNet$_{rest'}$ with that of its random counterpart (constructed from VoNet$_{rand}$). Figure 13 shows the the average feature entropy exhibited by the communities of a particular size (y-axis) versus the community size (x-axis) for both the real and the random version of VoNet$_{rest'}$. The curves in the figure makes it quite clear that the average feature entropy exhibited by the communities of VoNet$_{rest'}$ are substan-

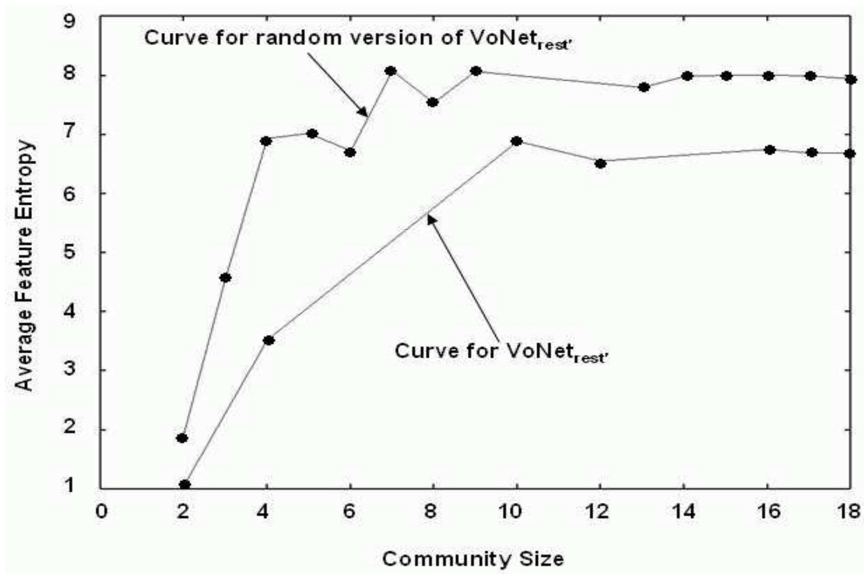


Fig. 13. Curves showing the average feature entropy of the communities of a particular size versus the community size for VoNet$_{rest'}$ as well as its random counterpart.

tially lower than that of the random ones. This result immediately reveals that it is again feature economy that plays a key role in the emergence of the communities of VoNet$_{rest'}$.

4. Conclusion

In this paper we explored the co-occurrence principles of the vowels, across the inventories of the world's languages. In order to do so we started with a concise review of the available literature on vowel inventories. We proposed an automatic procedure to capture the co-occurrence patterns of the vowels across languages. We also discussed the notion of feature entropy, which immediately allows us to validate the explanations of the organizational principles of the vowel inventories furnished by the earlier researchers.

Some of our important findings from this work are,

- The smaller vowel inventories (corresponding to the communities of VoNet$_{assort}$) tend to be organized based on the principle of maximal perceptual contrast;
- On the other hand, the larger vowel inventories (mainly comprising of the communities of VoNet$_{rest}$) reflect a considerable extent of feature economy;
- Co-occurrences based on robustness are prevalent across vowel inventories (captured through the communities of VoNet$_{rest'}$) and their emergence is again a consequence of feature economy.

Until now, we have mainly emphasized on analyzing the co-occurrence principles of the vowel inventories of the world's languages. An issue that draws attention is how the forces of perceptual contrast and feature economy have interacted causing the emergence of the human vowel systems. One possible way to answer this question is by having a growth model for the network, where the growth takes place owing to the optimization of a function (see [4] for a reference), which involves the above forces and also accounts for the observed regularities displayed by the vowel inventories. It would be worthwhile to mention here that though most of the mechanisms of network growth rely on preferential attachment-based rules [1], yet there are scenarios which suggest that additional optimizing constraints need to be imposed on the evolving network so as to match its emergent properties with empirical data [23, 24]. Such a growth model based on some optimization technique can then shed enough light on the real dynamics that went on in the evolution of the vowel inventories. We look forward to develop the same as a part of our future work.

References

[1] Barabási, A.-L. and Albert, R., Emergence of scaling in random networks, *Science* **286**, 509-512, (1999).

