Boundary effects in a three-state modified voter model for languages

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The standard three-state voter model is enlarged by including the outside pressure favouring one of the three language choices and by adding some biased internal random noise. The Monte Carlo simulations are motivated by states with the population divided into three groups of various affinities to each other. We show the crucial influence of the boundaries for moderate lattice sizes like $500 \times 500$. By removing the fixed boundary at one side, we demonstrate that this can lead to the victory of one single choice. Noise in contrast stabilizes the choices of all three populations. In addition, we compute the persistence probability, i.e., the number of sites who have never changed their opinion during the simulation, and we consider the case of "rigid-minded" decision makers.

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# 1 Motivation and Model

The political situation after the break-up of former communist powers and the emergence of new sovereign states in Europe and elsewhere, justify yet another look at linguistic practices as informed by geopolitical agendas with the ever growing asymmetric power relations and the ongoing struggle for the accumulation of linguistic and cultural capital. Pronounced language asymmetries with highly competitive behaviour have caused a situation in which no successor state can claim a one-and-only homogeneous "national language" without serious caveats. In complex decision making, there are typically no single agreements when large numbers of decision makers are expected to choose from a large set of alternatives [1]. Any attempts to tackle the intricacies of these phenomena at the more global level, stumble across a series of theoretical and methodological problems. However, more recent studies have shown that if individuals tend to share similar knowledge structures within a given choice domain, then a rather stable global choice behaviour is observed with about 90% probability [2].

In the present study, we shift the analysis of geographical [3], linguistic, sociological and political factors to another report [4], while looking here for a model which may describe a type of language competition observed in an environment populated by strong minorities facing several alternative choices and partially bordering on supporting states. We do not intend to discuss whether "dialects" would be a better name instead of "languages", however, we notice that recent linguistic analyses [5] could not trace any dialectal differences following national lines in many of the successor states in e.g. Southeast Europe. Instead, it has been increasingly argued [6] that all different groups in the region tend to use exactly the same idiom. However, the restructured political pictures lead to the emergence of completely new policies, such that the question of language has become a top political issue in a community which is linguistically homogeneous but politically divided [6]. We treat this problem as the one of opinion dynamics where everybody can adopt one of the three choices A, B, and C (each representing the opinion about the linguistic identity), with transitive [7] preference relations. Thus we model the evolution of the global choice behaviour in a tripartite system where due to particular economic and political alliances, languages may happen to be in a closer contact at one point in time and more divided at another. As a consequence, people may start adopting linguistic features or even full languages of their neighbours, if they have sufficient gains or are influenced by a set of social and/or political factors. This is especially valid for those languages which both belong to the same linguistic family and border with one another.

In our model, we assume that the simulated $L \times L$ lattice is bordered on top by the population preferring the linguistic choice A and on bottom by the population preferring the choice C. This we achieve through two boundary lines of only A on top and only C on bottom. Initially, in the middle of the lattice, the decision makers (DM) preferring the choice B are dominant, while on the top we have DM mostly preferring A and on the bottom mostly preferring C, with the concentrations of A, B, and C varying linearly with

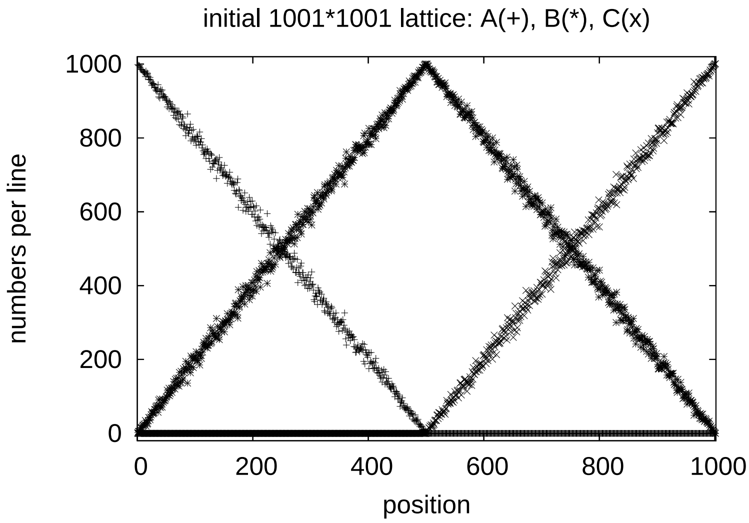


Figure 1: Initial distribution of A, B and C as a function of height, with concentrations varying linearly with height. Half of the people selects the B choice (mostly in the middle), one quarter selects A (mostly near one border) and one quarter selects the choice C (mostly near the other border).

height. Figure 1 shows for $L = 1000$ the initial distribution of choices A, B, and C as a function of height, i.e., we plot the numbers of A choices in each horizontal line while doing the same for the B and C choices.

