Dynamics of Naming Games in Random Geometric Networks

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***Abstract*— We investigate a simple agent-based model, the Naming Game, on random geometric networks. The Naming Game is a minimal model, employing local communications, capturing the emergence of shared communication schemes (languages) in a population of autonomous semiotic agents. Implementing it on random geometric graphs, local communications being local broadcasts, we can model the corresponding agreement dynamics in large-scale, autonomously operating wireless sensor networks. A potential application of the algorithm is encryption key creation for a community of agents for secure communications, visible only to the members of the community. The late-stage temporal behavior of the dynamics of the Naming Game can be understood in terms of the theory of coarsening, occurring in domain and phase ordering in physical and chemical systems.**

I. Introduction

Reaching agreement without global coordination is of fundamental interest in large-scale autonomous multi-agent systems. In the context of social systems, the objective is to understand and predict the emergence of large-scale population-level patterns arising from empirically supported local interaction rules between individuals (e.g., humans). Examples for such phenomena driven by social dynamics include the emergence and the evolution of languages [1] or opinion formation [2], [3]. From a system-design viewpoint in technological (e.g., sensor) networks [4], [5], the objective can be somewhat reversed, in that it is to construct local rules giving rise to a fast and efficient convergence to a global consensus, when needed.

In this paper we "borrow" and slightly modify a simple set of rules, referred to as Language or Naming Games (NG), originally proposed in the context of semiotic dynamics [6], [7]. Such problems have become of technological interest to study how artificial agents or robots can invent common classification or tagging schemes from scratch without human intervention [6], [7]. The original model by Steels [6], [8]–[10], was constructed to account for the emergence of shared vocabularies or conventions in a community of interacting agents. More recently a simplified version of the NG was proposed and studied on various network topologies by Baronchelli et al. [11]–[13] and Dall'Asta et al. [14]. The advantage of studying a minimal model is that one can gain a deeper understanding of the spontaneous self-organization process of networked autonomous agents in the context of reaching global agreement, and can extract quantitative scaling properties for systems with a large number of agents. They investigated this simplified version of the NG on fully-connected (FC) (also referred to as mean-field or homogeneous mixing) [11], [12], regular [13], and small-world (SW) networks [14].

In the FC network, each agent has a chance to meet with all others and compare their current local vocabularies before updating them. On regular networks, agents have only a limited and fixed number of neighbor with whom they can interact/communicate. The communication in both cases is "local", in that *pairs of agents* are selected to interact and to update their vocabularies. The basic algorithmic rules of the NG are as follows [11], [13]. A pair of "neighboring" nodes (facilitated by the underlying communication topology), a "speaker" and a "listener", are chosen at random. The speaker will transmit a word from her list of "synonyms" to the listener. If the listener has this word, the communication is a success, and both players delete all other words, i.e., collapse their list of synonyms to this one word. If the listener does not have the word transmitted by the speaker, she adds it to her list of synonyms without any deletion. It was found that employing the above local rules (*pair-wise* interactions), after some time, the

agents vocabularies converge to a unique vocabulary shared among all agents [11]–[14]. The major differences between the NG on FC and on regular (e.g., two-dimensional) networks arise in the memory needed to develop the common language before convergence occurs, and in the time to converge. In the FC network, the convergence process (reaching global agreement) is fast, but large memory is needed per agent [11]. For a regular two-dimensional network, spontaneous evolution toward a shared dictionary is slow, but the memory requirement is much less severe [13]. When the NG is implemented on SW networks, the agreement dynamics performs optimally in the sense that memory need is small, while convergence is much faster than on the regular networks [14].

The situation motivated above (i.e., the need for shared vocabularies) can also be quite realistic in the context of sensor networks [4], [5]. Envision a scenario where mobile or static sensor nodes are deployed in a large spatially extended region and the environment is unknown, possibly hostile, the tasks are unforeseeable, and the sensor nodes have no prior classification scheme/language to communicate regarding detecting and sensing objects. Since subsequent efficient operation of the sensor network inherently relies on unique object identification, the autonomous development of a common "language" for all nodes is crucial at the exploration stage after network deployment [4], [5]. For this task, however, there are more efficient and faster schemes, guaranteeing to reach global agreement on the naming (tagging) of an object. In particular, basic leader-election (LE) algorithms [15]–[20] could be employed to arrive at a common word among a community of agents which observed the object to be named: Upon observation, each agent "coins" a random tag (identification number) for the object. Following the observation, the observing agents participate in the leader election algorithm (with not the purpose of electing a leader, but choosing a unique identifier for the object). For example, in a two-dimensional regular network of N agents, the convergence to a unique identifier takes time of $\mathcal{O}(\sqrt{N})$ on average. Convergence time in the NG in two dimensions is of $\mathcal{O}(N)$, significantly longer than the LE algorithm when N is large. Thus, for the purpose of constructing a shared classification or tagging scheme in a sensor network, launching the LE algorithm is the preferable choice. Unlike social networks, sensor networks, although operating autonomously, are intelligently designed (by humans), who can make the choice a priori which algorithm to employ based on their efficiency.

