Dynamics of Internal and Global Structure through Linguistic Interactions

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Abstract. Development of category structure in communication is studied by a constructive approach. Individuals having a word relation matrix as their internal structure communicate by uttering and accepting sentences. Words in sentences uttered are situated in relation with other words by each individual. Words make clusters according to their interrelationships. The structures and dynamics of clusters are studied. Coexisting commonality and individuality of clustering structure is observed in an ensemble of individuals.

1 Introduction

A Constructive Approach to Dynamically Complex Systems. Increasingly, biological, ecological, brain, language and social systems have been studied as complex systems. One of the characteristics of these systems is that they show various spatio-temporal dynamics, such as coexisting stability and adaptability, collective behavior, diversification, differentiation, hierarchical level formation, emergence or open-ended evolution. Chaotic behavior is also sometimes observed in these systems. It is thought that these characteristics arise from dynamical many-to-many interactions and high dimensionality (Kaneo and Tsuda, 1997). Some researchers have pointed out that considerations of self-referential features or internal observers are also crucial to understanding such dynamically complex systems (Tsuda, 1991; Gunji et al., 1997).

The conventional manner of studying these systems is to attempt to describe them objectively. However, structural instability or undecomposability between observers and observed systems, both of which typically occur in complex systems, can make such objective descriptions very difficult. In such cases, constructive approaches can be effective for studying the systems (Kaneo and Tsuda, 1994). One example of such an approach is to build models with elements having internal dynamics that interact with each other, and to then observe the emergence of global behavior. However, we assert that observation of the emergence of a global order is not enough in itself, since the phenomena shown in...
complex systems are usually not static but dynamic, and sometimes show openended evolution. Therefore, the constructed models must demonstrate not only emergence but dynamics of global behavior through the dynamics of elements.

Perhaps the key point in modeling complex systems is the introduction of the dynamics of elements. Elements can change their internal states and their relationship to other elements. By the individual dynamics, global and local levels are dynamically coupled, and this dynamical coupling drives the systems to be dynamic also at the global level (Kaneko and Tsuda, 1994).

Language as Dynamics. Language also should be studied as a dynamical complex system. Language can be viewed in two ways: structurally and dynamically. The structural view is a static one in which language structure, e.g. syntax (Chomsky, 1957) or pragmatic rules (Grice, 1975; Sperber and Wilson, 1986), offers idealized approaches to language. The alternative view is dynamic. It concentrates on the actual use of language rather than abstract notions of how language ought to be. Language is considered as a collection of dynamical processes based on the individual creative acts of speaking, hearing, writing, and reading.

Bakhtin has termed these two views abstract objectivism and individualistic subjectivism, respectively (Vološinov, 1986). He sketched out these two views as follows. In the former view, language is understood as “a stable, immutable system on normatively identical linguistic forms.” In the latter view, in contrast, language is thought of as “activity, an unceasing process of creation realized in individual speech acts.” Bakhtin stressed the importance of identifying language from the individualistic subjective standpoint, stating that “What is important for the speaker about a linguistic form is not that it is a stable and always self-equivalent signal, but that it is an always changeable and adaptable sign.”

Clearly, the use of language is such a creative process, rather than the recognition of the “pre-established” forms. Words uttered are not connected with \textit{a priori} and constant meanings. Words have fundamental polysemy. Language users are continually creating sense in fluctuating contexts by situating words in relation to other words (Fukaya and Tanaka, 1996).

Another way to understand the value of the dynamical view of language is by considering the notion of the metaphor. Metaphorical expressions are creative or dynamic precisely because they can “bend” conventionally structured language. By producing or understanding metaphorical expressions, especially creative metaphors, our internal structure should change. We cannot say that creative expressions are valid or not valid, since they are novel and cannot be understood by this distinction of a conventional language structure. Rather, we should consider whether or not we accept the expressions. If we accept them, our internal structure changes and our language structure might also come to be modified.

A constructive approach is also useful for studying language systems (Steels, 1997). The emergence of global order as a language-like behavior is observed by modeling individuals in terms of their linguistic interactions. This way of study-
ing language constitutes a part of that of evolutionary linguistics. Evolutionary linguistics throws a bridge across the two viewpoints of language. The global structure, i.e. the structure common to individuals, described in the static view should emerged as a result of the dynamic processes of using language. It must be noted that the global structure in language should not be static. The change in internal structure resulting from the use of language induces the dynamics of the structure of language. In keeping with this point, we have previously studied a system of evolutionary grammars, which shows the emergence and dynamics of grammars common to communication networks (Hashimoto and Ijegami, 1996).