Opinion dynamics can be described in several different ways, by using e.g. the models of Deffuant et al. [8], Krause and Hegselmann [9], Sznajd [10], or some older approaches such as the model of Axelrod [11]. In the present paper, the basic dynamics is the traditional voter model [12], where at each iteration every site accepts the language of a randomly selected nearest neighbour. Then every individual with choice A having three individuals with choice B and one with A as neighbours is likely to give up the choice A and shift to B. Instead of this voter model we also could have used a Potts-type model [14], as did Lim et al. [3] without mentioning Potts, or an Ising model with spin $+1, 0, -1$. The voter model, however, seems to be simpler, and is a first step towards a more complete future work on this topic.

In addition, we assume that with low probability $p$ at each iteration each individual accepts the choice B because of "advertising campaigns" [13] from higher authority. Furthermore, we added noise to the whole process, in

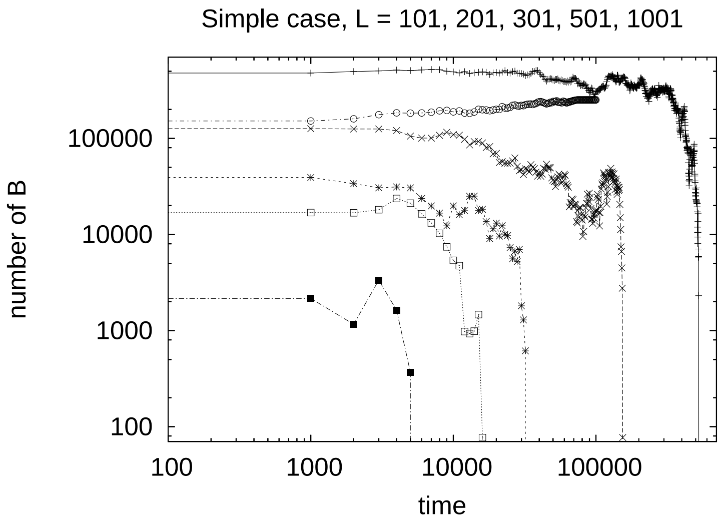


Figure 2: Double-logarithmic plot of B choices versus time in the simple voter model.

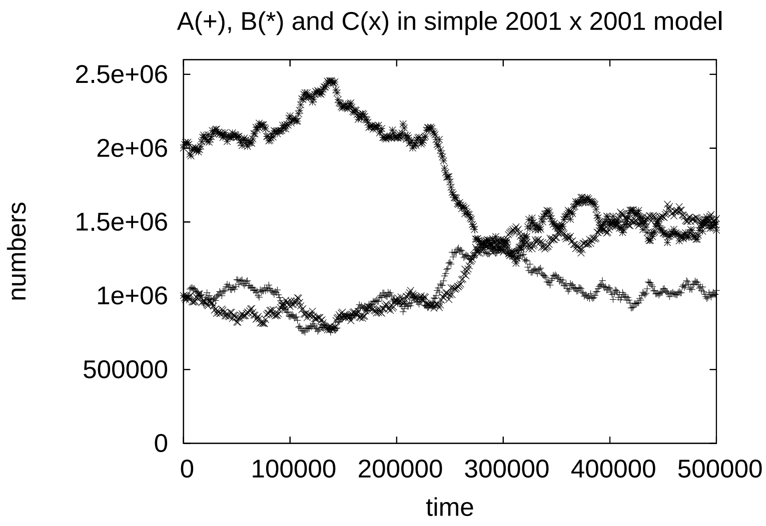


Figure 3: All three choices for the largest lattice in the simple voter model.