There are possible situations, however, when the NG algorithm, in addition to being interesting for its own merit in studying agreement dynamics on various networks, can also be beneficial from a system-design viewpoint. That can be the case when we do not intend the outcome of the agreement to be easily predictable. The actual process of electing a "leader" or coordinator among sensor nodes may actually be such a scenario. The leader must typically be a trusted node, with possible responsibilities ranging from routing coordination to key distribution [21]. The basic LE algorithms are essentially based on finding global extremum (e.g., maximum) through local communications [15]–[17]. Thus, the elections can be stolen by placing a node in the network with a sufficiently high ID (e.g., the largest number allowed by the number representation scheme of the sensor chips.)

Another possible application of the NG algorithm is autonomous key creation or selection for encrypted communication in a community of sensor nodes. Instead of having a centralized or hierarchial key management system with domain and area key distributors [21], group of sensor nodes can generate a shared "public" key (becoming visible to group members).

Sensor networks are both spatial and random. As a large number of sensor nodes are deployed, e.g., from vehicles or aircrafts, they are essentially scattered randomly across large spatially extended regions. In the corresponding abstract graph two nodes are connected if they mutually fall within each others transmission range, depending on the emitting power, the attenuation function and the required minimum signal to noise ratio. Random geometric graphs (RGGs), also referred to as spatial Poisson/Boolean graphs, capturing the above scenario, are a common and well established starting point to study the structural properties of sensor network, directly related to coverage, connectivity, and interference. Further, most structural properties of these networks are discussed in the literature in the context of continuum percolation [22]–[24].

The common design challenge of these networks is to find the optimal connectivity for the nodes: If the connectivity of the nodes is too low, the coverage is poor and sporadic. If the node connectivity is too high, interference effects will dominate and results in degraded signal reception [25]–[28]. From a topological viewpoint, these networks are, hence, designed to "live" somewhat above the percolation threshold. This can be achieved by adjusting the density of sensor nodes and controlling the emitting power of the nodes; various power-control

schemes have been studied along these lines [25], [28]. In this paper we consider random geometric graphs above the percolation threshold, as minimal models for the underlying network communication topology. The focus of this work is to study the NG algorithm *on* these well studied graphs.

II. Naming Games on Random Geometric Networks

A. Random Geometric Networks

As mentioned above in the Introduction, here we consider random geometric graphs [22]–[24] as the simplest topological structures capturing the essential features of ad hoc sensor networks. N nodes are uniformly random distributed in an $L \times L$ spatial area. For simplicity we consider identical radio range R for all nodes. Two nodes are connected if they fall within each other's range. An important parameter in the resulting random geometric graph is the average degree $\overline{k}$ (defined as the average number of neighbors per node), $\overline{k} = 2K/N$, where K is the total number of links and N is the number of nodes. In random geometrical networks, there is a critical value of the average degree, $\overline{k}_c$, above which the largest connected component of the network becomes proportional to the total number of nodes (the emergence of the giant component) [22]–[24]. There is a simple relationship between the average degree $\overline{k}$, the density of nodes ρ, and the radio range R of the nodes [22]–[24], $\overline{k} = \rho\pi R^2$, which can be used to control the connectivity of the network.

B. The Naming Game

We consider the Naming Game on random geometrical graphs. In the original context of the NG, agents try to reach agreement in finding a unique "word" for an object observed by them. In one of the above proposed potential applications, agents try to generate a shared public key for encrypted communication. For simplicity, we will use the term "word" for the latter as well when describing the algorithm.

