Modeling Endangered Languages:

The Effects of Bilingualism and Social Structure

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A recently developed dynamical system for modeling language endangerment allows the evolution of the numbers of monolingual speakers of two competing languages to be estimated. In this paper, we extend the model to examine the role of bilingualism and social structure, neither of which are addressed in the previous model. We adopt a simple strategy for language maintenance — the society is assumed to enhance the status of an endangered language whenever the number of monolingual speakers of it falls too low — to estimate the peak probability that both languages can be maintained, and to show the period within which such maintenance is possible at all. We show for social structures that are not fully connected that as the neighborhood of each speaker in the community becomes more localized, so the probability that maintenance can be achieved is increased.

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1. Introduction

The 6,000 or so languages spoken on our planet today are the products of numerous millennia of cultural evolution. They encapsulate the experience and knowledge of diverse peoples collected in widely different environments, and are a precious part of the human heritage. With the explosive expansion of a few dominant languages in recent decades, the great majority of languages are critically endangered in that they will soon have no speakers and become extinct (Krauss, 1992). Indeed, Pagel (1995) estimates that roughly 140,000 languages have ever existed (median estimate); thus it is the fate of the majority of languages to become extinct. Fishman (1991) argues that death of a language generally leads to death of the underlying culture to which it is linked. It is therefore an important challenge to understand such situations as precisely as possible, and to recognize if there are measures that can help us preserve some of this heritage.

Much work has been carried out on both theoretical and empirical issues of achieving language maintenance as evidenced by the numerous volumes on the subject (e.g. Fishman, 1991; Grenoble & Whaley, 1998; Crystal, 2000; Nettle & Romaine, 2000; Fishman, 2001; Bradley & Bradley, 2002). Fishman (2001:1) begins his recent volume on language maintenance by stating:

> What the smaller and weaker languages (and peoples and cultures) of the world need are not generalized predictions of dire and even terminal illnesses but, rather, the development of therapeutic understandings and approaches that can be adjusted so as to tackle essentially the same illness in patient after patient.

Toward this end, Abrams and Strogatz (2003) have recently suggested a quantitative perspective to this issue, proposing a dynamical system model for language death. The model predicts that whenever two languages compete for speakers, one language will eventually become extinct unless action is taken to enhance its status. Although the model appears to obtain good results when fitted to data for several endangered languages, it does not account for either bilingual individuals or the structure of the society in which the languages compete. Despite these shortcomings, the model does point the direction for a quantitative study of the effects of language maintenance strategies, a direction that we pursue here.

In this paper, we introduce an extension of the Abrams and Strogatz model that explicitly models bilingualism; the extended model also predicts that language death is inevitable, although the trajectories leading to this state differ. We then implement a multi-agent simulation of the system to investigate the stochastic nature of language death, estimating the probabilities that endangered languages can be maintained in various scenarios. In particular, we demonstrate circumstances under which a pair of competing languages can be maintained in a population comprising both monolingual speakers of each language and bilingual speakers. We also examine the role of social structure on the probabilities of maintenance, representing the social structure by a local-world network (Li & Chen, 2003), a recently developed network paradigm that we use to encapsulate individuals' lack of global knowledge of either the social structure or the language usage patterns of the other individuals comprising it.

The paper is laid out as follows: In Section 2, we discuss deterministic models of language death, briefly describing first the Abrams and Strogatz model for a population in which two languages compete for speakers. We then introduce an extension of the model that incorporates modeling of bilingualism. In Section 3, we go on investigate a simple strategy for language maintenance that is based on this deterministic model, showing circumstances in which an intervention to increase the status of an endangered language can bring about its maintenance. The deterministic models discussed in Sections 2 and 3 allow us to determine the most likely final state of the system. However they tell us little about the probability that the system converges to each of the competing states. Therefore, in Section 4 we implement a multi-agent simulation of the extended model to estimate the probabilities of convergence. We then investigate the effects of social structure on the behavior of the system in Section 5. The paper concludes in Section 6.

2. Deterministic Models of Language Death

2.1 The Monolingual Model of Abrams and Strogatz (2003)

Abrams and Strogatz (2003) have recently presented a simple dynamical system model of language death for a population in which two languages, X and Y, compete for speakers. The model allows the evolution of the system to be predicted, potentially allowing languages that are endangered to be identified at an early stage and appropriate action to maintain them planned.

In deriving their deterministic model, Abrams and Strogatz make a number of simplifying assumptions, in particular:

- each speaker is monolingual in either X or Y;
- the population has no underlying social structure.

In almost all cases, however, a population in which two languages compete for speakers will include some bilinguals who interact with the monolingual speakers of both languages. In this context, the first assumption can be interpreted as bilinguals having negligible influence on the language usage patterns of monolinguals. However, it seems likely that an endangered language has a greater chance of being maintained the more speakers it has, whether monolingual or bilingual. The second assumption means that each speaker has perfect knowledge of the language usage patterns of each other speaker. However, it is well known that societies organize themselves into clusters of relatively dense connectivity, with knowledge of the language usage patterns of others speakers distributed only locally. In the course of this paper, we will refine these two assumptions to render the model more realistic.

Abrams and Strogatz also assume that each language remains unchanged over time — in other words, language change is assumed to have negligible effect on the language usage patterns of monolingual speakers. Also, their model incorporates neither spatial distribution nor age distribution of its speakers. We do not address these issues in this paper.

The functional form of the dynamics of the system come from the further assumption that speakers adopt a language at a rate proportional to its attractiveness, which increases with both the proportion of speakers of that language and its perceived status. Writing the proportions of speakers of X and Y as $0 \le x \le 1$ and $0 \le y \le 1$, respectively, with $x+y=1$, and the status of language X as $0 \le s \le 1$ (the status of language Y being $1-s$), the attractiveness, $P_{\mathrm{YX}}(x,s)$, of language X to speakers of Y indicates the rate at which speakers of Y switch to speak X. The rate at which speakers of X switch to speak Y is denoted similarly by $P_{\mathrm{XY}}(x,s)$, which, by symmetry, is equal to $P_{\mathrm{YX}}(1-x,1-s)$. The dynamics of the model of Abrams and Strogatz are given by

$$\frac{dx}{dt} = y\,P_{\mathrm{YX}}(x,s) - x\,P_{\mathrm{XY}}(x,s). \tag{1}$$

Setting $P_{\mathrm{YX}}(x,s)$ to be a monotonic increasing function of both x and s, and assuming that no speaker of Y switches to speak X either when X has no speakers (i.e., $x=0$) or when X has zero status (i.e., $s=0$), the system has three equilibria, two of which are stable[1]: $x=0$ and $x=1$. In other words, the model implies that one language will eventually acquire all the speakers in the population, causing the language with which it is competing to become extinct.

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[1] Informally, an equilibrium, or fixed point, of a dynamical system is a state of the system that, when attained, remains unchanged over time. Hence, an equilibrium is a state of the system for which the rate of change is zero. An equilibrium is said to be stable when the system, after a small perturbation from the equilibrium state, returns back to that state. We shall refer to equilibria that are not stable as unstable. The set of all initial states from which the system converges to a particular stable equilibrium is called the basin of attraction of that equilibrium. More formal definitions of these and other terms relating to dynamical systems can be found, for example, in (Strogatz, 1995).

Abrams and Strogatz go on to suggest a specific functional form for the attractiveness, $P_{\mathrm{YX}}(x,s) = c\,x^a\,s$, reducing the model to

$$\frac{dx}{dt} = c\left[(1-x)x^a\,s - x(1-x)^a\,(1-s)\right], \tag{2}$$

where c>0 adjusts the overall rate of change of the system, and the exponent a≥0 controls the impact of the proportion of speakers of a language on its attractiveness. They fit the model to diachronic census data collected for three endangered languages: Scottish Gaelic, Welsh and Quechua. In each case, the resultant curve shows a close fit to the diachronic data; it remains to be shown, however, that the curves can be extrapolated to predict future behavior. The paper closes with the comment that maintenance of the two competing languages can be achieved by controlling the status, s, of the endangered language appropriately; we discuss the issue of language maintenance in Section 3.

2.2 An Extended Model incorporating Bilingualism

The monolingual model of Abrams and Strogatz appears to work very well in modeling changes in the patterns of language usage within a population in which two languages compete. However, in practice, we observe that typically a speaker does not suddenly give up one language completely in favor of another — it is extremely rare, for example, for children to loose the ability to communicate with their parents. Almost always, speakers will maintain the language acquired from their parents and, perhaps, learn additional languages to various degrees, particularly while young. Such speakers may switch languages back and forth, depending on the context, whether it be home, school, or workplace. The nature and extent of the bilingualism will depend on a variety of societal factors. Furthermore, as an intermediate step toward switching language, speakers attain a second language most often by interacting with bilingual speakers. We therefore believe that the incorporation of bilingualism is essential for realistic modeling of language death. To build this into the model that we now propose, we assume that there is a third class of speakers, Z, who are bilingual.

We extend the model of Abrams and Strogatz by assuming that monolingual speakers may acquire the competing language, so becoming bilingual, but may not switch language directly. We model the attractiveness of being bilingual by considering the gain conferred to a monolingual speaker who becomes bilingual. Suppose, for example, a monolingual speaker of Y becomes bilingual. In addition to being able to communicate with both Y monolinguals and Z bilinguals, the speaker can now also communicate with X monolinguals. We therefore assume that the attractiveness of switching from being monolingual in Y to being bilingual is an increasing function of the proportion, x, of monolingual speakers of X as well as of the perceived status, s, of X. Thus we assume that monolingual speakers of Y acquire the competing language X to become bilingual at the rate $P_{\mathrm{YZ}}(x,s)$, similar to the function $P_{\mathrm{YX}}(x,s)$ in the Abrams and Strogatz model.