In this paper, we study the development of category structure from the viewpoint of language as a dynamical phenomenon. We have constructed a model of an ensemble of individuals communicating with each other. Individual agents understand sentences uttered by situating words in relation to other words. This results in an internal structure, the word relation matrix, peculiar to each individual. In this way, we continue to investigate our previously proposed usage-based viewpoint, in which the meaning of words is understood in terms of how language is actually used (Hashimoto, 1997). The usage of words in communication thus reflects the word relation matrix of each agent.

2 Algorithm for Calculating Word Relationships

We study the dynamics of categorization by observing how words in sentences are situated in relation to each other through communication between artificial agents. Each agent has its own word relationship matrix as its internal structure, which develops according to the agent’s utterances and acceptances.

The algorithm for calculating the similarity between words in sentences is basically that of Karov and Edelman (Karov and Edelman, 1996) with a few modifications. They have presented an algorithm for word sense disambiguation based on similarity among words. A key concept in the algorithm is the mutual dependency between words and sentences. That is, similar words appear in similar sentences and similar sentences are composed of similar words. Similarities between words are calculated from similarities among sentences in which the words are used. The similarity between sentences is derived from that of words used in the sentences.

We make two modifications on their algorithm. One is that we consider the correlation of appearance between words in texts in deriving the relationship among words. A text is a set of sentences or, in the context of this paper, a conversation. If patterns of appearance of words in texts are similar, e.g., some words appear many times in a text but do not do so in the other texts, these words are considered to have high affinity. Conversely, different appearance patterns between words imply a disaffinity between them. The other revision is that the relationships are calculated immediately after uttering or accepting a sentence, since we are interested in the dynamics of word relationships in conversations.

126
The relationship between words and between sentences of an agent \( k \) are respectively defined by the following formulae:

\[
P^k_{t,n}(w_i, w_j) = \begin{cases} 
\alpha^w \frac{\sum_{s \in \mathcal{S}} K^k_{t,n}(s) A^k_{t,n}(s, w_j)}{\sum_{s \in \mathcal{S}} K^k_{t,n}(s)} + \frac{1}{2} \left( \frac{D^k_{t,n}(w_i, w_j)}{\sqrt{N^k_{t,n}(w_i) N^k_{t,n}(w_j)}} + 1 \right) & \text{if } i \neq j \\
1 & \text{if } i = j 
\end{cases}
\]

\[
P^k_{t,n}(s_i, s_j) = \begin{cases} 
\alpha^s \frac{\sum_{w \in \mathcal{W}} K^k_{t,n}(w) A^k_{t,n}(w, s_j)}{\sum_{w \in \mathcal{W}} K^k_{t,n}(w)} + \frac{1}{2} \left( \frac{D^k_{t,n}(s_i, s_j)}{\sqrt{N^k_{t,n}(s_i) N^k_{t,n}(s_j)}} + 1 \right) & \text{if } i \neq j \\
1 & \text{if } i = j 
\end{cases}
\]

(1)

(2)

Here, the subscripts \( n \) and \( t \) indicate a sentence and a text, respectively. A text is a conversation. The numbering system for sentences and texts is illustrated in Fig. 1. The binary relation \( w \in s \) means that a word \( w \) is used in a sentence \( s \), and \( s \ni w \) means that a sentence \( s \) uses a word \( w \). The first terms in (1) and (2) describe the usage similarity between words and between sentences, respectively, and the second terms the correlation between appearance of words and between that of sentences in texts, respectively. These two terms are combined with the coefficients \( \alpha^w \) and \( \alpha^s \) for words and sentences, respectively.