We modify the communication rules to make them applicable for sensor networks. Instead of pairwise communications, nodes will initiate *broadcast* (to all neighbors) in a continuous-time asynchronous fashion. First we consider the initial condition when the "vocabulary" of each node is empty. At every elementary time step, a node is chosen randomly out of N nodes (mimicking Poisson asynchrony for large N). This node (the "speaker") will broadcast a word from her list of "synonyms"; if her list of synonyms is empty, the speaker randomly invents a word; if she already has several synonyms, it randomly chooses one. Her neighbors (the "listeners") compare their vocabularies with the word transmitted by the speaker. If a listener has this word, she considers the communication a success, and she deletes all other words, collapsing her list of synonyms to this one word. If a listener does not have the word transmitted by the speaker, he adds it to his list of synonyms without any deletion. If at least one listener had the word transmitted, the speaker considers it (at least a partial) success, and (somewhat optimistically) collapses her list of synonyms to this one word. At every step, the "success" rate S is defined as the fraction of listeners who were successful (i.e., those that had the word transmitted by the speaker). From the above it is clear that the listeners have to report the outcome of the "word matching" to the speaker, hence the elementary algorithmic step requires $(\overline{k}+1)$ broadcasts. (For actual communication implementations, for a significant reduction of the $\overline{k}$ responses from the listeners, see Section III.) In this paper time t is given in units of $(\overline{k}+1)$ broadcasts per node (during which, on average, N word matching have been attempted). The main difference between the above algorithm and the one by Baronchelli et al. [13] is the *broadcast* (instead of pairwise communications) and the underlying network (RGG in this paper) to capture the essential features of the NG in sensor networks.

Other initial conditions may also be feasible. For example, for key generation, instead of starting from "scratch" (empty list of words for each node), each agent can have a pre-generated (possibly long) list of words. The different initial conditions will only have an effect on the early time behavior of the system (see next Section).

When starting from empty vocabularies, agents invent words randomly. After time of $\mathcal{O}(1)$ [on average of order $(\overline{k}+1)$ broadcast per node], $\mathcal{O}(N/(\overline{k}+1))$ different words have been created. Following the early-time increase of the number of different words $N_d(t)$, through local broadcasts, agents slowly reconcile their "differences", and eventually will all share the same word. First, a large number of small spatial clusters sharing the same word develop. By virtue of the random *diffusive* motion of the interfaces separating the clusters, more and more of the small clusters are being eliminated, giving rise to the emergence of larger clusters, eventually leading to one cluster in which all nodes are sharing the same word. As suggested by Baronchelli et al. [13], this late-time process is analogous to coarsening, a well-known phenomenon from the theory of domain

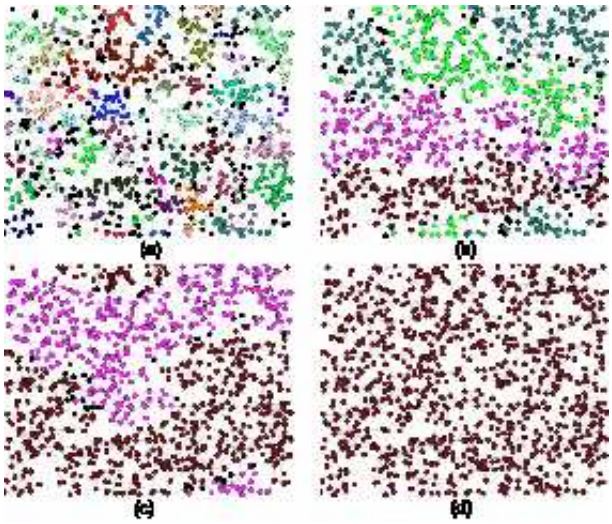

Fig. 1. (Color figure) Snapshots of the time evolution of the contents of the agents' word lists during the process of reaching global agreement for $N = 1000$ nodes at time (a) $t = 1$; (b) $t = 56$; (c) $t = 167$; (d) $t = 235$. The average degree is $\overline{k}\approx 12$ Initially, the word lists are empty for all agents. Time is measured in units of $(\overline{k}+1)$ broadcasts per node. Different colors correspond to different words, with black indicating nodes with multiple words. After the early-time increase in the number of different words in the systems, small spatial clusters sharing the same word quickly form, then subsequently "coarsen" until eventually only one global cluster prevails.

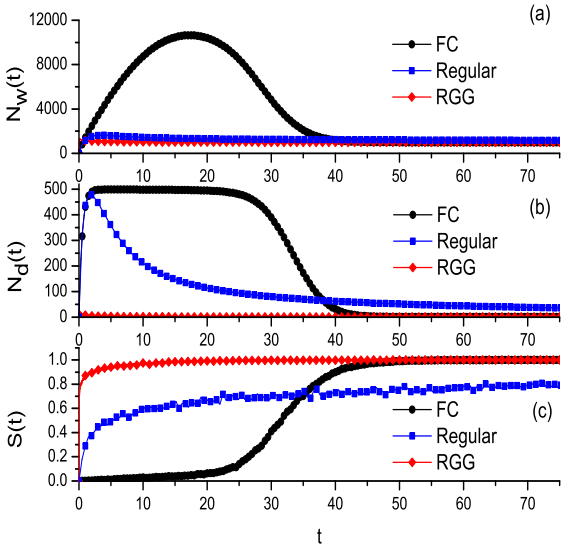

Fig. 2. Time evolution of the relevant observables in the Naming Game in the fully connected (FC), two-dimensional ($2d$) regular, and random geometric networks (RGG) for N=1024, averaged over 1000 independent realizations of the game; (a) the total number of words in the system $N_w(t)$; (b) the number of different words $N_d(t)$; (c) the average success rate $S(t)$. The average degree of the underlying RGG is $\overline{k}\approx 50$. Data for the FC and $2d$ regular networks are reproduced by our simulations, following Refs. [11], [13], for comparison.