On the contrary, bilingual speakers may become monolingual by giving up one language when few monolingual speakers of that language remain or when its status is low. So, for example, a bilingual speaker who gives up language X to become monolingual in Y gives up the ability to communicate with X monolinguals. For a given status, s, the attractiveness of doing so will be maximal when no other X monolinguals remain and minimal when all other speakers are monolingual in X. Hence we assume that bilingual speakers give up language X to become monolingual in Y at some rate $Q_{\mathrm{ZY}}(1-x,1-s)$, which decreases monotonically with both x and s.

We obtain the following model for language change:

$$
\begin{aligned}
\frac{dx}{dt} &= z\,Q_{\mathrm{ZX}}(1-y,s) - x\,P_{\mathrm{XZ}}(y,1-s) \\
\frac{dy}{dt} &= z\,Q_{\mathrm{ZY}}(1-x,1-s) - y\,P_{\mathrm{YZ}}(x,s)
\end{aligned}
\tag{3}
$$

where x, y and z are the proportions of monolingual speakers of X and Y, and of bilinguals, respectively, with $0 \leq x,y,z \leq 1$ and $x+y+z=1$.

We assume the same functional form for the rate of acquiring an additional language as Abrams and Strogatz assume for switching language, i.e., $P_{YZ}(x,s) = cx^a s$. Adopting this same form for the rate of transition from a bilingual state to a monolingual state, but with the constant of proportionality set to $c'>0$, i.e., $Q_{ZY}(1-x,1-s) = c'(1-x)^a(1-s)$, we obtain the dynamical system model

$$\begin{aligned} \frac{dx}{dt} &= c'(1-x-y)(1-y)^a s - c\,x\,y^a(1-s) \\ \frac{dy}{dt} &= c'(1-x-y)(1-x)^a(1-s) - c\,y\,x^a s \end{aligned} \tag{4}$$

The rates of transitions between states X, Y and Z are summarized in Figure 1.

[Figure 1 about here]

The resultant system has three equilibria, U*, X* and Y*, as indicated in the direction fields[2] displayed in Figure 2 for several values of parameter a: 0.5, 1.0, 1.5 and 2.0 (s=0.4, c=0.1, c'=0.1). Equilibrium X* is located at x=1, y=0, z=0 for all values of a (except the degenerate case a=0), and indicates death of language Y. Likewise, Equilibrium Y*, located at x=0, y=1, z=0, indicates death of language X. The position of the third equilibrium, U*, however, varies according to the values of parameters a, s, c and c'. While a full analysis of the stability of the system is beyond the scope of this paper, we observe that X* and Y* are stable, and U* is unstable for all $a \geq 1$ (Figures 2b–d). For certain values of $a<1$ the stability of the equilibria may be reversed, with X* and Y* unstable, and U* stable, leading to maintenance of a bilingual

[2] The direction field of a dynamical system is a diagram that indicates the direction of change of the system at various states. Trajectories of the system are tangential to the slope at each state, and so are easily inferred.

society — such is the case for a=0.5, shown in Figure 2a. However, based on diachronic data collected for several endangered languages, Abrams and Strogatz estimate the value of parameter a in their model to lie in the range 1.31 ± 0.25 (Abrams & Strogatz, 2003); thus the attractiveness of each language grows faster than linearly with respect to its number of speakers. In the experiments that follow we therefore assume that the value of a is not less than 1, and conclude that maintenance of the two competing languages is impossible without intervention.

[Figure 2 about here]

Figure 3 shows the direction field for several values of the status s: 0.2, 0.3, 0.4 and 0.5 (a=1.0, c=0.1, c'=0.1). As the status of language X is reduced, so the unstable equilibrium U* moves closer to the stable equilibrium X*. The directions fields show clearly that a language with low status must have many speakers for it to be maintained without intervention. For example, for s=0.2 (Figure 3a), U* is positioned at x=.988, y=.001 — for a population of 1,000 speakers, this corresponds to language Y having just 12 speakers: 1 monolingual plus 11 bilingual. If more than these numbers of monolingual and bilingual speakers of Y were present in the population, X would eventually die. For s=0.5 (Figure 3d), U* is positioned at x=0.632, y=.172; hence a far greater range of initial states will lead to X being maintained (albeit at the expense of death of Y).

[Figure 3 about here]

The values of c and c' adjust the relative rates of transition between the monolingual and bilingual states. c controls the rate of at which monolinguals adopt a second language to become bilingual. Increasing the value of c relative to c' shifts the position of the unstable equilibrium U* away from the curve x+y=1, which indicates that no bilinguals are present in the population. Our experiments suggest that minor differences in the magnitudes of c and c' (for example c=0.2 and c'=0.05) produce no qualitative change in the stability of the system.

We conclude then that for $a \geq 1$, one of the two competing languages will eventually acquire all the speakers, and the other language will die out, as predicted by the Abrams and Strogatz model.

3. Modeling Language Maintenance

It is well known that maintenance of an endangered language can sometimes be achieved by top down processes like legislation or by bottom up movements like ethnic pride. Fishman has proposed the 8-stage Graded Intergenerational Disruption Scale (GIDS) by which the prospects for maintaining an endangered language can be assessed (Fishman, 1991; Fishman 2001). The GIDS can be used to identify the contexts in which the endangered language is spoken, ranging from nationwide usage throughout the mass media and governmental operations, to the language being taught in literacy schools but not compulsorily, to sparse usage by socially isolated elderly people, with partial reconstruction of the language perhaps required. The GIDS therefore allows language maintenance strategies to be focused on appropriate tasks.

In a similar vein, Crystal (2000) has identified six main mechanisms of intervention by which a language may be maintained:

- Increasing the prestige of its speakers;
- Increasing the wealth of its speakers;
- Increasing the power of its speakers;
- Improving its presence in the educational system;
- Ensuring that the language can be written down;
- Providing access to electronic technology to its speakers.

Such interventions may very well create a stable state in which a set of competing languages are all maintained. For some cases, this may be a desirable situation in which the endangered language is preserved.

To incorporate the modeling of language maintenance into the system, we consider the effect on the system of adjusting the status of the endangered language. We make no attempt here to determine an optimal strategy for language maintenance — there is little value in doing so since the relationships between the various maintenance mechanisms and the status of a language are unknown. Rather, we investigate conditions under which intervention that brings about an increase in the status of an endangered language can lead to that language being maintained. In this context, we refer to a language as endangered at some time instant if the state of the system at that time is located within the basin of attraction of the equilibrium that corresponds to the death of that language — in other words, an endangered language is one that the model predicts will eventually die out without action being taken to maintain it.

We set the status, $s(x)$, of an endangered language X to be a function of the proportion of monolingual speakers, x, of that language. For simplicity, we assume that a community will intervene to maintain X by enhancing its status whenever its proportion of speakers falls below some threshold:

$$s(x) = \begin{cases} s_0 & : \; x \geq th_x \\ s_0 + \delta s & : \; x < th_x \end{cases} \tag{5}$$

where $0 \leq th_x \leq 1$ is the threshold proportion of speakers below which the community intervenes to maintain X, $0 \leq s_0 \leq 1$ is the status of X prior to intervention, and $0 \leq s_0 + \delta s \leq 1$ is the enhanced status of X after intervention. A graph of the status function is shown in Figure 4. Larger values of δs model stronger, potentially more effective intervention, while larger values of th_x model earlier intervention. Note that setting either δs or th_x to zero corresponds to no intervention. Although, in practice, significant enhancement of the status of a language cannot be brought about instantaneously, a smoother functional form for $s(x)$ that models a less abrupt increase in status, such as that indicated by the dashed line in Figure 4, gives rise to the same qualitative behavior as we now describe.

[Figure 4 about here]

Figure 5a shows an example of the direction field of the resultant system with intervention (s=0.4; δs=0.2; th_x=0.3). Comparing this figure to Figure 5b, which corresponds to the system with no intervention, we observe that two additional equilibria have been introduced: an unstable equilibrium, V*, and a stable equilibrium, Z*. This third stable equilibrium corresponds to a state in which both languages are maintained, with some speakers bilingual. This change in the qualitative behavior of the system comes about because the dynamics differ depending on whether or not the proportion of speakers of the endangered language exceeds the threshold — when $x \geq th_x$ the system has the same dynamics as system (4) with the status set to 0.4, as shown in Figure 5b; however, when $x < th_x$ the system has the same dynamics as system (4) with the status set to 0.6, as shown in Figure 5c. The stable equilibrium Z* lies on the line of transition between the two dynamics: $x = th_x$.