The affinity of sentences to words and that of words to sentences are defined using the relationship between words and between sentences as follows:

\[
A^k_{t,n}(s, w) = \frac{\sum_{s' \ni w} K^k_{t,n}(s') P^k_{t,n}(s, s')}{\sum_{s' \ni w} K^k_{t,n}(s')} 
\]

(3)

and

\[
A^k_{t,n}(w, s) = \frac{\sum_{w' \ni s} K^k_{t,n}(w') P^k_{t,n-1}(w, w')}{\sum_{w' \ni s} K^k_{t,n}(w')} 
\]

(4)

respectively. In the four equations, (1) \sim (4), weights for similarities and affinities define the contribution made by the appearance probability of each word and sentence and by the length of each sentence. The weight factors are defined as

\[
K^k_{t,n}(s) = \frac{P^k_{t,n}(s)}{l_g(s)} 
\]

(5)

and

\[
K^k_{t,n}(w) = \frac{1}{P^k_{t,n}(w)} 
\]

(6)

where \( l_g(s) \) is the length of a sentence \( s \), which is defined by the number of words included in the sentence; \( P^k_{t,n}(w) \) and \( P^k_{t,n}(s) \), to be defined later, are the appearance probablity of a word \( w \) and that of a sentence \( s \) in all sentences from the first to the \( n \)th sentence, respectively. These weight factors mean that
the more a word is used, the less informative it is; that the more a sentence is used, the greater its contribution is; and that a word in a longer sentence is less important than one in a shorter sentence.

Due to the symmetry of words and sentences in the functions \( D_{i,n}^k \) and \( N_{i,n}^k \), we can use a symbol \( x \) to denote a word \( (w) \) or a sentence \( (s) \) in the following equations. In order to consider the dynamics of internal structure through conversations, the correlation of words and sentences should be incrementally updated per each utterance and acceptance of sentences. The functions \( D_{i,n}^k \) and \( N_{i,n}^k \) are recursively defined as

\[
D_{i,n}^k(x_i, x_j) = D_{i-1,n^*(t-1)}^k(x_i, x_j) + T_{i,n}^{k,2}(p_{i,n}^k(x_i) - P_{i,n}^k(x_i))(p_{i,n}^k(x_j) - P_{i,n}^k(x_j)),
\]

and

\[
N_{i,n}^k(x_i) = N_{i-1,n^*(t-1)}^k(x_i) + T_{i,n}^{k,2}(p_{i,n}^k(x_i) - P_{i,n}^k(x_i))^2,
\]

where \( n^*(t) \) is the last sentence in the \( t \)th text.

The appearance probability of \( x \) in all sentences from the first to the \( n \)th sentence \( P_{i,n}^k(x_i) \) is defined as

\[
P_{i,n}^k(x_i) = \frac{\sum_{t'=1}^{t-1} h_{t',n^*(t')}^k(x_i) + h_{i,n}^k(x_i)}{\sum_{t'=1}^{t-1} T_{t',n^*(t')}^{k,2} + T_{i,n}^{k,2}},
\]

and the appearance probability of \( x \) in a text \( t \) by the \( n \)th sentence is defined as

\[
p_{i,n}^k(x) = \frac{h_{i,n}^k(x_i)}{T_{i,n}^{k,2}},
\]

where \( h_{i,n}^k(x_i) \) is frequency, i.e., the number of appearances, of a word or a sentence, and \( T_{i,n}^{k,2} \) is the length of a text counted by \( x_i \), i.e., the number of all words or that of all sentences, in a text \( t \) by the \( n \)th sentence (see Fig. 1).

By uttering or accepting a sentence, the word-sentence affinity \( (4) \) is calculated from the word relation matrix at uttering/accepting the previous sentence; and the correlations among words and among sentences are updated. Then, these formulae are calculated in the order of \( (2) \rightarrow (3) \rightarrow (1) \). The relationship of a word with itself is set to be \( R_{i,0}^k(w_i, w_j) = \delta_{i,j} \) for new words before the calculation. In the first text, \( D_{1,n}^k(x_i, x_j) \) and \( N_{1,n}^k(x_i) \) are 0, but the correlation of a word or a sentence with itself is always 1, i.e., \( D_{i,n}^k(x_i, x_j) / \sqrt{N_{i,n}^k(x_i)N_{i,n}^k(x_j)} = \delta_{i,j} \).

3 Modeling for Conversations

In our model there are \( M \) topics \((0 \sim M)\), which are placed on a one-dimensional space with the periodic boundary. A randomly selected topic with zero, one or two adjacent topics is presented to agents at the start of a conversation. The presented topics are collectively called a situation and a conversation is called a text.
In the initial state, agents do not know any words and do not have a name for any of the topics. At the beginning of each text, two agents are randomly selected to converse. A speaker agent randomly selects a topic from the presented situation and utters a sentence that includes the name for the topic. The speaker then updates its word relation matrix according to the uttered sentence. In the case that an agent has not had a name for the selected topic at the time of utterance, it creates a word by combining characters as the topic name.