and phase ordering in physical and chemical systems [29]. Figure 1 shows snapshots of vocabularies of the nodes at different times. For later times, group of nodes which already share the same word, slowly coarsen, until eventually only one domain prevails. This behavior is also captured by Fig. 2(b), tracing the number of different words as a function of time $N_d(t)$, eventually reaching global agreement, $N_d = 1$

Initializing the system with a possibly long list of words for all agents, motivated by key generation, after some early time reduction (after which almost all agents have one word in their list, except the ones at the interface between clusters), the system again exhibits the same coarsening behavior described above.

Before turning to the detailed discussion of our simulation results, we first sketch the framework of coarsening theory [29], applicable to the observed late-time dynamics of the NG on regular d-dimensional lattices [13]. While RGG is a random structure, it is embedded in two dimensions, and we also attempt to employ elementary scaling arguments from coarsening theory. According to Ref. [13], on regular d-dimensional lattices, the typical size of domains (each with already agreed upon one word) is governed by a length scale $\xi(t) \sim t^{\gamma}$ with γ=1/2, analogous to that of domain formation in systems with a non-conserved order parameter [29]. Thus, in d dimensions, the total number of *different* words N_d at time t scales as the typical number of domains

$$N_d(t) \sim \frac{N}{\xi^d(t)} \sim \frac{N}{t^{d\gamma}} \, . \qquad (1)$$

Further, the total number of words N_w, at this late coarsening stage, can be written as the number of nodes N plus the number of nodes with more than one (on average, between one and two) words, separating the different domains: of order of typical number of domains times the typical length of the interface of one domain, yielding

$$N_w(t) - N \sim \frac{N}{\xi^d(t)} \xi^{d-1}(t) \sim \frac{N}{\xi(t)} \sim \frac{N}{t^{\gamma}} \, . \qquad (2)$$

Similarly, the "failure rate" for word matching, $1-S(t)$, (where $S(t)$ is the success rate) scales as the fraction of nodes at the interfaces separating domains with different words

$$1 - S(t) \sim \frac{1}{\xi(t)} \sim \frac{1}{t^{\gamma}} \, . \qquad (3)$$

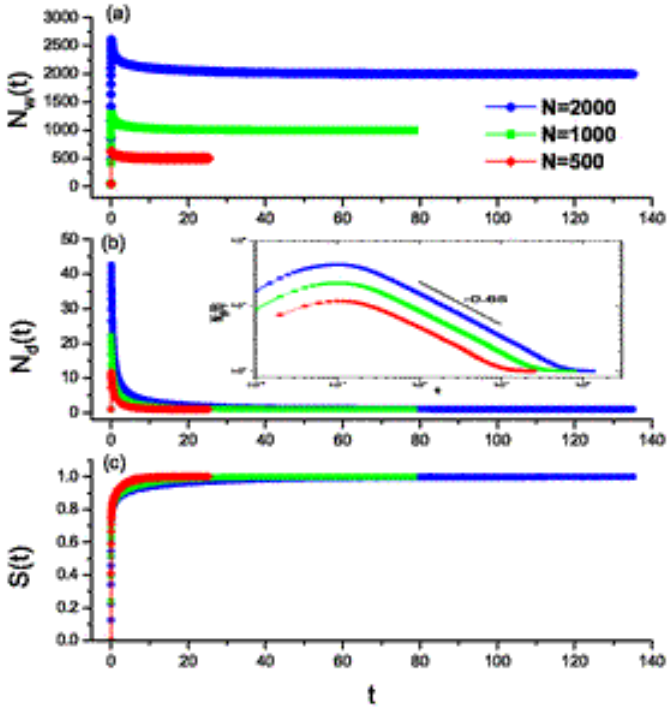

Fig. 3. Time evolution of the relevant observables in the Naming Game in random geometric networks (RGG) for three system sizes, averaged over 1000 independent realizations of the game; (a) the total number of words in the system $N_w(t)$; (b) the number of different words $N_d(t)$; (c) the average success rate $S(t)$. The average degree of the underlying RGGs is $\overline{k}\approx 50$. The inset of (b) shows $N_d(t)$ on log-log scales, displaying the late-stage coarsening and the corresponding power-law decay, approximately $N_d(t)\sim t^{-0.65}$.