[Figure 5 about here]

This behavior can be generalized as shown in Figure 6, which summarizes the qualitative behavior of the system for different values of the threshold th_x. The system has up to five equilibria: those corresponding to the system prior to intervention — X* and Y*, which are both stable, and U*, which is unstable — and those brought about by the intervention — Z*, which is stable, and V*, which is unstable. In order that the stable equilibrium Z* appear, and that language maintenance be possible, intervention should be undertaken neither too soon (Figure 6a) nor too late (Figure 6c). Rather, intervention should be undertaken within an intermediate window, as shown in Figure 6b.[3] Note that Z*, when present, always lies on the

[3] While we have been unable to determine an analytical expression for the range of values of the threshold th_x for which maintenance is possible, the range corresponds roughly to the proportions of monolingual speakers of the endangered language associated with the unstable equilibria V* (lower bound) and U* (upper bound). Analytical expressions for the positions of equilibria U* and V* have been obtained for the case that a=1; we exclude them here for brevity.

line $x=th_x$. Whether or not maintenance is actually achieved depends on the initial state of the system.

[Figure 6 about here]

Our model implies that intervention taken too early may lead to an endangered language having its status increased to such an extent that the erstwhile prestige language itself becomes threatened by extinction (Figure 6a), so requiring further intervention to maintain it. This would lead to oscillations in the status of each language, with regular maintenance of each language in turn being necessary. It is often the case, however, that endangered languages are only identified when their numbers of speakers have become low relative to those of the languages with which they compete. Thus intervention too late, leading to death of the endangered language, rather than intervention too early, is typically the more immediate concern in planning strategies to maintain actual languages. Nevertheless, in the sections that follow, we continue to analyze both the lower and upper bounds for language maintainability to better understand the dynamics of the model so that situations in which status oscillation occurs can be avoided.

4. Stochastic Models of Language Death

We now examine the impact and relative efficiency of this maintenance strategy in more detail, introducing a stochastic model for language death, which allows us to estimate the probability that the system converges to each stable state.

The dynamical system just discussed is deterministic — given an initial state of the system, the final state of the system is uniquely determined (even if we are not always able to establish an analytical expression for it). The system of differential equations (4) allows us to determine the rate of change of the system at any state. For example, at some state, the proportion of monolingual speakers of an endangered language X might be calculated to increase by 10% per unit of time, while the proportion of monolingual speakers of the competing language Y might be calculated to decrease by 5% per unit of time. But which bilinguals switch to become monolingual speakers of X, and which monolingual speakers of Y switch to become bilinguals is not addressed — the model does not distinguish the states of individual speakers, only the proportions of speakers having each state. Thus the dynamical system model encapsulates the expected behavior of the system.

Typically, the expected behavior of an appropriately defined dynamical system approaches the actual behavior of the underlying system as the population size is increased. However, the relevant population sizes in the context of language death and language maintenance are typically small, often of the order of hundreds or thousands of individuals. For such small populations, fluctuations in the language usage patterns of certain individuals may lead to dynamics that diverge significantly from the expected behavior. The deterministic model based on the system of differential equations (4) may therefore be insufficient to capture the full range of possible behavior of the system. Even when we consider the maintenance of an endangered or minority language having, say, a few million speakers (as is the case, for example, for Bai, a minority language of China spoken in Yunnan Province), it may often be the case that many of the speakers live in small, relatively isolated communities or else form cliques within larger communities with which they have comparatively little interaction.

In order to encapsulate such variation in the dynamics, we adapt our model to investigate the stochastic nature of the dynamics of language death by implementing the system as a multi-agent simulation, an approach that has found frequent application in quantitative modeling of language evolution (e.g. Hurford, 1989; Nowak, Plotkin & Krakauer, 1999; Wang, Ke & Minett, 2004). We do so by re-interpreting the transition rates of change between states that drive the dynamics of the deterministic system, shown previously in Figure 1, as transition probabilities.

The simulations are implemented as follows: A set of n agents, each agent modeling a single speaker in the population, is initialized. Each agent is assigned one of the three states, X, Y or Z, according to the specified initial proportions of speakers of each type; we denote the initial proportions of monolingual speakers of X and Y by x_0 and y_0, respectively. In all the experiments that follow, we set parameters c and c', which control the overall rate of change of the system, to the value 0.1. Having also selected the values of parameters a, s_0, δs and th_x, we set the simulation running. The transition probabilities are then calculated for each agent from the proportions of neighbors having each state. During each iteration of the simulation, the probability that a monolingual speaker of X switches to being bilingual, say, is given by $cy^a(1-s)$; such a speaker remains monolingual in X with probability $1-cy^a(1-s)$. The transition probabilities are therefore

$$
\begin{aligned}
\Pr(X \to Z) &= c\, y^a (1-s), \\
\Pr(Z \to X) &= c' (1-y)^a s, \\
\Pr(Z \to Y) &= c' (1-x)^a (1-s), \\
\Pr(Y \to Z) &= c\, x^a s.
\end{aligned}
\qquad (6)
$$

During each iteration, each agent samples the states of all its neighbors — for now we assume that the agents interact within a fully connected social network, and so sample the states of all the agents. Each simulation is run for 1,000 iterations, after which we identify the stable state (if any) that emerges: all agents monolingual in X only, all agents monolingual in Y only, or a stable mixture of both monolingual and bilingual agents.

We now explain our approach to estimating the probability of convergence of the system to each stable state by means of an example. Figure 7a shows the evolution of the system during one run of the simulation for a population of 1,000 agents with parameter values x_0=750, y_0=250, a=1.0, s_0=0.4, δs=0.2 and th_x=0.3. The system quickly converged to a stable equilibrium at x=0.3, $y\approx$0.5, $z\approx$0.2 about which it then oscillated. This represents an endangered language X being maintained with about 30% monolingual and 20% bilingual speakers. Notice that the trajectory, shown in Figure 7b, follows the direction field, on which it is superimposed, to a large degree, indicating that the behavior of the stochastic system has not diverged significantly from that of the deterministic system.

[Figure 7 about here]

The same initial conditions, however, sometimes led to the system converging to a state in which only one language had any speakers, as shown for a second run in Figure 8a. Despite converging to a different stable state, the trajectory again follows the direction field closely, as shown in Figure 8b. This variation in the final state of the system is due to the initial state (x_0=750, y_0=250) lying close to the boundary between the basins of attraction of the stable equilibria X*, corresponding to death of Y, and Z*, corresponding to maintenance of both X and Y. Slight perturbations away from the expected trajectory, indicated by the direction field, lead to the system converging to different stable equilibria: in some runs, the system falls into the basin of attraction of X*, in others it falls into the basin of attraction of Z*. By calculating the relative frequency of convergence to each stable equilibrium over many runs of the simulation, the probability that the system converges to each equilibrium can be estimated.

[Figure 8 about here]

4.1. Language Maintenance

As we discussed in Section 3, a major concern is to what extent should the status of an endangered language be enhanced and when should this be done in order that it be maintained — we have already demonstrated that intervention should take place neither too soon nor too late for maintenance of both competing languages to be possible. We now apply the language maintenance strategy described in Section 3 to the stochastic model for language death to investigate how the probability of maintenance depends on different sets of parameter values.

We begin by examining the performance of the maintenance strategy when the initial state of the system lies near the boundary between the two basins of attraction (x_0=750, y_0=250, as for the runs considered in Figures 7 & 8). Figure 9 shows the effects of a population intervening to enhance the relative status of an endangered language X from s=0.4 to s=0.5 (δs=0.1) whenever its proportion of speakers falls below various thresholds, th_x. The figure indicates that if the population takes action to maintain X after its proportion of speakers, x, has fallen below 0.3 (th_x<0.3), then only one language can be maintained, Y being significantly more likely than X. However, if the community decides to take action before the proportion of speakers of X falls below 0.3 ($0.3 \leq th_x \leq 0.7$), a bilingual community becomes possible. The emergence of a bilingual community is most likely (≈40%) when th_x is approximately equal to 0.5. If the intervention is made before the proportion of speakers of X falls below 0.7 ($0.7 \leq th_x$), only language X will be maintained, threatening the competing language Y with extinction.

[Figure 9 about here]

Figure 10 shows the effects of intervention when the relative status is enhanced from s=0.4 to s=0.6 (δs=0.2). In order to maintain both languages, intervention must take place before the proportion of monolingual speakers of X falls below 0.2, otherwise one language will inevitably die. Both languages may be maintained when the threshold lies in the range $0.1 \leq th_x \leq 0.7$, the peak probability of such maintenance (≈40%) occurring in the range $0.3 \leq th_x \leq 0.5$.

[Figure 10 about here]

Comparing Figures 9 and 10, we observe the following qualitative relationships between the status enhancement, δs, the enhancement threshold, th_x, and the probability of maintenance:

- increasing δs has no significant effect on the <u>upper</u> bound on th_x for which maintenance of both languages is possible;
- increasing δs significantly reduces the <u>lower</u> bound on th_x for which maintenance of both languages is possible;
- increasing δs <u>expands</u> the “window” of peak probability of maintenance, but has no significant effect on the peak probability itself.

We observe the same qualitative behavior for other combinations of parameter values.

5. The Impact of Social Structure on Language Death

Our simulations in the previous section were based on the assumption that each agent has complete knowledge of the states of all other speakers — in effect, this is equivalent to assuming that the underlying social structure is fully connected such that each speaker has perfect knowledge of and interacts with each other speaker.

We now investigate the impact of social structure on the probability of language maintenance. In selecting a paradigm with which to encode the social structure of a population of speakers, we turn to the so-called "local-world" network paradigm, recently developed by Li and Chen (2003). Local-world networks integrate into a single paradigm both random networks and scale-free networks (Barabási & Albert, 1999), which, together with "small-world" networks (Watts & Strogatz, 1998; Watts, 1999), have begun to find application in studies of social systems (e.g. Moody, 2001; de Bot & Stoessel, 2002).