A hearer agent accepts the uttered sentence if there are zero unknown words or one unknown word in the sentence. After acceptance, the hearer's word relation matrix is updated according to the accepted sentence, and then the agents take turns alternating their roles of speaker and hearer. The new speaker agent makes a sentence in reply to the accepted sentence. If the uttered sentence is not accepted by the hearer, the agents do not take turns and the speaker again selects a topic at random from the presented situation. When 5 sentences are not accepted or the number of sentences accepted by both agents becomes 100, the text is concluded, and then another pair of agents and a new situation are selected for a new text.

To make a sentence for utterance, a speaker arbitrary selects a focus word from the words in the accepted sentence. At the commencement of a text, the name for the selected topic serves as the focus word. The speaker adopts a sentence with the highest affinity to the focus word from the word-sentence affinity matrix (4) of the agent. If the adopted sentence is the same as the accepted sentence, the second highest affinity sentence is adopted. If the adopted sentence is the same as the sentence uttered by the speaker in the immediately previous conversation, then the third highest affinity sentence is adopted.

An agent modifies the adopted sentence to be uttered in proportion to the creativity rate $c$. There are three means of modification: addition, replacement, and deletion of a word. One of the three is selected depending on the sentence modification rates, $m_{\text{add}}$, $m_{\text{rep}}$, and $m_{\text{del}}$, respectively. A word for addition or replacement is selected from already known words. The word itself is also modified by addition, replacement, or deletion of a character depending on the word modification rates, $m_{\text{add}}^w$, $m_{\text{rep}}^w$, and $m_{\text{del}}^w$, respectively.

4 Simulation Results and Discussion

First, to demonstrate the internal dynamics of individuals, we will summarize the results of simulated conversations between two agents in §4.1 and §4.2, and then describe that of simulations in an ensemble of agents in §4.3. The simulations are made using the following parameters. The maximum of topics $M$ is 9. The set of characters is \{a, e, i, o, u\}. The creativity rate $c$ is 0.1. The sentence modification rates are $m_{\text{add}} = 0.2$, $m_{\text{rep}} = 0.7$, and $m_{\text{del}} = 0.1$. The word modification rates are $m_{\text{add}}^w = 0.33$, $m_{\text{rep}}^w = 0.34$, and $m_{\text{del}}^w = 0.33$. The weight coefficients for similarity and correlation in (1) and (2) are $\alpha_w = \alpha^w = 0.4$ in the tw-agent simulations and 0.6 in the ensemble simulations. The results described
here are for these specific parameter values; however nearly the same results can
be observed using a wide range of parameter values.

An example of a conversation is as follows:

TEXT: 25  SITUATION:  0 1 2
TOPIC: 2
agent1.name[2]= ea   agent0.name[2]= i
1: ea-i                      0: ea-eii-iii-iiuo-eii
1: ea-eii-eii-eii           0: ea-i
1: ea-eii-iii-iiuo-eii      0: iiuo
1: eoi                       0: i-eoi
1: i-eoi                     0: eoi
1: i-ei-eoi                  0: i-eoi
1: eoi                       0: i-ei-eoi
1: i                         0: i-eii
1: eii-eii-eii              0: eii-eii
1: eii-eii                  0: eii-eii-ii-iiuo
1: eii-eii-eii              0: eii-eii
1: eii-eii                  0: eii-eii-eii

This example is the beginning part of text t = 25 of a simulation. The situation
is composed of the topics 0, 1 and 2. The agent 1 selects the topic 2. The agent
1 has already named the topic 2 ’ea’ and the agent 0 has named it ’i.’ As in this
example, agents in a conversation can adopt different names for the same topic.
The agent 1 utters a sentence “ea-i” based on the focus word ’ea,’ which is the
name for topic 2. The agent 0 accepts the sentence and replies with the sentence
“ea-eii-iii-iiuo-eii” based on the focus word ’ea.’