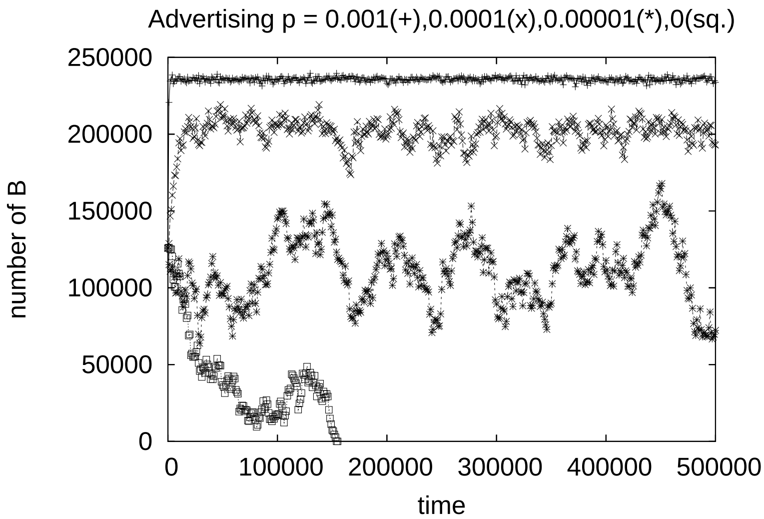


Figure 4: A small amount of "advertising" $p$ in favour of B stabilises the B choice; a larger $p$ increases it, though not to 100 percent = 251001. No "noise" is included yet.

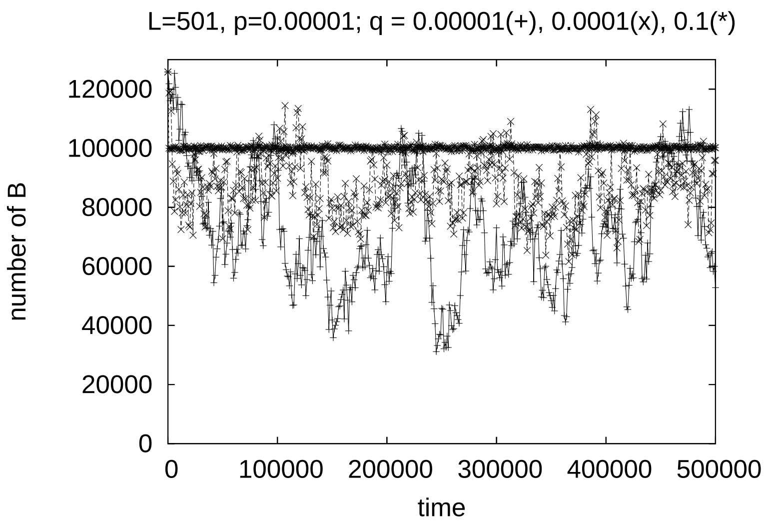


Figure 5: Addition of noise $q$ to voter model plus advertising reduces fluctuations.

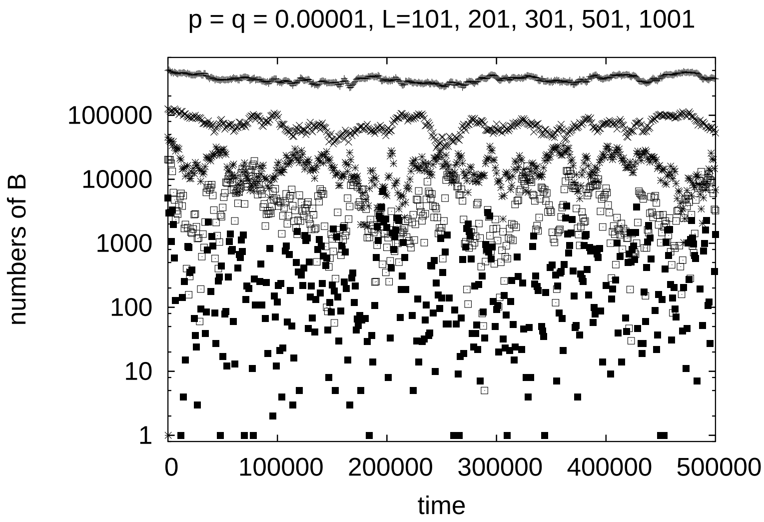


Figure 6: Dependence of B choice on lattice size, with small advertising and small noise.