Local-world networks are constructed recursively, adding nodes (here representing speakers) to the network one at a time, connecting them to a certain number of extant nodes. As with scale-free networks, they are constructed using preferential attachment — when a node is added to the network, it is assigned a greater probability of being connected to extant nodes having numerous connections than to nodes having few connections. In other words, speakers prefer to interact with those speakers who themselves interact with many speakers. Unlike in scale-free networks, however, when a node is added to a local-world network, it is connected preferentially only to nodes within a randomly selected subset of all extant nodes, its local-world. Thus speakers have local rather than global knowledge of the language usage patterns of other speakers in the population and only interact with a fraction of other speakers in their locality.

We extend the concept of preferential attachment to account for the language usage patterns of the speakers. Speakers are unlikely to interact frequently with other speakers with whom they share no common language. We therefore assign an interaction weight, $0 \leq w \leq 1$, that adjusts the likelihood that two speakers will interact despite not sharing a common language. Setting w=1 means that speakers ignore the language usage patterns of other speakers when selecting their neighbors, whereas setting w=0 means that speakers will only interact with other speakers with whom they share a common language (unless they consequently find too few speakers with whom to interact). Reducing the value of w therefore acts to increase the degree of isolation between the subgroups of monolingual speakers of the two competing languages. The procedure we use for constructing the local-world networks is described in Appendix A.

We now analyze the effect of the initial proportions of monolingual speakers on the probability of maintenance, starting with fully connected social structures. Figure 11 shows the behavior of the system for a population of 1,000 agents, with parameter values set to a=1.0 and s_0=0.4, with no intervention. The figure plots the estimated probability of convergence to each state as a function of the initial proportion of monolingual speakers of X; the remaining speakers initially all speak Y. The figure clearly indicates that a state in which both languages are maintained can be achieved only with negligible probability. Furthermore, for most initial proportions of speakers, $x \leq 0.7$ or $x \geq 0.8$, the system behaves in the same manner as the deterministic system (4), with just one language acquiring all speakers with probability 1. However, when the initial proportion of monolingual speakers of X lies in the range $0.7 \leq x \leq 0.8$, there is a gradual transition in the probabilities. This transition reflects the stochastic aspect of the interactions among the agents. As the population size increases, so the transition zone contracts, and the behavior of the system converges to that of the deterministic system (4) for all initial conditions.

[Figure 11 about here]

Figure 12 shows the behavior of the system for a locally connected population, again without intervention — the size of the local-world is set to 50 nodes; 20 connections are made between an incoming node and the local-world; the interaction weight is set to 1. The behavior is indistinguishable from that for the fully connected network. We observe the same lack of impact of social structure on the probability of maintenance for other values of parameters n, a and s_0. We therefore conclude that, in the absence of intervention, social structure has no significant influence on which language is maintained and which language dies.

[Figure 12 about here]

When the population intervenes to attempt to maintain both competing languages, however, we find that the underlying social structure does affect the behavior. Figures 13–15, below, show the behavior for a population of 1,000 agents, with parameter values set to a=1.0, s_0=0.4, δs=0.2 and th_x=0.5; that is, the population intervenes to increase the status of the endangered language X from 0.4 to 0.6 whenever the proportion of monolingual speakers of X falls below 0.5. Figure 13 highlights the behavior for a fully connected population, clearly indicating the range of initial proportions of monolingual speakers of the endangered language that allow both languages to be maintained, $0.2 \leq x_0 \leq 0.8$, with maintenance being virtually certain for $0.3 \leq x_0 \leq 0.7$. The same qualitative behavior, in which a single stable state emerges with probability 1 over a broad range of initial conditions, is observed for other values of parameters n, a and s_0 for a fully connected population.

[Figure 13 about here]

For a locally connected population, however, the behavior is less regular. Figure 14 shows the graph for a local-world size of 50 agents, with the number of connections, e_{LW}, between each incoming node and its local-world set to 20. We observe that the range of values for which both languages can possibly be maintained is the same as for the fully connected population: $0.2 \leq x_0 \leq 0.8$. However, the peak probability is somewhat less than 90% for x_0=0.3. The probability of maintenance decays gradually for larger initial proportions of monolingual speakers of the endangered language to about 50% for x_0=0.7. The probability then decays rapidly to zero as x_0 approaches 0.8, as for the fully connected population. From this behavior we infer that maintenance is more difficult to achieve within societies having an underlying local-world structure. Furthermore, the probability of maintenance appears to be maximal when intervention is undertaken “at the last moment”, but not so late that the opportunity is missed — status enhancement is best implemented when the state of system is closest to the position of the stable equilibrium that would be introduced by such enhancement; doing otherwise increases the risk that the system diverges from this equilibrium.

[Figure 14 about here]

Figure 15 makes this point stand out even more sharply, showing the behavior for a society having an underlying scale-free structure, for which the local-world encompasses the entire network (n_{LW}=1000; e_{LW}=20; w=1). In this case, a non-negligible probability of achieving maintenance is attained for the same range of initial proportions of monolingual speakers of the endangered language: $0.2 \leq x_0 \leq 0.8$. However, the peak probability is further reduced to about ⅔ for $x_0 \approx 0.3$, and the decay to significantly lower probabilities as x_0 increases is rapid — for example, there is less than a 20% probability of maintenance being achieved for x_0=0.5. The same qualitative behavior is also observed for other local-world and scale-free networks.

[Figure 15 about here]

The contrast in the behavior of the system for a society with either local-world or scale-free network structure may be related to the degree distribution of the two structures.[4] The <u>degree</u> of a particular node is defined as the number of edges that connect that node to other nodes; the <u>degree distribution</u> of a network is then defined as the frequency distribution of the degrees of all the nodes in the network, and is typically drawn as a graph of frequency against degree. Nodes that are connected to a large number of other nodes are called <u>hubs</u>. Scale-free networks

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[4] We note that the broader window of peak probability of maintenance observed for the fully-connected network structure is due to the agents having many more connections to other agents than for either the locally-connected or scale-free network structures.

have relatively few hubs; nodes having fewer connections are observed in increasing number according to the power law distribution (Barabási and Albert, 1999), which appears as a straight line when plotted on a log-log scale. Local-world networks, however, tend to have more hubs with an intermediate number of connections, despite having fewer hubs of such large degree as observed in a scale-free network (Li & Chen, 2003). The result seems to be that information can diffuse more rapidly across a local-world network. In the context of language maintenance, this may lead to social networks with local-world structure allowing easier maintenance of an endangered language.

We have also considered the role of the interaction weight, w, which adjusts the likelihood that agents having no common language will interact. Setting the interaction weight to less than 1 — indicating that agents prefer to interact with other agents with whom they share a common language — typically acts to reduce the upper bound on the maintenance window, but does not significantly impact the peak probability of maintenance. Thus the endangered language may be maintained, but is more likely to endanger the language with which it is competing. This is exemplified in Figure 16, which shows results for a locally connected society having the same parameters as were used to generate Figure 14, but with the interaction parameter set to w=0.25. (Notice that the peak probability of maintenance is not significantly altered.) This suggests that maintenance of two competing, isolated languages (w<<1) is more difficult when the monolingual speakers of the endangered language are initially numerous.

Until the predictions of the model have been fitted to empirical data, we hesitate to claim that the probabilities quoted here are truly representative of the likelihood of language maintenance being achieved within an actual linguistic community. However, we do claim that the qualitative behavior observed from our analysis of the estimated probabilities tells us a great deal about the qualitative effects of social structure on language maintenance.

[Figure 16 about here]

6. Conclusion

We have introduced an extension of the Abrams and Strogatz (2003) model for the dynamics of language death in which we explicitly model bilingualism and social structure. In the absence of intervention, the qualitative dynamics of the system are, in most cases, identical to those of the Abrams and Strogatz system — when two languages compete for speakers, eventual extinction of one language is inevitable. However, we have shown that by appropriate increase in the status of an endangered language, the dynamics can be altered such that both languages are maintained with non-negligible probability. Such intervention should be undertaken within a window of opportunity, enhancing the status of the endangered language before it becomes moribund but not so soon that the language with which it is competing itself becomes endangered. For all but the simplest (least realistic) social structures that we have modeled, the peak probability of successful maintenance is obtained by implementing the maintenance strategy as late as possible.

A number of aspects of the model proposed here can be refined. As we mentioned in Section 2, we have not modeled either the geographic distribution or the age distribution of the speakers. Network models have been proposed that account for the distances between nodes (e.g., Kleinberg, 2000). However, the geographic distribution of speakers within some community is not necessarily an indication of the likely social connections that obtain among them, particularly in urban environments. A recently proposed modification of the local-world paradigm (Gong, Ke, Minett, & Wang, 2004), in which the local-world of each node is re-assessed at each time step, may serve to model the dynamic aspects of interaction among speakers within a community, obviating the need to explicitly model geographic distribution. Situations in which two or more relatively isolated communities interact, each initially having distinct language patterns of its own, may be conveniently modeled by merging multiple networks, one for each community.

Age distribution and inter-generational learning may prove to be more significant factors that determine the likely fate of an endangered language and the likelihood of its being maintained. Fishman's GIDS scale (1991) makes explicit the role of the language capabilities of speakers of different ages: for example, a society in which only the aged speak the endangered language corresponds to Stages 7 and 8 in the scale, suggesting that maintenance may only be achieved with great difficulty. But once children have achieved competence in the endangered language via compulsory education, Stage 4, maintenance may become more likely. In the context of the model presented here, the maintenance strategies adopted by the agents are heterogeneous, with some agents more able to adjust the status of their language and more able to achieve bilingualism than others.