4.1 Word Clusters

Flat and Gradual Clusters. Words make clusters according to their interre-
lationship. Figure 2(a) gives an example of a cluster structure. This figure is a
scattered diagram drawn using the results of the principal coordinate analysis of
the word relationship matrix $R(w_i, w_j)$ (1). We can see four clusters of words in
the figure. There are roughly two types of clusters. The first is a cluster in which
words are distributed over rather small regions, such as the words 1 ~ 4 and 9
~ 12 in the figure. The other is a cluster in which words lie in a long and narrow
region, such as the words 5 ~ 8, here. The former type is called a flat cluster,
for if we draw word relation matrix directly, as in Fig 2(b), this type of cluster
is seen as having a flat top. In the latter type, the word relationship gradually
changes from one end of the cluster to the other. Since the relationship between
words gradually decays, as shown in Fig 2(b), we term this a gradual cluster.

Near the start of the simulations, a few flat clusters exist rather independ-
ently. Flat clusters extend their boundaries with new words and come to have
gradual edges. Some sentences connect separated clusters. The structure of the
word relationship is usually made up of a combination of the two types of clus-
ters, as in Fig 2.
Fig. 2. (a) An example of scattered diagram of the word relation matrix $R(w_i, w_j)$ processed by the principal coordinate analysis. The X and Y axes are the first and second principal coordinates obtained by the analysis, respectively. Each number in the diagram indicates a word. (b) The word relationship matrix $R(w_i, w_j)$ shown in (a) is directly drawn in this graph. Words are listed in the X and Y axes. The number for each word corresponds to that of in (a). The Z axis is the value of $R(w_i, w_j)$.

Clustering as Categorization. Clustering can be regarded as a categorization of words by an agent through conversations, since words in a cluster have a stronger relation with each other and a weaker relation with words out of the cluster. The two major types of cluster structure, flat and gradual, correspond to two different concepts of a category. Because the boundaries of flat clusters are sharp, it is easy to distinguish whether or not an entity is a member of the cluster. This is a conventional concept of a category in which membership is rigidly determined by necessary and sufficient conditions. In contrast to the flat cluster, a gradual cluster shows a graduated change in relationship from large to small. This corresponds to a new concept of categorization. In the new concept, the degree to which words are included in a category is a matter of the gradient.

In actual simulations, these two types of clusters present in combination. That is, a cluster is likely to have a flat top and a graded boundary. In terms of categories, the flat top portion corresponds to central members of the category, and words having only a slight relation to the central words are peripheral members. This structure resembles that of a prototype category (Lakoff, 1987; Taylor, 1995).

4.2 Dynamics of Word Relationships and Clusters

Change of Word Relationships by New Usages. Word relationships change with conversations. We exemplify the transitions of the relationship of a word to other words in Fig. 3(a). This constitutes a portion of dynamics of the internal structure of an agent. Each line is sustained at a particular value with some fluctuations for some period of texts and with occasional large and abrupt changes. Simultaneous and large changes in relationships to many words are sometimes
observed, as \( t = 21 \) in the figure. The high value of some words are lowered, and the small values of other words are rapidly increased.

![Fig. 3. (a) An example of a transition of word relationship per each text is depicted. The Y axis represents the relationship of the word \( aa \) to the other words \((R(aa, w))\) of an agent at the end of each text. The X axis is the number of texts. Since the word \( aa \) appears at \( t = 9 \), the graph starts from that text. Lines arising from the X axis indicate the appearances of new words. (b) This is a transition of word relationship of the word \( aa \) to the other words per each uttered/accepted sentence in the text \( t = 21 \). The X axis is the number of uttered/accepted sentences (\( n \)) and the Y axis is the word relationship \( R(aa, w) \). Between \( n = 2070 \) and \( n = 2080 \), many words switch their relationships rapidly and simultaneously.

The simultaneous and large changes in word relation are induced by appearances of rare or new usages of words. Uttering or accepting a sentence in which a new word is used or a word is used in an unusual way may vary the word relationship largely and rapidly. We describe the dynamics of \( R(aa, w) \) per each utterance/acceptance of sentences in the text \( t = 21 \) in Fig. 3(b). A simultaneous and large change is observed between \( n = 2070 \) and \( n = 2080 \). The utterance of a sentence containing words in rare way of use at \( n = 2074 \) results in a large and rapid change in dynamics. Words used in this text (\( t = 21 \)) develop their relationship, and words that are not used in this text but that are used in other texts diminish their relationship. However, changes in the relationship to ubiquitously used words as a result of this appearance of rare usage are small.