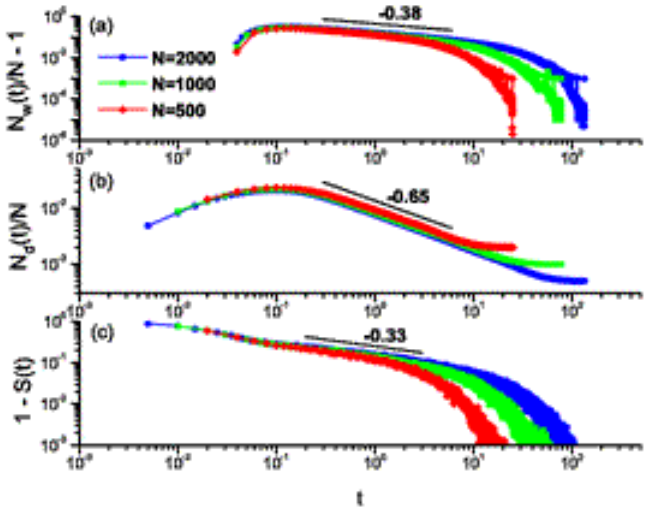

Fig. 4. The scaled version of the same data shown in Fig. 3 on log-log scales; (a) the total number of words in the system $N_w(t)/N-1$; (b) the number of different words $N_d(t)/N$; (c) the average success rate $1-S(t)$. The straight line segments correspond to the best-fit power-law decays $N_w(t)/N-1\sim t^{-0.38}$, $N_d(t)/N\sim t^{-0.65}$, $1-S(t)\sim t^{-0.33}$ for (a), (b), and (c), respectively.

The main feature of the above power-law decays (up to some system-size dependent cut-offs) is that the number of different words N_d, the total number of words N_w, and the success rate $S(t)$ only depend on t through the characteristic length scale $\xi(t)$. Further, for the typical time t_c to reach global agreement or consensus, one has $\xi^d(t_c)\sim N$, i.e.,

$$t_c \sim N^{1/(d\gamma)} \ . \qquad (4)$$

C. Simulation Results

Relevant quantities measured in the simulations are the total number of words in the system $N_w(t)$ (corresponding to the total memory used by the agents for word allocation at time t), the number of different words $N_d(t)$, and the average rate of success $S(t)$ of the word-matching attempts. Figure 2 displays the time evolution of these three quantities for the RGG, compared to the fully connected (FC) and to the $2d$ regular networks. Here, for the comparison, we reproduced the corresponding data of Refs. [11], [13]. The behavior of the NG on RGG is qualitatively very similar to that of the NG on $2d$ regular graphs. After time of $\mathcal{O}(1)$, $\mathcal{O}(N/(\overline{k}+1))$ different words have been invented [Fig. 2(b) and 3(b)]. $N_w(t)$ also reaches its maximum in time of $\mathcal{O}(1)$ [Fig. 2(a) and 3(a)]].

Focusing on the late-time behavior of the systems, plotting $N_d(t)/N$, $N_w(t)/N-1$, and $1-S(t)$ vs t on log-log scales, confirms the power-law decays associated with the underlying coarsening dynamics, predicted by Eqs. (1), (2), and (3), respectively.

Further, from Figs. 4(a) and (c) we find, within errors, two consistent estimates for the scaling exponent for typical length scale [see Eqs. (2) and (3)]: $\gamma\approx 0.38$ and $\gamma\approx 0.33$, respectively. The number of different words, according to Eq. (1), in turn, should scale as $N_d(t)/N\sim 1/t^{2\gamma}$, close to our measured exponent $2\gamma\approx 0.65$ [Figs. 3(b) and 4(b)]. The time to global agreement scales as $t_c\sim N^{1.23}$, shown in Fig. 5, in reasonable agreement with the one predicted by Eq. (4) with the exponent $1/(2\gamma)$.

In addition to the average convergence time t_c, we also measured the standard deviation Δt_c [Fig. 5], and constructed the probability density $P(t_c)$ for this observable [Fig. 6]. The data collapse of the scaled probability densities for different system sizes [inset Fig. 6], supports the underlying coarsening picture, governed by a single length scale $\xi\sim t^{\gamma}$ which reaches the linear system size $N^{1/d}$ at $t=t_c$. These results also indicate some weakness of the NG from a system-design viewpoint: the standard