We have made no attempt to model code-switching, which often leads to the endangered language adopting features of the language with which it competes. Rather, we have assumed that such language shift, when it occurs, does not impact the attractiveness of a language. Code-switching and language shift might be incorporated into the model by treating the languages as consisting of multiple components, e.g. basic lexical items or syntax, each having its own status and attractiveness, and each of which may be learned independently by each speaker. The death of individual components may then be traced as the system evolves. It is likely that such a model would predict the fall of an endangered language, slowly at first as the first few components of the endangered language come to be replaced by their more prestigious counterparts, and then increasingly rapidly as the remnants of the dying language vanish. It is uncertain, however, whether the qualitative behavior of this model would differ from that of the model presented here, and, consequently, whether we would learn any new strategies for achieving maintenance of actual languages.

Indeed, we have not yet shown this model to have practical application to actual languages, having focused on the theoretical performance of a simple maintenance strategy in various situations. To do so, we shall require diachronic data for the numbers of both monolingual and bilingual speakers of sets of competing languages, rather than for just monolingual speakers as is the case with the Abrams and Strogatz model. We intend to collect such data in the near future for Bai, which competes with Mandarin, and She, which competes with Yue (Cantonese), both minority languages in China. The data can then be fitted to the model and the future evolution of the system predicted.

Furthermore, to test that our modeling of language maintenance strategies is valid, the diachronic data for two consecutive time spans will be fitted separately to obtain independent estimates of the status of the endangered language in each period, enabling us to estimate the probability that both languages will ultimately be maintained, or else to indicate what action must be taken to achieve such maintenance.

We have shown that social structure plays a significant role in the likelihood of maintenance of an endangered language. Studying the social structure of both Bai-speaking and She-speaking communities may allow us to refine our model for social structure further, enabling more effective strategies for language maintenance to be planned.

Appendix A — Algorithm for Constructing Local-World Networks

Here we describe the algorithm that we use to construct local-world networks. The algorithm is applied recursively, adding one node at a time, starting with a single node.

First, select the parameters of the linguistic community to be modeled. The parameters are the total number of agents, n, which corresponds to the number of nodes in the network, and the initial numbers of agents monolingual in X, x_0, and Y, y_0. Then, select the size of the local-world, n_{LW}, the number of nodes in the local world to which each new node should be connected, e_{LW}, and the interaction weight, w.

Our network-building algorithm then proceeds:

1. Randomly assign the language spoken by each agent — to be represented in the network by a node — according to the initial proportions of monolingual speakers of each language, x_0 and y_0.
2. Start with a single node and no connections:
    a) Initialize the number of nodes that have been added to the network to m=1.
3. Add a new node to the network:
    a) Randomly select n_{LW} extant nodes to be the local-world of the new node.
    If fewer than n_{LW} nodes are extant, set the local-world size to $\lceil m \times n_{\mathrm{LW}} / n \rceil$.
    b) Connect the new node to e_{LW} nodes within its local-world by preferential attachment, i.e., the node connects to e_{LW} extant nodes in its local-world with probability proportional to their modified degree — the <u>modified degree</u> of an extant node with respect to a new node is calculated by multiplying the degree of the extant by the interaction weight, w, if it shares no language with the new node, or else leaving the degree unmodified if it does.
    If fewer than n_{LW} nodes are extant, connect preferentially to just $\lceil m \times e_{\mathrm{LW}} / n \rceil$ nodes.
    c) Increment the number of nodes by one, i.e. $m = m + 1$.

d) Repeat Step 3 until the network is completed, i.e. $m = n$.

Scale-free networks can be constructed using the same algorithm by setting the size of the local-world to the number of nodes in the entire network, i.e. $n_{\mathrm{LW}} = n$.

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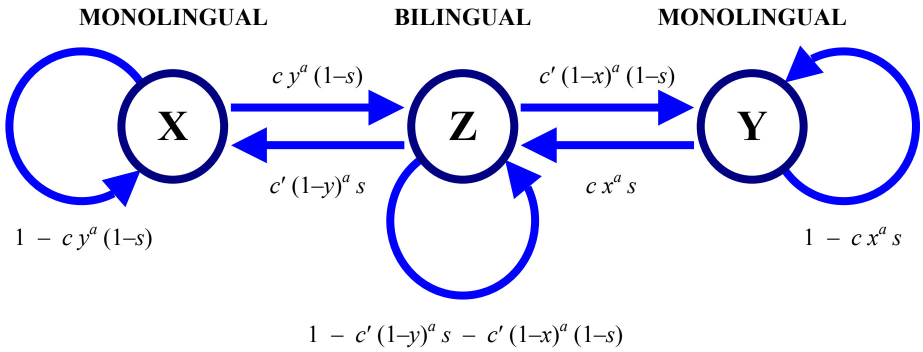
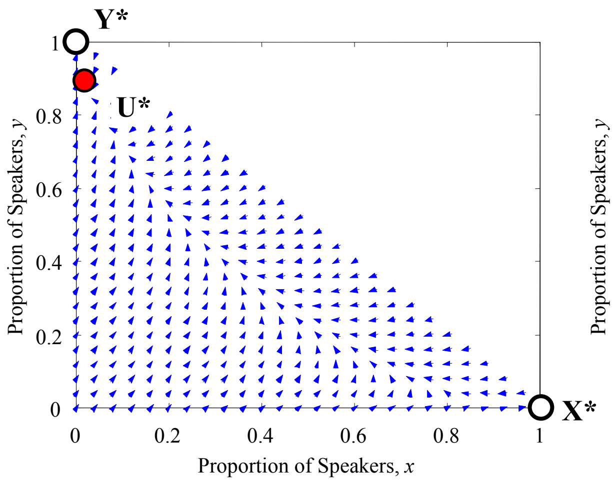
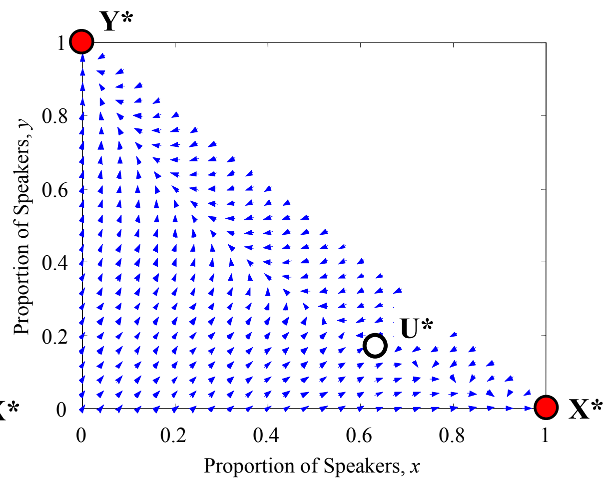


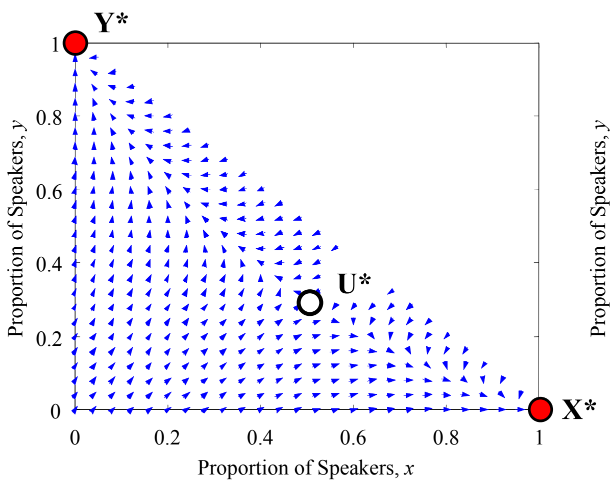
Figure 1 Transition rates for the bilingual model.