**Cluster Structure Dynamics.** We next inquire how large changes in word relationships affect the cluster structure by observing word movement in scattered diagrams. The sentence-word affinity matrices \( A(s_i, w_j) \) at before and after the large change (\( n = 2074 \) and \( n = 2077 \)) are combined to form one matrix, and this matrix is processed using principal component analysis (PCA). The first and second principal components are used to draw the scattered diagram in Fig. 4. Corresponding words between before and after the large change are connected by arrows.
From this diagram, it can be seen that words in a cluster move in almost the same directions before and after this change: therefore, the whole structure of clusters does not change drastically. But the word *aa*, which is used in a rare way at *n* = 2074, moves in a different direction from words in the cluster at *n* = 2074. It changes the cluster to which it belongs. A large change in the word relationships between a particular word and others does not restructure the whole system of clusters, but this change effects the word relationship incrementally with each subsequent step. Thus the entire clustering structure may slowly transform.

**Dynamical Stability and Adaptability.** By uttering or accepting sentences in which usage of words are new or rare, some words change their positions in the clustering structure. But most words move coherently in the same direction, and thereby avoiding a sudden transformation of the whole structure. It can be said that the structures of clusters have the characteristics of stability and adaptability.

The dynamical stability and adaptability often seen in complex systems must be equipped for language systems. If a language is too rigid, its users will not be able to formulate new expressions to describe diverse experiences, and if it is too unstable, no structurization will be possible at either the individual of global level will be possible, and hence no communication will take place. Geeraerts explains this point as it pertains to categorization: “To prevent the categorical system from becoming chaotic, it should have a built-in tendency towards structural stability, but this stability should not become rigidity, lest the system stops being able to adapt itself to the ever-changing circumstances of the outside world (Geeraerts, 1985).”

**4.3 Global Level**

In the above sections, we have sketched out the development of the internal structure in individuals and its dynamics through conversations. In this section, we
consider communication in an ensemble of agents. The focus issues are whether a structure common to the ensemble emerges and how individual dynamics effect the common structure, i.e., how commonality and individuality develop.

**Definition of Distance Rate.** Representing the disparity between agents numerically provides clues to understanding the degree to which the agents have a common internal structure and how the common structure behaves. We consider the Euclidean distance between word relation matrices of two agents as the distance between the agents. But since agents generally know different words, the matrices must be reconstructed for the words which either agent knows. In terms of a symbol \( W_{l,n}^k \), denoting a set of words which an agent \( k \) knows at the \( n \)th sentence in a text \( t \), a word relation matrix of an agent \( k \) is formally described as \( R_{l,n}^k(w_i, w_j | w_i, w_j \in W_{l,n}^k) \). The reconstructed word relation matrix of an agent \( k \) to another agent \( k' \) is defined for words which either agent knows \((w_i \in W_{l,n}^k \cap W_{l,n}^{k'})\). For known words by the agent \( k \) \((w_i \in W_{l,n}^k)\), elements of a reconstructed matrix of an agent \( k \) are the same as those of the original matrix \( R_{l,n}^k \). Those for words unknown by the agent \( k \) but known by the other agent \( k' \) \((\{w_i | w_i \notin W_{l,n}^k \land w_i \in W_{l,n}^{k'}\})\) are set to zero. In summary, the reconstructed matrix of an agent \( k \) to another agent \( k' \) is defined as

\[
R_{l,n}^{k|k'}(w_i, w_j | \forall w_i, w_j \in W_{l,n}^k \cup W_{l,n}^{k'}) = \begin{cases} 
R_{l,n}^k(w_i, w_j) & \text{if } w_i, w_j \in W_{l,n}^k \\
0 & \text{if } (w_i \notin W_{l,n}^k \land w_i \in W_{l,n}^{k'}) \lor \\
& (w_j \notin W_{l,n}^k \land w_j \in W_{l,n}^{k'}) 
\end{cases}
\]  

(11)

The distance between two agents \( k \) and \( k' \) is defined with the reconstructed matrices as

\[
\rho_{l,n}^{k, k'} = \sqrt{\sum_i \sum_j (R_{l,n}^{k|k'}(w_i, w_j) - R_{l,n}^{k|k'}(w_i, w_j))^2} \ .
\]  

(12)

Since the number of known words always increases in the simulation, the dimension of the matrix \( R_{l,n}^{k|k'} \) becomes bigger and bigger and the distance \( \rho_{l,n}^{k, k'} \) seldom decreases. We take the average of the distance by the number of words,

\[
\bar{\rho}_{l,n}^{k, k'} = \frac{\rho_{l,n}^{k, k'}}{|W_{l,n}^k \cup W_{l,n}^{k'}|} \ .
\]  