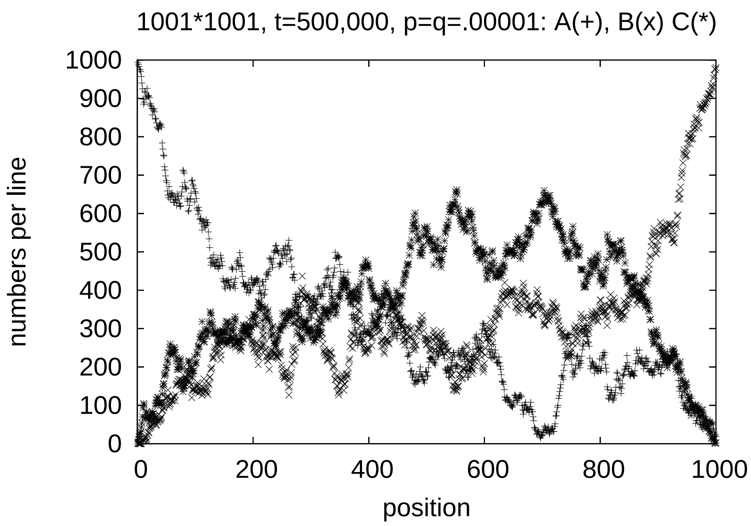


Figure 7: Geographic distribution of A, B and C opinions after equilibration as a function of the line number in the lattice; $p = q = 10^{-5}$.

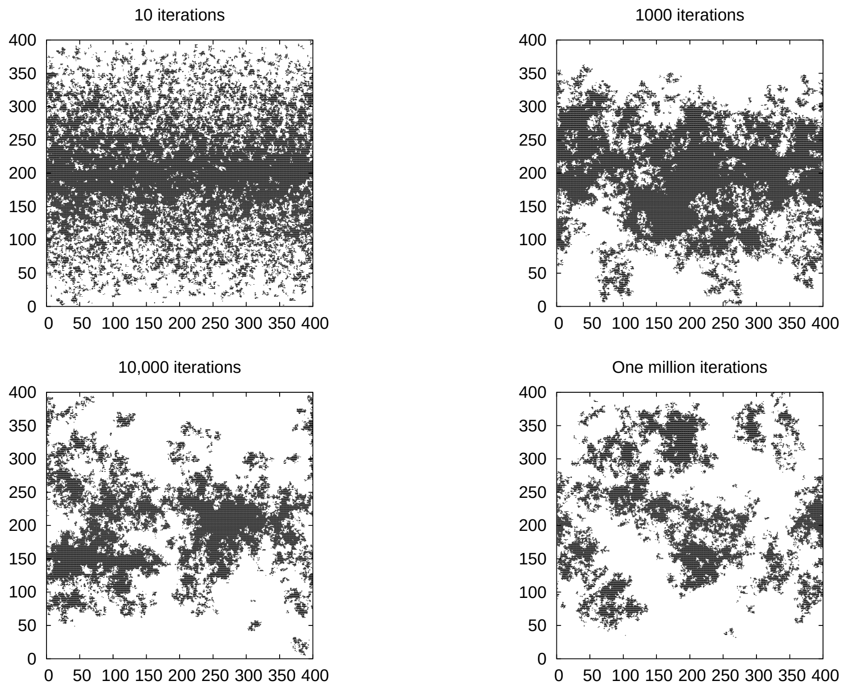


Figure 8: Two-dimensional distribution of the B opinion as a function of time, with small noise and small advertising, $p = q = 10^{-5}$.

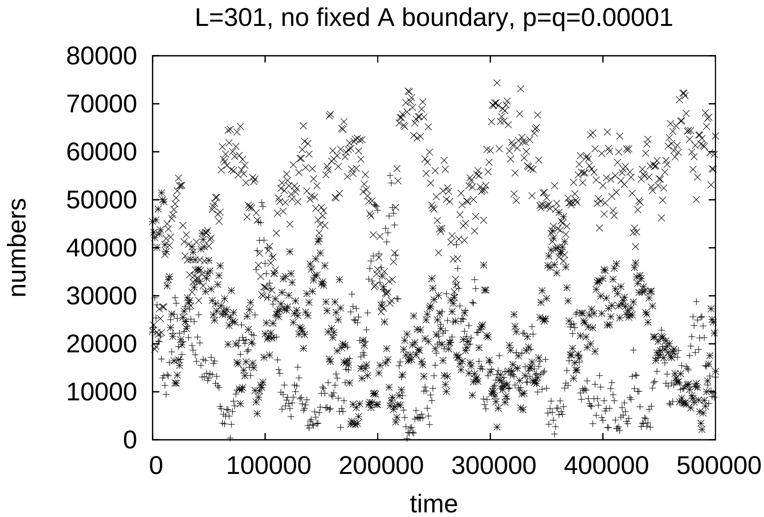


Figure 9: C is likely to win if rigid A boundary at A side is removed. For smaller $L$ this victory is more pronounced, for larger $L$ less.