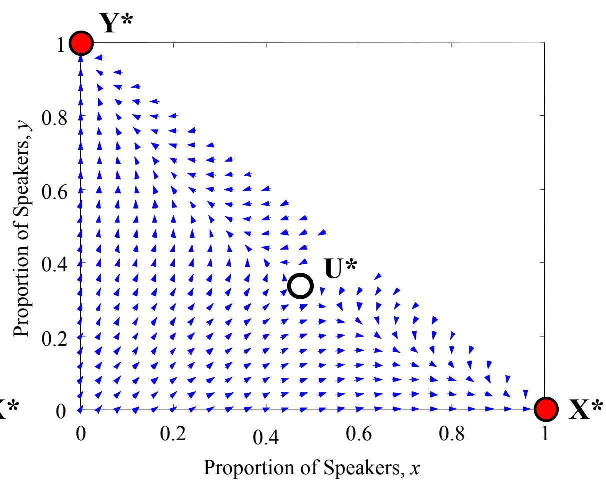
a) a=0.5



b) a=1.0

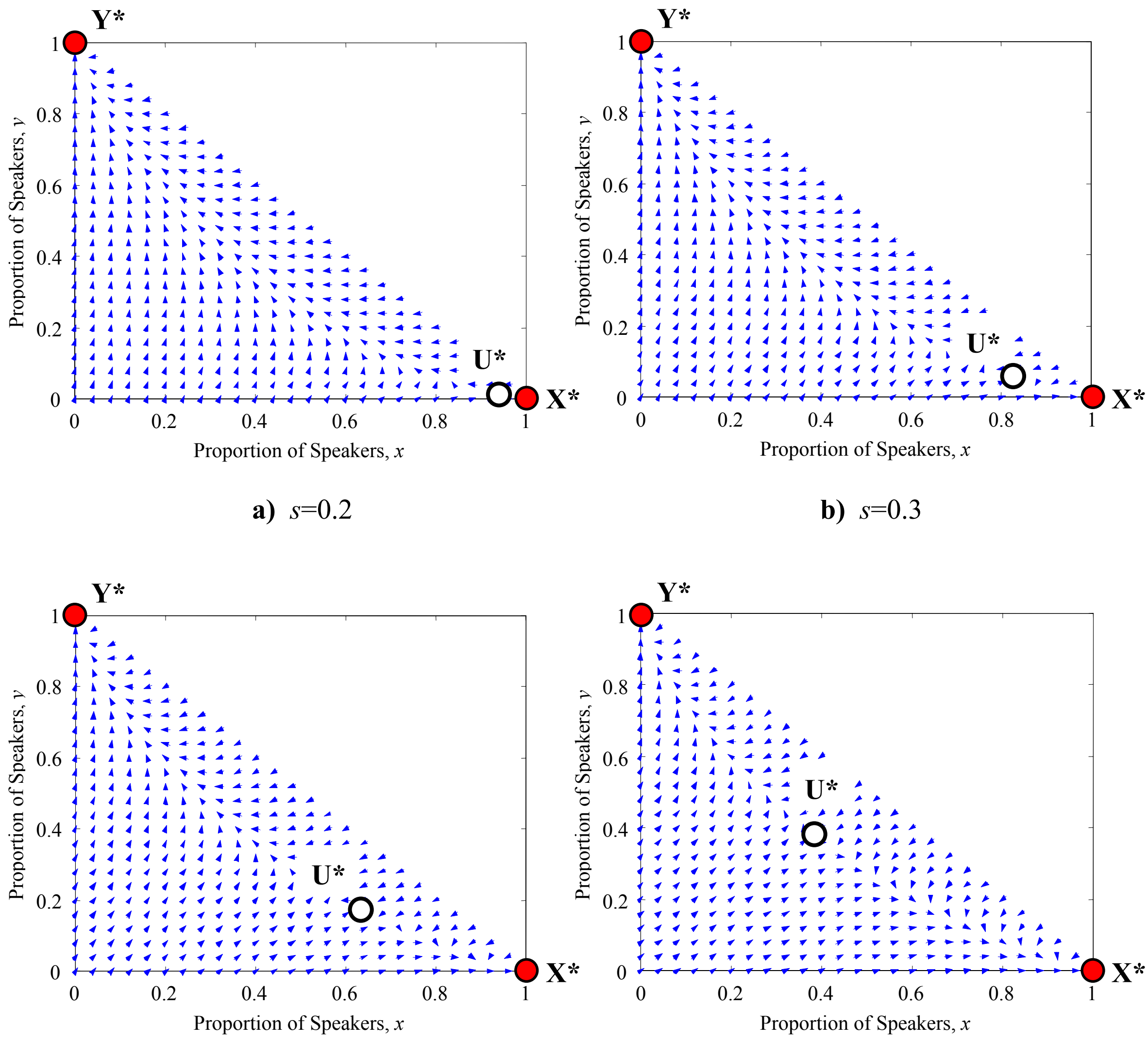


c) a=1.5



d) a=2.0

Figure 2 Direction field for the bilingual model for various values of a.
For $a \geq 1$, there are only two stable equilibria, corresponding to all members of the population being monolingual speakers of a single language, either X (equilibrium X* at x=1, y=0) or Y (equilibrium Y* at x=0, y=1). A third equilibrium at U* is unstable. For a<1, the stability of each equilibrium is reversed. The number, z, of bilingual speakers is indicated by the vertical (or horizontal) distance from the line $x+y=1$. Arrows show the direction of change of the system. Filled/unfilled circles indicate stable/unstable equilibria.
(a=0.5, 1.0, 1.5, 2.0; s=0.4)



a) s=0.2 b) s=0.3

c) s=0.4 d) s=0.5

Figure 3 Direction field for the bilingual model for various values of s.
(a = 1.0; s = 0.2, 0.3, 0.4, 0.5)

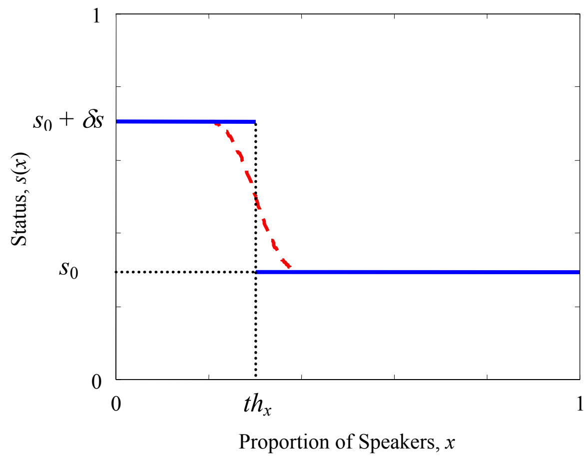
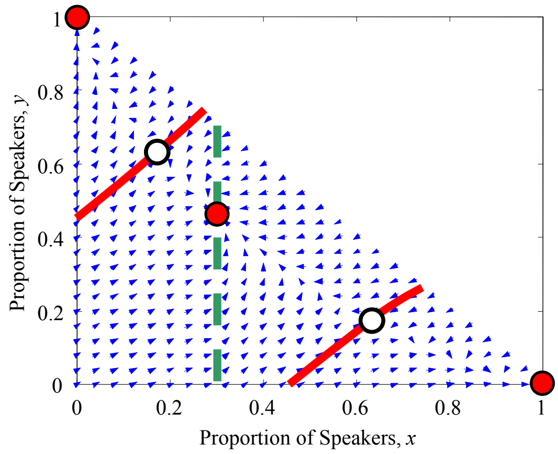
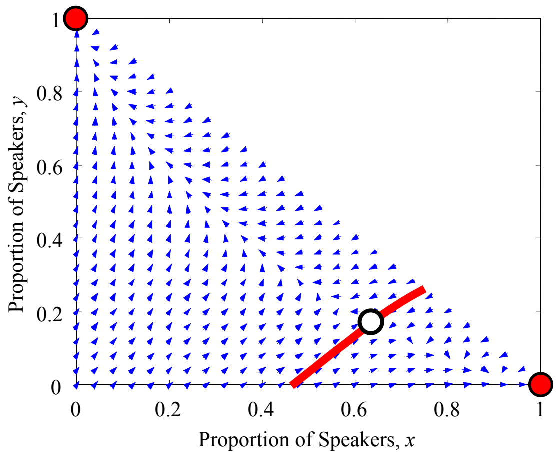


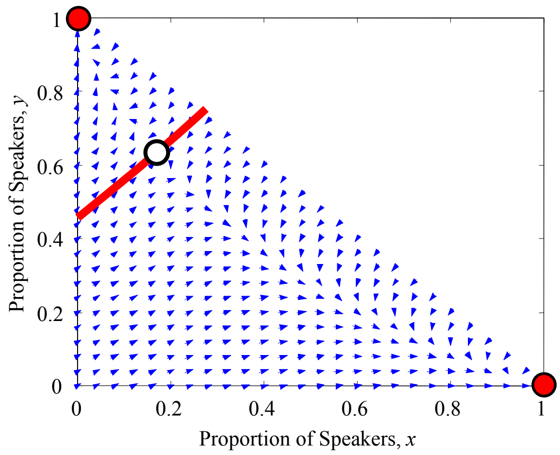
Figure 4 Functional relationship between proportion, x, of speakers of an endangered language and its relative status, $s(x)$. The solid line models a population intervening to bring about a discrete increase in the status of the endangered language, from s_0 to $s_0+\delta s$, as its proportion of speakers falls below the threshold, th_x — this functional relationship is assumed in the experiments throughout this paper. The dashed line models a more gradual intervention to increase the status of the endangered language.



a) With intervention (s_0=0.4; δs=0.2; th_x=0.3). The dashed line marks the threshold of the transition from the dynamics for s=0.4 (right-hand side, panel b) to the dynamics for s=0.6 (left-hand side, panel c). A stable equilibrium has been introduced at x=th_x (with basin of attraction bordered by the two solid lines) — all initial states within the basin of attraction tend towards the equilibrium state, corresponding to both competing languages being maintained.