(13)

where \( |W_{l,n}^k \cup W_{l,n}^{k'}| \) is the number of words in \( W_{l,n}^k \cup W_{l,n}^{k'} \). We call \( \bar{\rho}_{l,n}^{k, k'} \) the distance rate between agents \( k \) and \( k' \) at the \( n \)th sentence in a text \( t \). The average for all pairs of agents is

\[
\langle \bar{\rho}_{l,n} \rangle = \frac{\sum_{\text{all pairs}} \bar{\rho}_{l,n}^{k, k'}}{K C_2} \ ,
\]  

(14)

where \( K \) is the number of agents in an ensemble and \( K C_2 \) is the number of pairs of agents.
Development of Common Structure. We report the results of simulations in which the number of agents \( K \) is 5. The coefficients in (1) and (2) are \( \alpha^w = \alpha^e = 0.6 \). The other parameters are the same as in the simulations in the previous subsections.

All agents share an identical initial state, that is, they do not know any words or sentences. Two agents are selected randomly to converse. Note that only one pair makes a conversation in one text, and that the agents that do not participate in the conversation cannot hear sentences uttered by the participants in the conversation. Therefore, each agent has its own experience of conversations and its own way of developing its internal structure. However, the difference between agents can be reduced by joining a conversation, since the agents conversing hear the same sentences.

Figure 5 shows the transition of the distance rates, including the distance rate for for each pair and the average rate for all pairs, at the end of texts per each text. At first the average distance rate is large, and then it converges at around \( \langle \bar{d} \rangle \approx 0.4 \). This means that each agent has both common and not common parts in its internal structure.

To see how the shared structure between a pair of agents is organized through conversations, we define the difference matrix as

\[
E_{k,k'}^{i,n} = |\hat{p}_{k,k'}^{i,n} - \bar{p}_{k,k'}^{i,n}|
\]

In Fig 6, we depict scattered diagrams of the difference matrices for a pair of agents. Figure 6(a) is at the end of text \( t = 9 \). This is the first text in which the pair of agents converse. Figure 6(b) is at the end of text \( t = 35 \) which is the fifth text of a series of conversations between the two agents. Since the agents converse with the other agents before the text \( t = 9 \), they develop their own structure. After the five conversations between them, the difference is more organized than at the end of first text. The common words, i.e. those that share a common relationship, and the not common words are clearly separated.

The ratio of elements for which the difference between two agents in a pair is larger than 0.5 to all elements, namely, the percentage of words for which \( E_{k,k'}^{i,n} > 0.5 \) in all elements, decays exponentially as shown in Fig. 7(a). The number of texts in which the pair converse is taken as the X axis in the figure. The degree of commonality between the agents increases through conversations.

Although the similarity among the internal structure of agents seems to increase with conversations, the agents in a pair converse with the other agents,
Fig. 6. Structurization of difference between two agents through conversations. These graphs are scatter diagrams of difference matrices at the end of texts, $E_{t,n^*(t)}^{k,k'}$, processed by the principal coordinate analysis. Each circle indicates a word. (a) The end of the first text in which the agents converse. (b) The end of fifth text in a series of conversations between them.

Fig. 7. (a) The ratio of elements for which the difference between two agents larger than 0.5 ($E_{t,n^*(t)}^{k,k'}(w_i, w_j) > 0.5$) for a pair of agents to all elements in a difference matrix vs. the number of texts in which the pair converses. (b) Transition of the ratio of elements for which the difference between two agents becomes larger ($C_{t,n^*(t),n^*(t')}(w_i, w_j) > 0$) for a pair of agents to all elements in a change matrix through conversations.

and therefore disparity between the agents can again be increased. The manner by which differences between two agents change between two texts is expressed by the change matrix defined as

$$ C_{t,n,n',n'}^{h,k'} = E_{t,n'}^{h,k'} - E_{t,n}^{h,k'} . \quad (16) $$

If $C_{t,n,n',n'}^{h,k'}(w_i, w_j)$ is positive, the difference between two agents becomes larger, and it becomes smaller in the opposite case. Figure 7(b) shows the transition of the ratio of elements which the difference of word relationships between two agents increases with texts. The number of texts in which the pair participates in conversations is taken as the $X$ axis. Each point shows the percentage of the elements $C_{t,n^*(t),n^*(t')}(w_j, w_j) > 0$ at between the end of the text and the end of the next text in which the pair converses. This quantity does not decay monotonically but shows oscillatory behavior. Although the common part
evolves through conversations, the non-common part does not simply become small.