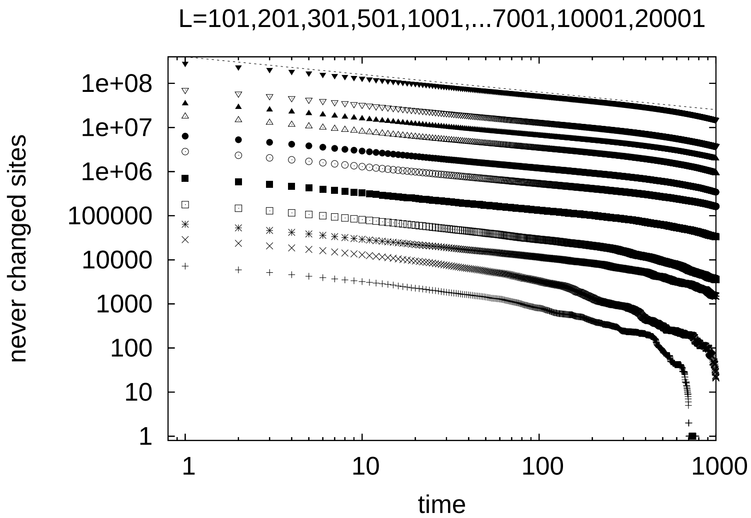


Figure 10: Persistence: Number of sites which have not changed since the beginning of the simulation. For longer times this number decays faster, reaching zero before or near 20,000 iterations.

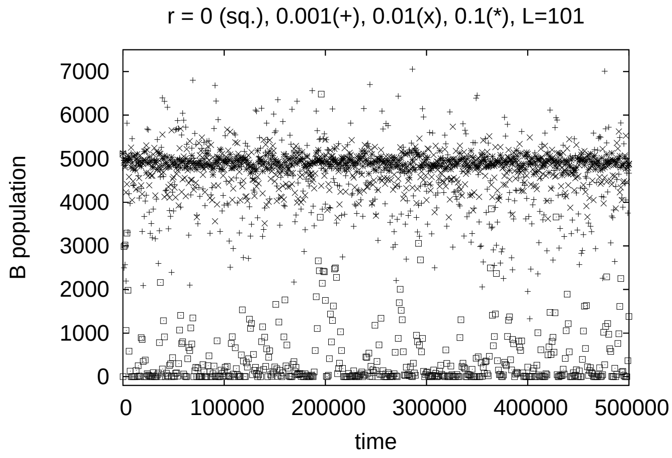


Figure 11: Persistence: B choice if a random quenched fraction of 0.1, 1 and 10 % never changes in a $101 \times 101$ lattice; $p = q = 10^{-5}$. For comparison we repeat from Fig. 6 the data without such a fraction (squares at the bottom).

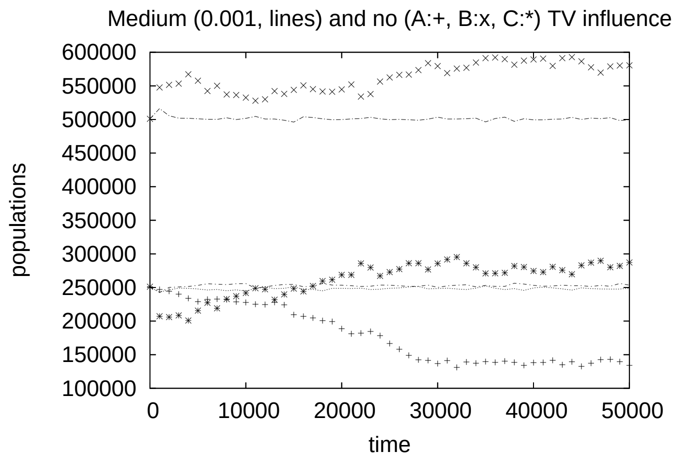


Figure 12: Axelrod-type generalisation, $p = q = 10^{-5}$, with influence of age, money and politics added. The separate symbols correspond to the three choices without local mass media effects, the lines show the stabilising effect of mass media with a convincing probability of 0.001 per iteration.

order to mimic perturbations and random fluctuations in a person's switching behaviour among choice items over time due to e.g. mixed marriages, sudden preference changes of latent origin, or other personal reasons of individual DM. More precisely, with low probability $q$ each individual at each iteration shifts to another randomly selected choice: C shifts equally often to A and B, B shifts to A three times more often than to C, and A shifts to B three times more often than to C. We use this particular update rule for the opinion dynamics because we wish to model a situation in which there is a virtual i.e., political barrier splitting the lattice into two major pieces such that C dominates on one side of the barrier, while A and B are predominantly preferred on the other (e.g. through the separation into two political entities). Furthermore, each of the three choices A, B, and C has some special property: Only the population with B choice has no border with fixed sites, while populations preferring A and C choices partially border on two areas from which they sporadically obtain additional political support. In addition, C choice is rather disliked by those individuals choosing A and B. Thus, we assume that individuals' information is not arbitrary but is constrained by the individuals' context-based mental model of the respective choice domain.