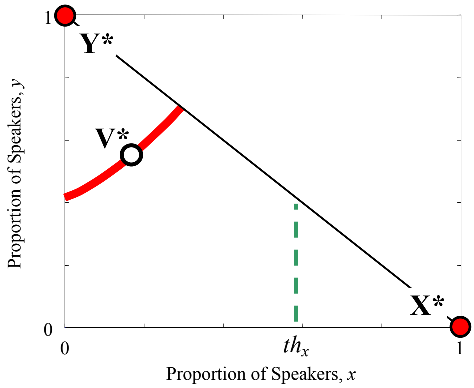


b) Without intervention (s=0.4) — language death is inevitable

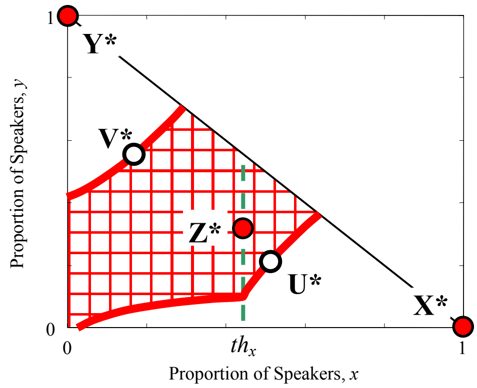


c) Without intervention (s=0.6) — language death is inevitable

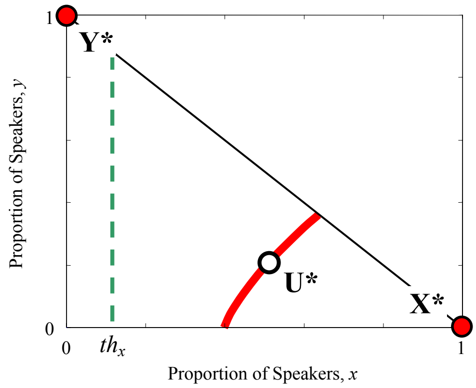
Figure 5 Direction field of the system with intervention, showing the introduction of a third stable equilibrium corresponding to both competing languages being maintained. (a=1; s=0.4; δs=0.2; th_x=0.3)



a) Intervention too early — Y dies



b) Both languages maintained



c) Intervention too late — X dies

Figure 6 Intervention must be undertaken neither too soon nor too late otherwise language death is inevitable. Successive panels show the stability of the system for decreasing value of the threshold, th_x. Stable equilibria are marked X*, Y* and Z*; unstable equilibria are marked U* and V*. The basin of attraction of the equilibrium, Z*, for which both languages are maintained, is shown hatched.

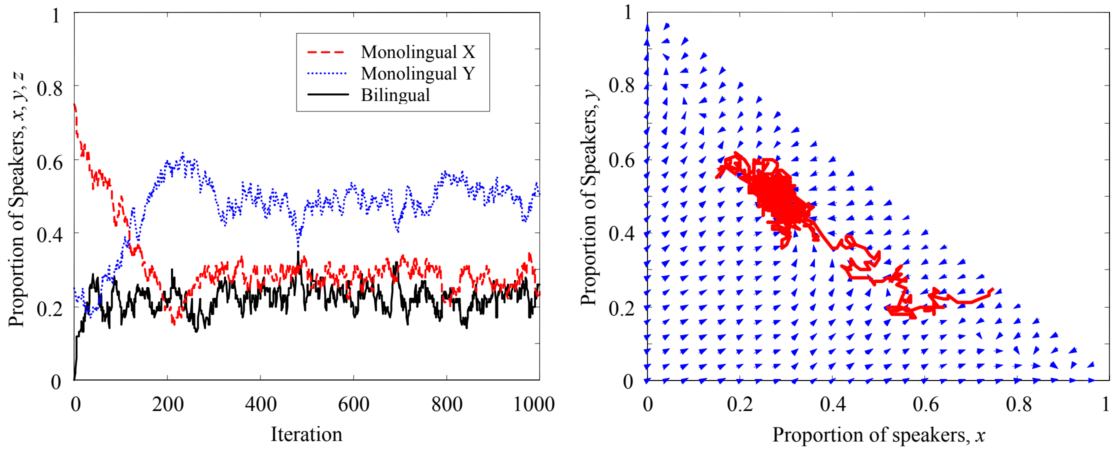


Figure 7 One run of the simulation in which the system converges to a stable state at $x\approx0.3$, $y\approx0.5$, $z\approx0.2$ after 200 iterations. Thereafter, the system oscillates about this state; both languages are maintained.

(n=1000; x_0=750; y_0=250; a=1.0; s_0=0.4; δs=0.2; th_x=0.3)

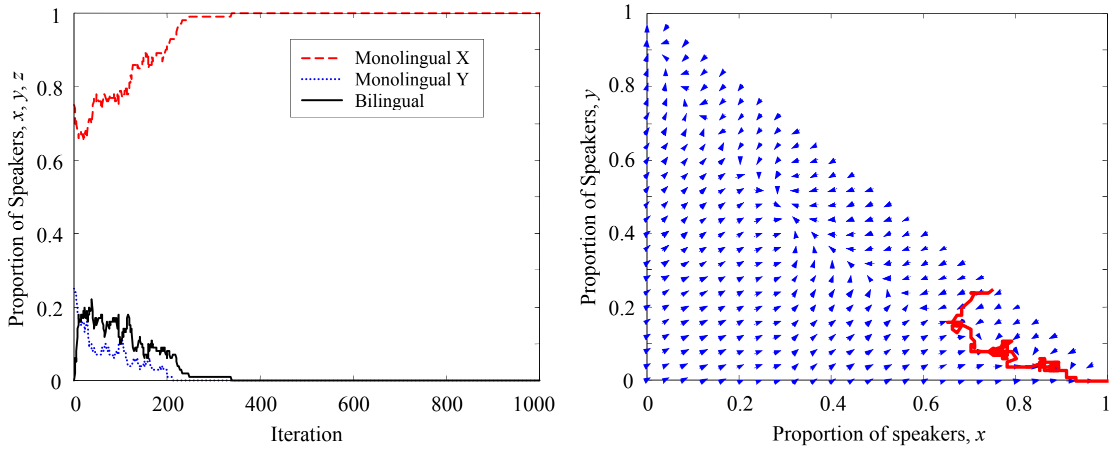


Figure 8 A second run of the simulation in which the system converges to different stable state from that observed in Run 1, shown in Figure 7 — language X quickly acquires all speakers and language Y dies.

(n=1000; x_0=750; y_0=250; a=1.0; s_0=0.4; δs=0.2; th_x=0.3)

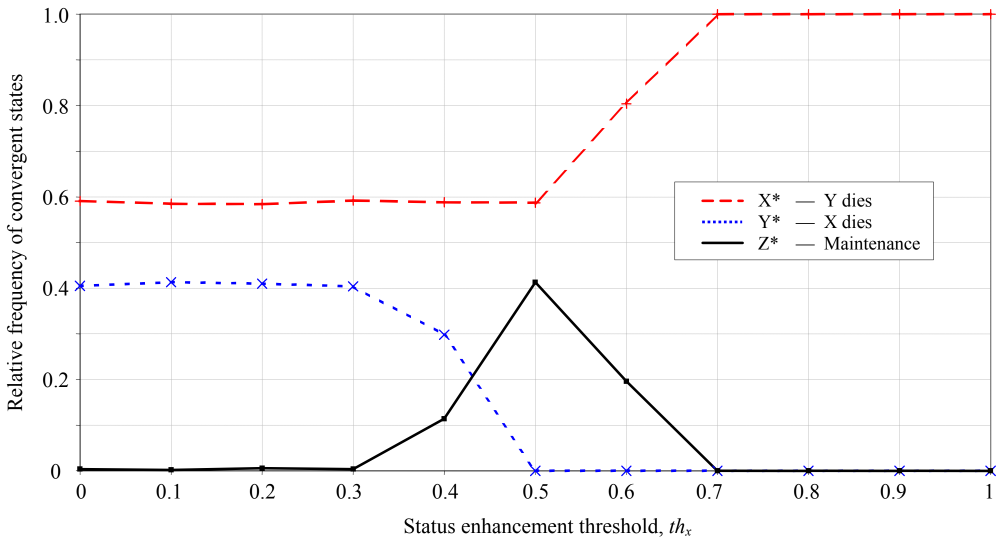


Figure 9 The effect of status enhancement δs=0.1 on the probability of maintenance. A bilingual community emerges with non-negligible probability only for th_x in the range [0.3,0.7]; emergence of a bilingual community is most likely (≈40%) when th_x≈0.5. (n=1000; x_0=750; y_0=250; a=1; s=0.4; c=0.1)

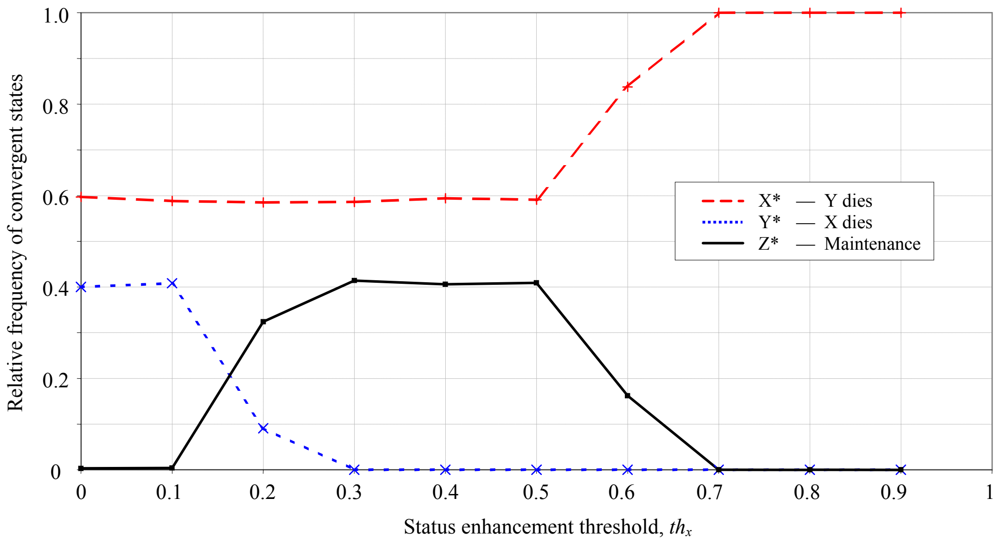


Figure 10 The effect of status enhancement δs=0.2 on the probability of maintenance. A bilingual community emerges with non-negligible probability only for th_x in the range [0.1,0.7]; emergence of a bilingual community is most likely (≈40%) when th_x lies within the interval [0.3,0.5].