[2] de Boer, B., Self-organisation in vowel systems, *Journal of Phonetics*, **28**(4), 441–465, (2000).

[3] Boersma, P., *Functional phonology*, Doctoral thesis, University of Amsterdam, The Hague: Holland Academic Graphics, (1998).

[4] Ferrer i Cancho, R. and Solé, R. V., Optimization in complex networks, *arXiv:cond-mat/0111222*, (2001).

[5] Choudhury, M., Mukherjee, A., Basu, A. and Ganguly, N., Analysis and synthesis of the distribution of consonants over languages: A complex network approach, *Proceedings of COLING–ACL*, 128–135, Sydney, Australia, (2006).

[6] Clements, N., Features and sound inventories, *Symposium on Phonological Theory: Representations and Architecture*, CUNY, (2004).

[7] de Groot, A. W., Phonologie und Phonetik als funktionswissenschaften, *Travaux du Cercle Linguistique de*, **4**, 116–147, (1931).

[8] Hinskens, F. and Weijer, J., Patterns of segmental modification in consonant inventories: A cross-linguistic study, *Linguistics*, **41**, 6. (2003).

[9] Jakobson, R., *Kindersprache, aphasie und allgemeine lautgesetze*, (Uppsala, 1941), reprinted in *Selected Writings I. Mouton*, (The Hague, 1962), 328-401.

[10] Ke, J., Ogura, M., and Wang, W.S.-Y., Optimization models of sound systems using genetic algorithms, *Computational Linguistics*, **29**(1), 1–18, (2003).

[11] Ladefoged, P. and Maddieson, I., *Sounds of the worlds languages*, (Oxford: Blackwell, 1996).

[12] Liljencrants, J. and Lindblom, B., Numerical simulation of vowel quality systems: the role of perceptual contrast, *Language*, **48**, 839–862, (1972).

[13] Lindblom, B., Phonetic universals in vowel systems, *Experimental Phonology*, 13–44, (1986).

[14] Lindblom, B. and Maddieson, I., Phonetic universals in consonant systems, *Language, Speech, and Mind*, Routledge, London, 62–78, (1988).

[15] Maddieson, I., *Patterns of sounds*, (Cambridge University Press, Cambridge, 1984.)

[16] Martinet, A., *Ѐconomie des changements phonétiques*, (Berne: A. Francke, 1955).
[17] Mukherjee, A., Choudhury, M., Basu, A. and Ganguly, N., Modeling the co-occurrence principles of the consonant inventories: A complex network approach, *arXiv:physics/0606132 (preprint)*, (2006).
[18] Mukherjee, A., Choudhury, M., Basu, A. and Ganguly, N., Self-organization of the Sound Inventories: Analysis and Synthesis of the Occurrence and Co-occurrence Networks of Consonants. *arXiv:physics/0610120 (preprint)*, (2006).
[19] Newman, M. E. J., The structure and function of complex networks, *SIAM Review*, **45**, 167–256, (2003).
[20] Radicchi, F., Castellano, C., Cecconi, F., Loreto, V., and Parisi, D., Defining and identifying communities in networks, *PNAS*, **101**(9), 2658–2663, (2003).
[21] Schwartz, J-L., Boë, L-J., Vallée, N., and Abry, C., The dispersion-focalization theory of vowel systems, *Journal of Phonetics*, *25*, 255–286. (1997).
[22] Shannon, C. E., and Weaver, W., *The mathematical theory of information*, (Urbana: University of Illinois Press, 1949).
[23] Solé, R. V., Pastor-Satorras, R., Smith, E. and Kepler, T., A model of large-scale proteome evolution, *Adv. Complex Syst.*, **5**, 43–54, (2002).
[24] Vázquez, A., Flammini, A., Maritan, A., and Vespignani, A., Modeling of protein interaction networks, *Complexus*, **1**, 38–44, (2003).
[25] Wang, W. S.-Y., The basis of speech. Project on linguistic analysis reports, (University of California, Berkeley, 1968), reprinted in *The Learning of Language*, (1971).