We further investigate the case in which one of the two fixed boundaries in the model is removed (e.g. when the neighbouring side weakens or disrupts its support), assuming that this might lead to the victory of one of the opinion alternatives within the tripartite system. We also compute the persistence probability $s(k,t)$ for different lattice sizes, where $k$ is the number of opinions in the original voter model, and $p = q = 0.00001$.

We do not take into account the population size dynamics (birth vs. death rates). Of course, the question of how exactly individuals aggregate their preferences over $k$ candidate languages involves much more fine-granularity than considered in the present model. Indeed, the opinion switching dynamics might be far more complex than described by the update rule in the present study. For instance, not only single individuals, but moreover, larger amounts of population members might drastically change their attitudes towards linguistic (and political) campaigns such that they no longer participate in the opinion switching process. Thus, they might just "tune out" [15] at one or more points in time, thereby considerably affecting the overall evolution of preferences or choices. At such times, the individual state remains frozen, until a particular event causes the decision maker to re-enter the switching process. In order to reflect on this type of behaviour in our simulations, we included a situation in which a particular random (quenched) fraction of population never changes its opinion (fixed set of "stubborn" decision makers). However, we assume here that some aspects of the human opinion dynamics are independent of the detailed micro-processes of decision making, relying more on some overall, universal properties. These microscopic quantities are besides not always directly accessible to the observer. On the other hand, there are macroscopic quantities, defined by parameters that are fixed from the outside. The main goal of statistical physics is exactly to provide a link between these microscopic and the more global, macroscopic aspects of an investigated system.

# 2 Results

For the simple voter model without advertising and noise, Fig.2 shows how the choice B decays due to the influence of the fixed boundaries with A and C only. The larger the lattice, the longer does the B choice survive. The single increasing curve for $L = 501$ shows the B choice if the two boundary lines are all B, instead of all A and all C, respectively. This figure thus indicates the importance of the lattice boundaries. For the same simple model but a larger lattice, Fig.3 shows the time dependence of the A, B and C opinions.

If advertising in favour of B is added to the voter model, the B opinion can be prevented from dying out and can even increase, enlarging thus its initial majority; see Fig.4. Very small advertising probabilities $p > 10^5$ suffice.

All following figures include both advertising with $p = 10^{-5}$ and noise with varying small $q$. Figure 5 shows that somewhat counter-intuitively, the noise reduces the fluctuations if its strength $q$ increases. For $q = 0.1$ already the results agree well with theoretical expectations of 40 % A, 40% B, 20% C, the stationary solution of the rate equations for noise only. Again, small lattice sizes let the B opinion decay; however now it happens that after B has died out it is resurrected via advertising and noise; see Fig.6. Fig.7 shows what happens in the long run with the initial distribution of A, B and C, as a function of the line number of the lattice. Fig.8 shows as a function of time how the voter process first leads to clustering: A prefers to be near A, B near B, and C near C. Later, the decrease in preference for the B opinion is seen until it is stabilized by noise and advertising.

If the rigid boundary of pure A at one end is omitted, then both B and A are strongly reduced and C wins for small $L$; see Fig.9.

The number of sites which never changed during a simulation decays initially as about $1/t^{0.4}$ for $t < 10^3$, and then stronger down to zero, see Fig. 10 and straight line there. This power law differs from the behaviour of other voter models with different noise or without noise [16].

If we assume a small fraction $r$ of sites never to change, then they help the B population to survive better, and for larger $r$ they reduce fluctuations, as seen in Fig.11.

When we allow people to migrate, by exchanging residences with randomly selected people anywhere in the lattice, the B choice goes down (not shown). This effect vanishes if people migrate only once.

Finally, we expanded the above model of Figs.6-8 into a modified three-choice three-trait Axelrod [11] model with three additional binary variables: age (young or old), money (rich or poor), politics (left or right). In the sense of Axelrod [11], the switch to the language of a randomly selected neighbour happens only if one agrees with this neighbour in at least one of the three new aspects. Also, old people only switch half as often as young people. Initially, 3/4 of people with choices A or B are rich and 1/4 is poor, while for those with choice C the inverse is assumed.