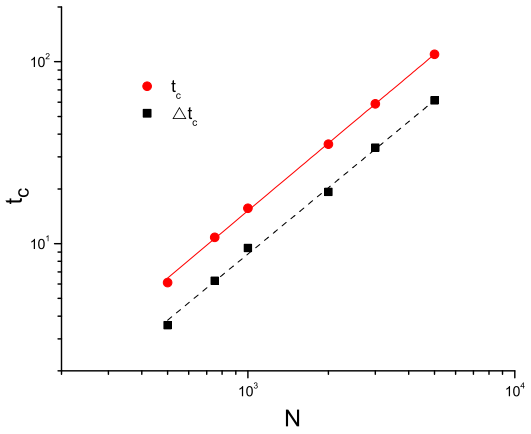

Fig. 5. Average and the standard deviation of the convergence time t_c until global agreement is reached, as a function of the number of nodes on log-log scales, averaged over 1000 independent realization of the game. The average degree of the underlying RGGs is $\overline{k}\approx 50$. The straight lines correspond to the best-fit power-laws with exponents 1.23 and 1.21, for the average (solid line) and for the standard deviation (dashed line), respectively.

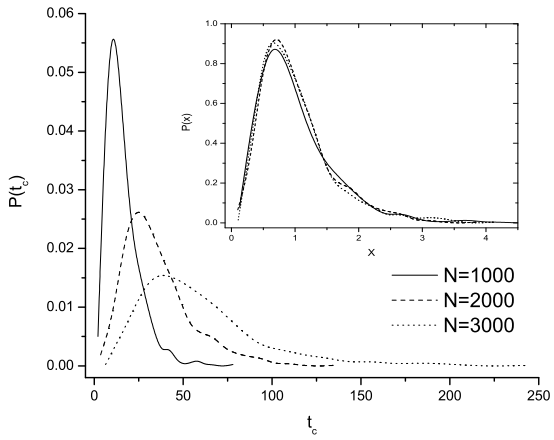

Fig. 6. Probability densities of the convergence time for three systems sizes. Data are gathered from 1000 independent realizations of the game. The average degree of the underlying RGGs is $\overline{k}\approx 50$. The inset shows the probability densities for the scaled variable $x = t_c/\overline{t_c}$ for the same data.

deviation, within error, scales in the same fashion with the number of nodes as the average itself, $\Delta t_c \sim N^{1.21}$, [Fig. 5]. The lack of self-averaging for large systems (strong dependence on the individual runs) of the NG is inherently related to the coarsening dynamics, having a single interface wondering at the latest stage (and not to the underlying random structure). Suppressing large average convergence times and the corresponding large standard deviations will be addressed in the next section.

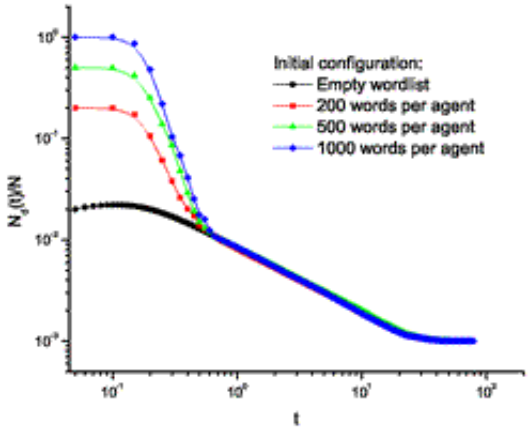

Fig. 7. Time evolution of the (scaled) number of different words for three initial size of the word list on log-log scales. The number of nodes is $N = 1000$ with average degree $\overline{k}\approx 50$. For comparison, the same quantity is also shown for the "empty" word list initial condition.

To model random key selection from a pre-existing list of keys, we also considered the NG with a different set of initial conditions, where each node has the same list of keywords. The results for $N{=}1000$ agents and for three different initial length of the word list is shown in Fig. 7, together with the previous "empty word list" initial condition for comparison. As can be seen, agents' choices quickly converge locally to small spatial clusters of nodes having the same word, followed by the *late-stage* coarsening dynamics, identical to the one observed with the "empty word list" initial condition. The scaling properties, and the average time to convergence, hence are identical to those discussed earlier, regardless of the different initial conditions.

III. Conclusion and Outlook

In this paper, we have explored the Naming Games on Random Geometric Graphs, a simple model for agreement dynamics in a large-scale sensor networks. Such a scheme can be applicable in scenarios when the outcome of the agreement process (in terms of the value of the IDs, tags, or keys) is preferred not to be biassed toward extremal values. We have found that qualitatively similar to two-dimensional regular networks, the NG on RGG can be reasonably well described by the physical theory of coarsening. In particular, local clusters of nodes sharing the same word quickly form, followed by slow coarsening of these clusters in the late stage of the dynamics. The typical linear size (diameter) of the clusters scales as $\xi(t)\sim t^{\gamma}$. Our simulation results, $\gamma\approx 0.38$, indicate that, at least for the range of finite system sizes

studied here (up to $N{=}5000$), the characteristic length scale for the NG in the RGG grows slower than in its regular two-dimensional counterpart. In turn, the average time to reach global agreement increases somewhat faster on RGGs.