**Commonality and Individuality.** In the present model, achieving a mutual understanding in communication means that the words used in a conversation are situated in relationship to other words in the same way by partners, i.e., that these words form an equivalent structure in word relation matrices to other agents. Actually, as we have seen, although commonality to agents develops with conversations, the whole matrix does not come to be identical. This is because an agent interacts with not only one agent but several, and its internal structure changes accordingly. This results in an ever-changing relation among agents.

Mutual understanding is, of course, important. Of great significance in communication through language, however, is the openness with which an individual construe the utterances of others, i.e., the diversity of interpretation of utterances. Such openness drives the language to be dynamic. Coping with both commonality and individuality at the local level allows for stability and dynamics of language. The characteristics of a category system that has both dynamical stability and adaptability, as discussed in the previous section, may manage the compatibility of commonality and individuality.

Here we have seen coexisting common and non-common parts of agents, but the global-level structure has yet to be clarified. Due to computational limits, both the number of agents in an ensemble and the number of texts are small. Further inquiry into the structure and dynamics of larger systems will be needed.

## 5 General Discussion

**Dynamical Models for Origins of Languages** We can often understand what a sentence means even if it does not obey “rules of syntax” or “the precise word meanings found in dictionaries.” For example, even if one of the speakers in a conversation is a foreigner speaking in a faltering manner, a mutual understanding can still be reached. Such “ill-formed” usages may induce changes in a language and in the dynamics of the internal worlds of language users. Using language is a dynamical process, and is not limited to the mere recognition of idealized rules of grammar or word meanings. It is difficult to study the dynamical nature of language by abstracting these idealized aspects from an already “well-formed” language.

In this study, we have proposed one of the possible methods for modeling dynamical language phenomena. Agents in the model do not share explicit grammar, rather they share a way to infer relations among words used. As has been demonstrated, shared and non-shared categorical structures develop through conversations and these structures are modified by “new” usage of words.

The final aim of our studies is to inquire into not only development but the origins of language. Therefore, the agents in our model are given no explicit linguistic knowledge, such as of grammar or word use or meaning. And in fact,
at the start of the trials they know no words at all. To this end, the conventional, i.e. descriptive, studies for languages and their development are, of course, important. But there is a limit to which we can expand our knowledge of language origins using only the descriptions of present languages. The constructive approaches described here may give possible scenarios for the origins and development of languages. These two ways of studying should be complementarily developed.

**External World** To investigate the dynamics of categorical structure in conversation, we have focused primarily on inter-individual interaction. Actual language phenomena, however, are not divorced from the environments surrounding individuals. We often use language in unauthorized ways to describe new situations brought on by environmental changes.

Our model should be developed to incorporate the dynamics of the external worlds of language users. It is important to clarify how shared and non-shared structures of language develop depending on the relations between external and internal worlds.

**Application to Discourse Analysis** Karov and Edelman (1996) have presented the algorithm to calculate similarity among words for word sense disambiguation in corpora. We have made two revisions to the algorithm. The first is that we consider correlation of appearance between words in a series of conversations. The second is that we define the new algorithm such that it can update word relation matrices per each utterance and acceptance.

These modifications improve the algorithm’s ability to treat “contextual” information, as well as its ability to handle the dynamic feature of conversations. The original algorithm is to process a corpus, a static database of sentences, but conversations are dynamic streams of utterances. Although its details must be fine-tuned for natural language processing, our new algorithm may ultimately be applied to word sense disambiguation in conversations and discourse analysis.

### 6 Conclusion

We have studied the dynamics of categorization based on a dynamical view of language. We model the comprehension of word meaning in communication by situating them in relationship to other words. The relationship is derived from the usage of words in sentences in conversations. The agents develop categorical structure of words as clustering through conversations. There are central and peripheral members in each cluster, and the extent to which words belong to a cluster changes gradually. The cluster structure has both stability and adaptability. Common structure to an ensemble of agents is organized by communication. Individuality in agents is also maintained. This coexistence of commonality and individuality is an outgrowth of the diverse experience of communication.
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