Fig.12 shows that now the populations fluctuate strongly but close to their initial values. If, on the other hand, the mass media in the upper quarter of the lattice (where A dominates initially) convince people to choice A with a probability of 0.001, with the same happening in the lowest quarter for choice C and in the central half for choice B, then the three populations are stabilised to their initial values of 25, 50 and 25 percent out of the total of the one million decision makers.

# 3 Discussion

In the present paper, we proposed the generalization of the standard three-state voter model for a linguistically homogeneous but politically divided population with subgroups of different mutual affinities. Starting with this tripartite initial configuration, the Monte Carlo simulations were conducted for different lattice sizes, different amounts of "advertising" (outside pressure) and asymmetric noise distribution. These simulations lead us to several, non-trivial conclusions.

First, the individual preferences for the initially most dominant opinion are more prone to decay given the rather unstable and fuzzy borders with respect to the two other populations with more fixed boundaries and opinions different from the majority. Thus, we found a substantial importance of the boundary effects for the investigated population when placed on smaller lattices. However, increasing the size of the lattice prolongs the survival of the most dominant opinion.

Second, adding some "advertising" in favour of the dominance state prevents the opinion of the majority from extinction, while considerably increasing its size. In the model, the "advertising" probabilities refer to the different outside pressures of e.g. the international community in favor of the largest population due to particular reasons. Somewhat counter-intuitively, when noise was added to the process, it reduced the previously observed fluctuations in choice behaviour. The noise parameter in our model can relate to a set of different factors such as marriages and relationships between culturally and linguistically different individuals, unexpected changes of preferences for different political parties with different language policies, or other personal reasons of individual decision makers.

In addition, we investigated a particular case of removing a rigid boundary at one of the previously stable sides, thus yielding the victory of only one single opinion for small lattice sizes ($L = 301$). The decay function for individuals who never changed their opinion during the simulations, tends to follow a power law which differs considerably from the behaviour documented in other studies of persistence probabilities, where different types of noise were used or no noise was applied at all [16]. Finally, if for some reasons a particular fraction $r$ of population "tunes out" from any further opinion switching, this leads then to a better than the previously observed stabilization of the most dominant state, with additional reductions of fluctuations for larger $r$.

The update rule reported in this study was used in order to mimic situations in which three populations are not just geographically, but also virtually, i.e., politically divided (e.g., via formation of two separate entities following an inter-cultural conflict). Further research in this direction should therefore include a situation where the political barrier is removed, thus necessitating different update rules in the model.

While considering the relevance of social effects on language change [27], we extended our voter model in the sense of Axelrod [11], resulting in a strong fluctuating behaviour of all three states but still close to their initial values (Fig.12). This indicates a global tendency of all three populations to form distinct and yet stable linguistic choice domains. This stability additionally increases with a sufficient mass media effect on all three populations.

Our investigation was motivated by a phenomenon in which three groups of a given population speak one and the same language (linguistic homogeneity), but have different opinions about their own linguistic identity (political diversity), thus each claiming to speak different language from the "other". In such situations, the name which people usually attach to "their language" is the one of their supporting neighbouring or other states. As a consequence, individuals can increasingly start to adopt linguistic features from languages of their supporting neighbours, especially if they were previously in a conflict with their within-population neighbouring members. If no supporting neighbouring states are available, people will try to invent or dig up half-forgotten words that function as shibboleths (dialect identifiers). Eventually, certain aspects of pronunciation that people are conscious of could be changed "by decision", while grammar is expected to remain unaffected, since people are not conscious of most of the grammar they use [17]. Indeed, as recent scientific studies have shown, an immensely large fraction of our everyday behaviour is unconscious, i.e., zombie-like [18], and consequentially, under rather uncertain conditions, we tend to act more as irrational decision makers [19]. For instance, one opinion might often be preferred to another, even if the probability of yielding the worse outcome with it is one. Often irrational and particularly politically-driven "swings" cause people to make more (unnecessary) decisions about their language and the language of others. From the linguistic perspective, one can predict in the aforementioned case that only highly salient linguistic features can change more rapidly.