(n=1000; x_0=750; y_0=250; a=1; s=0.4; c=0.1)

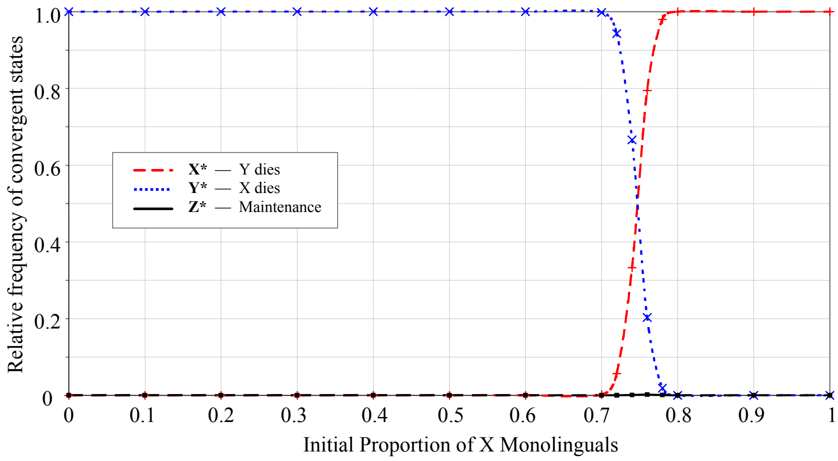


Figure 11 Probabilities of convergence for a population of 1,000 agents without intervention for a fully connected social structure.
(a=1; s_0=0.4)

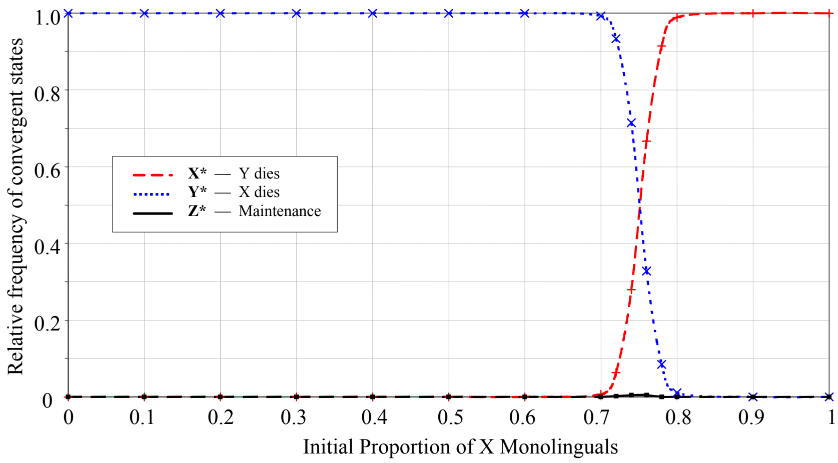


Figure 12 Probabilities of convergence for a population of 1,000 agents without intervention for a locally connected social structure

(a=1; s_0=0.4; n_{LW}=50; e_{LW}=20; w=1)

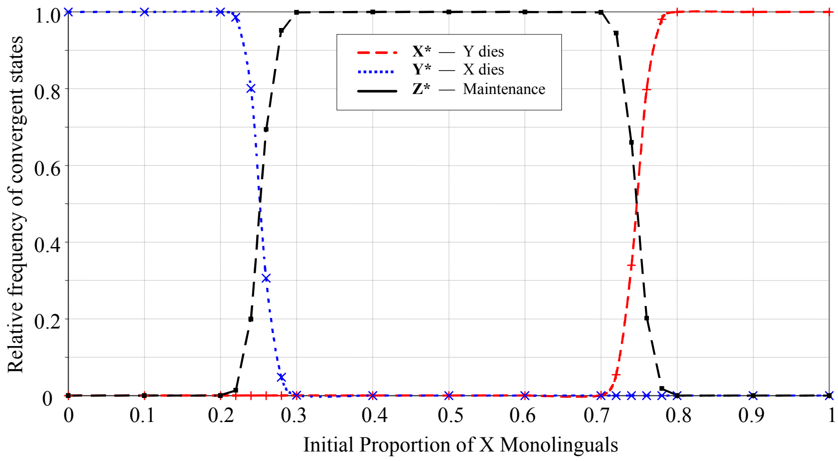


Figure 13 Probabilities of convergence for a population of 1,000 agents with intervention for a fully connected social structure.

(a=1; s_0=0.4; δs=0.2; th_x=0.5)

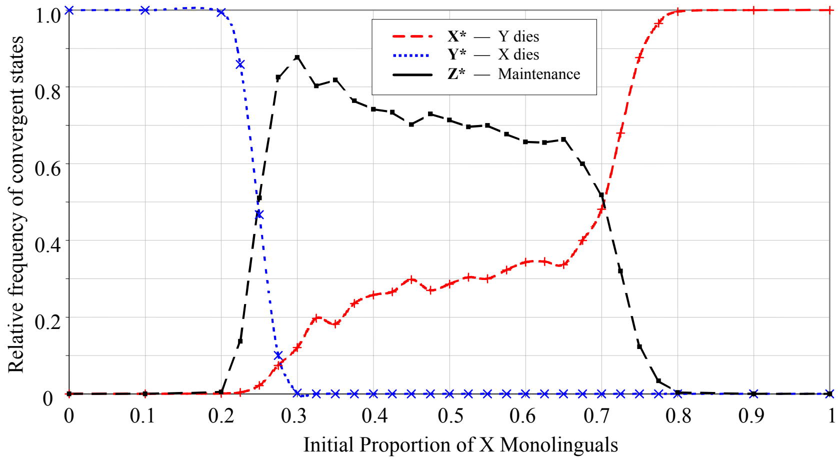


Figure 14 Probabilities of convergence for a population of 1,000 agents with intervention for a locally connected social structure.

(a=1; s_0=0.4; δs=0.2; th_x=0.5; n_{LW}=50; e_{LW}=20; w=1)

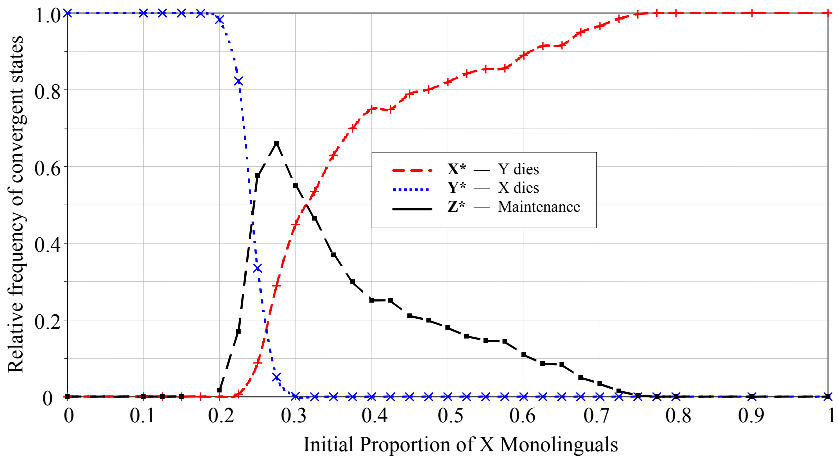


Figure 15 Probabilities of convergence for a population of 1,000 agents with intervention for a scale-free social structure.

(a=1; s_0=0.4; δs=0.2; th_x=0.5; n_{LW}=1000; e_{LW}=20; w=1)

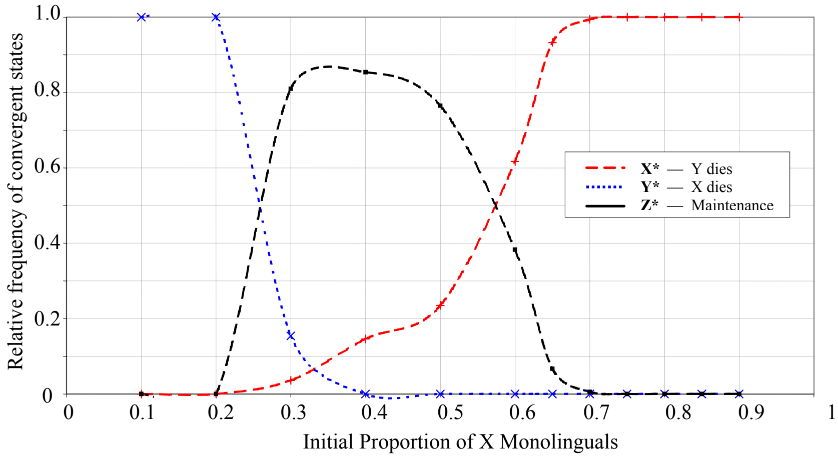


Figure 16 Probabilities of convergence for a population of 1,000 agents with intervention for a locally connected social structure — the competing languages are isolated, i.e. w<<1. (a=1; s_0=0.4; δs=0.2; th_x=0.5; n_{LW}=50; e_{LW}=20; w=0.25)