In addition to the typical time required to reach global agreement $\mathcal{O}(N^{1.23})$ (for *fixed* average degree), we can also estimate the typical message complexity of the NG for an efficient implementation in sensor networks. This quantity directly corresponds to the overall power consumption until global agreement is reached, which is of utmost importance in sensor networks. After just a few, of $\mathcal{O}(1)$, broadcasts per node, coarsening begins, and locally agreeing spatial clusters form. Nodes inside the clusters have reached agreement with all their neighbors, of which they are readily aware, hence, they no longer have to initiate broadcasts any longer. The number of "active" nodes, however, found at the interfaces between these cluster and which have at least one neighbor with different word(s), will initiate broadcast for word matching, but their number [typical number of nodes at the interfaces, Eq. (2)] decays as N/t^{γ}. Hence, the the total number of broadcasts during the game can be written as $(\overline{k}+1)\sum_{t=1}^{t_c} Nt^{-\gamma} \sim (\overline{k}+1)N\int_1^{t_c} dt\ t^{-\gamma} \sim (\overline{k}+1)Nt_c^{1-\gamma} \sim (\overline{k}+1)\mathcal{O}(N^{1.76})$, where in the last step, we used our empirically found scaling $t_c{\sim}N^{1.23}$ and the exponent $\gamma{\approx}0.38$. The prefactor $(\overline{k}+1)$ in the above scaling form of the total number of broadcasts can be further reduced by eliminating a progressively increasing fraction of responses (and the corresponding broadcasts) by the listeners: only those listeners that are *not* successful respond. Then, after listening for an appropriate default period, the speaker node would know the outcome of her word matching attempt by default: no response means success for the particular listener. As the fraction of active nodes decays, this implementation will cut significantly the above prefactor from $(\overline{k}+1)$ to $(\text{const.} \times \overline{k}/t^{\gamma}+1)$.

In light of recent results on NG on SW networks [14], one can consider accelerating the agreement process by adding a small fraction of possibly long-range "random" communication links between certain nodes of the RGG. Such networks have long been known to speed up the spread of local information or epidemics to global scales [30]–[32], with applications ranging from synchronization problems in distributed computing [33] to alarm-detection schemes in wireless sensor networks [34]. For sensor networks, this can be implemented either by including a small fraction of sensors equipped with long-range unidirectional antennas ("physical" long-range connections) or by establishing designated multi-hop transmission patterns (logical long-range connections) between certain nodes [35]. The NG on RGG with random long-range links is expected to exhibit scaling behaviors identical to that of the NG on SW networks [14]. In particular, the time to global agreement in the resulting SW-like sensor network shall be reduced from $\mathcal{O}(N^{1.23})$ to $\mathcal{O}(N^{0.5})$ [14]. We also expect the standard deviation of the agreement time to decrease. The nodes participating in the the required long-range or multi-hop communications may consume more power than the majority of the ordinary nodes. Globally, however, it may be (not only in terms of temporal reduction but also energetically) beneficial to pay the increased local costs for a small fraction of the sensor nodes, provided global agreement is reached much faster. We are currently investigating such potential trade-offs. Our future works will also address NG on more realistic communication topologies, relevant to wireless sensor networks, in particular, random spatial networks with heterogeneous range distribution, and also networks with dynamically changing connectivity.

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REFERENCES

[1] F.A. Matsen and M.A. Nowak, "Win-stay, lose-shift in language learning from peers", Proc. Natl. Acad. Sci. USA **101**, 18053–18057 (2004).

[2] G. Deffuant, D. Neau, F. Amblard, and G. Weisbuch, "Mixing beliefs among interacting agents", Adv. Compl. Syst. **3**, 87–98 (2000).

[3] E. Ben-Naim, "Opinion dynamics: Rise and fall of political parties", Europhys. Lett **69**, 671–677 (2005).

[4] Y. Lee, T.C. Collier, C.E. Taylor, E.E. Stabler, "The role of population structure in language evolution", Proceedings of the 10th International Symposium on Artificial Life and Robotics (2005).

[5] T.C. Collier and C.E. Taylor, "Self-Organization in Sensor Network", Journal of Parallel and Distributed Computing 64, 866-873 (2004).

[6] L. Steels, "The synthetic modeling of language origins", Evolution of Communication **1**, 1–34 (1997).

[7] S. Kirby, "Natural Language from Artificial Life", Artificial Life **8**, 185–215 (2002).

[8] Luc Steels, "The Origins of Ontologies and Communication Conventions in Multi-Agent Systems", Autonomous Agents and Multi-Agent Systems 1, 169-194 (1998).