People can start adopting other linguistic features or even fully switch to other languages due to many reasons: due to the use of different languages for different purposes (diglossia), because they decide to migrate to a foreign country, because the status of the non-native language is higher, or if their country is conquered by the linguistically different population. However, it was demonstrated that contrary to predictions of some models [20], a stable co-existence of languages in competition, regardless of their status, is still possible if they are sufficiently similar to each other [21]. Typical instances of such "linguistic areas" where several languages compete and share a number of features are found in Southeast Europe [4]. For example, in the region of Sanjak, several different languages share many of their features. The area of Sanjak is politically divided between the states of Serbia and Montenegro, and geographically and linguistically surrounded by four different states or areas (Serbia, Montenegro, Bosnia-Herzegovina, Kosovo-Metohia). In parts with Bosniak population in Sanjak, individual speakers claim to speak Bosnian language, however, the laws in Serbia allow Bosnian only as an "elective" in primary and secondary education, while Montenegrins "solved" this "problem" by labeling the language exclusively as "mother tongue" without any reference to any particular national language. Most recently, the new Montenegrin constitution introduced the new and the official Montenegrin language in the country, with other languages (Bosnian, Serbian, Albanian and Croatian) as additional, but secondary official languages of the state. This was preceded by a high disagreement about how the official language should be called, with competing opinions between those who viewed themselves as of different cultural and ethnic origins.

In Bosnia-Herzegovina, we have a related case, where strong minorities decided to introduce the standard languages of the neighbouring countries as their official idioms [6]. It seems therefore that individuals, and more importantly, much larger population clusters, indeed tend to consciously introduce several novel features in their languages just to distinguish themselves from "others". As we have found in our simulations, the final outcome of this process might strongly depend upon the character of the boundaries defined between different areas, but also on the size of the populated region, the internal random noise, and the outside pressure from some higher authority. We also wish to stress at this point that despite of the obvious structural similarity and mutual intelligibility of many of the Southeast European languages, these two factors should not be taken as decisive (or misused) in language planning or policy making, especially because some other world languages are mutually intelligible to a large degree just as e.g. Croatian and Bosnian (such as Urdu and Hindi), but are recognized as separate standard languages.

A counterexample to the Bosnian case would be Estonian [22], where the placement of verbs at the end of sentences was *consciously avoided* in order to differentiate the language from German, a language which it had been under the influence of. But this was of course an attempt guided by linguists. It is in fact not quite clear whether the word order had been influenced from German or whether verb-final order was just a natural, internal Estonian development. Somewhat more cooperatively, in the northwest Amazon region [23], there is a marriage system according to which people have to marry someone speaking a language that is different from their own: linguistic exogamy. Thus, the language which defines your identity is that of your father. In this situation people are careful not to borrow words from other languages, but they have influenced one another at the level of grammar to a great extent.

Explanations for provocative phenomena of this kind are best investigated within the framework of opinion dynamics; the voter model is a traditional example and is a good place to begin. We believe that further refinements and applications of our generalized voter model, carried over more realistic input while simultaneously considering a more complex set of intervening variables, might tease apart the multiple forces (latent) and practices (observables) through which individual decision makers shift to other languages or opinions. This can further shed light on the related psychological mechanisms responsible for integration and preference dynamics of these practices. For example, from the psychological point of view, one can investigate whether some collective phenomena of this type are better explained by means of modeling the individual *indifference classes* rather than *preferences* among various choices [24]. Thus, in rather conflicting and culturally highly mixed areas, agents are usually better aware of what they don't like, than what would they prefer more.

Additionally, more degrees of freedom of individual "voters" (using complex networks instead of lattices, allowing for a co-evolution of the network and voter states), as well as population size dynamics should be incorporated in future modeling. It would also be ideal to integrate the opinion dynamics models with those of language evolution and learning [25, 26]. Models of opinion dynamics can also help in future conflict resolution in the aforementioned regions, by suggesting e.g. possible modes of "softer" and scientifically-valid language policies, or by (re)defining particular linguistic norms as opposed to those driven by political factors.

We finally argue that the advantages of our generalized three-state voter model span beyond mere linguistic applications, since the model is able to simulate and characterize the nature of both collective political and linguistic states (i.e. their co-evolution). Moreover, through future comparison with realistic data, we might be able *to predict* whether and under which conditions various highly-similar languages can stably co-exist, and whether and how the political stability in a given region changes with the ever growing "linguistic diversity". Physicists and other computational modelers can significantly contribute to this research domain, by developing tools capable of establishing more isomorphic relations between the model parameters and the realistic traits of languages or opinions in competition.

# 4 Acknowledgements

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