[9] Luc Steels, "A self organizing spatial vocabulary", Artificial Life **2**, 319-332 (1995).

[10] Luc Steels and Angus McIntyre, "Spatially Distributde Naming Games", ECAL 97 (1997).

[11] A. Baronchelli, M. Felici, E. Caglioti, V. Loreto, and L. Steels, "Sharp Transition towards Shared Vocabularies in Multi-Agent Systems", arXiv:physics/0509075 (2005a).

[12] A. Baronchelli, L. Dall'Asta, A. Barrat, V. Loreto, "Strategies for fast convergence in semiotic dynamics", arXiv:physics/0511201.

[13] A. Baronchelli, L. Dall'Asta, A. Barrat, V. Loreto, "Topology Induced Coarsening in Language Games", Phys. Rev. E **73** (2006) 015102(R); arXiv:physics/0512045.

[14] L. Dall'Asta, A. Baronchelli, A. Barrat, V. Loreto, "Agreement dynamics on small-world networks", Europhys. Lett. **73**, 969–975 (2006); arXiv:cond-mat/0603205.

[15] D. Angluin, "Local and global properties in networks of processors", in *Proceedings of the 12th ACM Symposium on Theory of Computing*, 82–93 (1980).

[16] D.S. Hirschberg, and J.B. Sinclair, "Decentralized extrema-finding in circular configurations of processors", in *Communications of the ACM* **23**, 627–628 (1980).

[17] G. LeLann, "Distributed systems, towards a formal approach", in *IFIP Congress Proceedings*, 155–160 (1977).

[18] N. Malpani, J. Welch and N. Vaidya", "Leader Election Algorithms for Mobile Ad Hoc Networks", in *Proc. Fourth International Workshop on Discrete Algorithms and Methods for Mobile Computing and Communications*, 96-103 (2000).

[19] S. Vasudevan, B. DeCleene, N. Immerman, J. Kurose, D. Towsley, "Leader Election Algorithms for Wireless Ad Hoc Networks", in *Proc. of the 3rd DARPA Inf. Survivability Conf. and Exposition (DISCEX- III)* (2003).

[20] S. Vasudevan, J. Kurose, and D. Towsley, "Design and Analysis of a Leader Election Algorithm for Mobile Ad Hoc Networks", in *Proc. of the 12th IEEE International Conference on Network Protocols* (2004).

[21] B. DeCleene et al., "Secure group communications for wireless networks", in *Proc. of MILCOM*, VA, 2001.

[22] R. Meester and R. Roy, *Continuum Percolation* (Cambridge University Press, 1996).

[23] M. Penrose, *Random Geometric Graphs* (Oxford University Press, 2003).

[24] J. Dall and M. Christemsen, "Random geometric graphs", *Phys. Rev. E* **66**, 016121 [9 pages] (2002).

[25] P. Gupta and P.K.Kumar, "The Capacity of Wireless Networks", IEEE Trans. Inf. Theor. **IT-46**, 388–404 (2000).

[26] F. Xue and P.R. Kumar, "The number of Neighbors Neeeded Connectivity of Wireless Networks" Wireless Networks 10, 169–181 (2004).

[27] B. Krishnamachar, S.B. Wicker, and R. Béjar, "Phase Transition Phenomena in wireless Ad Hoc Networks", Symposium on Ad-Hoc Wireless Networks, IEEE Globecom, San Antonio, Texas, November (2001);

[28] W. Krause, I. Glauche, R. Sollacher, and M. Greiner, "Impact of Network Structure on the Capacity of Wireless Multihop Ad Hoc Communication", Physica A **338**, 633–658 (2004).

[29] A. Bray, Adv. in Phys. **43**, 357–459 (1994).

[30] D.J. Watts and S.H. Strogatz, "Collective dynamics of small-world networks", *Nature* **393**, 440–442 (1998).

[31] D.J. Watts, "Networks, dynamics, and the small-world phenomenon", *Am. J. Soc.* **105**, 493–527 (1999).

[32] M.E.J. Newman, "The structure and function of complex networks", *SIAM Review* **45**, 167–256 (2003).

[33] G. Korniss, M.A. Novotny, H. Guclu, Z. Toroczkai, and P.A. Rikvold, "Suppressing Roughness of Virtual Times in Parallel Discrete-Event Simulations", *Science* **299**, 677–679 (2003).

[34] Q. Lu, G. Korniss, and B.K. Szymanski, "Threshold-Controlled Global Cascading in Wireless Sensor Networks", Q. Lu, G. Korniss, and B.K. Szymanski, to appear in the Procedings of INSS 2006.

[35] A. Helmy, "Small worlds in wireless networks", *IEEE Comm. Lett.* **7**, 490–492 (2003).