Attractors in the Development of Communication

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Abstract

The development of communication in a population of agents is viewed as the behavior of a dynamical system. A deterministic communication system is shown experimentally to have point attractors that correspond to perfect communication. However, the determinism required for the presence of point attractors impedes the development of communication. A corresponding attractor type for the stochastic system is defined, and the existence of these attractors in a stochastic version of the system is demonstrated.

Introduction

The development of communication in a population of agents can be viewed as the behavior of a dynamical system, see e.g. (Steels, 1997, Hashimoto, 1998). Here, an analysis based on this viewpoint is performed. It is demonstrated by perturbation experiments that a deterministic communication system has point attractors that correspond to perfect communication. However, the determinism that is required to satisfy the conditions of attractorhood hinders the spontaneous development of communication. The stochastic system is preferable in this respect, and has points playing the same role as the attractors in the deterministic system. However, the concept of an attractor has no mathematically rigid equivalent in stochastic systems[1]. Therefore, a modification of the definition of a point attractor is used to analyze the behavior of the particular stochastic system at hand. The stochastic version of the system is shown to have attractors satisfying this definition, and to develop communication more reliably than the deterministic system.

Section 1. briefly describes the algorithm used by individual agents to adapt their communicative behavior. By a process of self-organization, the local adaptations defined by this algorithm lead to a globally coherent system of communication. In section 2., a dynamical system model of the deterministic system is developed. Section 3. experimentally investigates the presence of attractors in this system. The investigation of attractors in the stochastic version of the system is described in section 4. Finally, section 5. concludes.

[1] Measure theory does provide the concept of convergence of measures though, which is close in spirit.

1. The Development of Communication

In this section, an algorithm will be briefly described that, when used by individual agents, leads from an initially random situation to a state where agents refer to the situations in their environment with the same words. The stochastic algorithm is a slightly adapted version of that introduced in (De Jong, 1999), and is described in my Ph.D. thesis (De Jong, 2000). Here, the behavior of the stochastic system is analyzed by relating it to a deterministic version.

1.1 Internal Structure of the Agents

Each agent has a set of *meanings*. These meanings represent situations in the environment that are important to distinguish, and the current meaning can be determined directly from sensor data. They have been developed by an autonomous concept formation method, see (De Jong, 1999), and may in principle differ from agent to agent. Here however, the conditions of the experiments have been chosen such that all agents develop ideal conceptual systems, and because of this the concepts of the different agents are identical. For each meaning, an agent has a set of *associations* between some word and the meaning. The set of words is open, and determined by the words that the agent has received in a situation represented by that meaning. An association between a meaning μ and a word σ consists of a *use* component $a_u(\mu, \sigma)$ and a *success* component $a_s(\mu, \sigma)$, both real valued numbers. The total strength $a(\mu, \sigma)$ of the association is a weighted sum of its use and success components. Finally, for each word σ associated with a meaning μ, an estimate of the conditional probability $P(\mu|\sigma)$ is maintained.

1.2 Word Production

At each time step, after receiving sensor information, an agent determines its meaning that corresponds to the sensor data. It then selects a word from the set of words associated with this meaning, based on the association strengths. In the deterministic version of the system, the agent selects the word with the highest association. In the stochastic version, it uses the Boltzmann distribution to choose a word:

$$P_{prod}(\sigma_i) = \frac{e^{a(\mu,\sigma_i)/T}}{\sum_{j=1}^{n_s} e^{a(\mu,\sigma_j)/T}} \qquad (1)$$

where n_s is the number of words associated with the current meaning μ, and T is the temperature of the distribution. Low temperatures favor strongly associated words. High temperatures encourage exploration up to the point where, in the limit, the elements are equiprobable.

1.3 Word Interpretation

When the agents have produced a signal, they receive the words produced by all of the agents. Given this new information, they may decide that their sensor based situation determination was unlikely or stay with their initial determination. Based on the action value estimates of the chosen situation, an action is selected. In the first, signal based case, the reward following the action is compared against this estimate. If the reward conforms with the expectation, the signal based situation determination is deemed correct, and the *success* component of the association with the most often received word is increased and those of the associations with other words are decreased; otherwise it is decreased. Thus, no feedback evaluating production behavior is given, and the feedback evaluating word interpretation is determined by the agent itself based on its own imperfect information. Hence it does not reflect whether the agent interpreted the word correctly (i.e. according to its use by the population), but only whether the association *appears* to be correct. Furthermore, independent of the situation determination, the *use* values of the associations with words that were received are increased, and the use values of the other words associated with the situation are decreased.

2. Language as Dynamics

In this paper, the dynamical systems perspective on language is brought to bear; the tools of the domain will be used to analyze a system of agents that adapt their communication behavior. The approach of using concepts from dynamical systems theory has proven fruitful before in other language learning and development work. In (Pollack, 1991) for example, language induction was found to correspond to a phase transition in the weights of a learning network. The dynamical systems perspective has also been found useful in other areas of adaptive behavior research, see (Beer, 1997, Port and Van Gelder, 1995).

The communicative behavior of the agents is governed by their internal state, in particular by the associations between words and meanings of each agent. The whole of the associations of all agents therefore determines the language spoken by the agents. This language is not static, but continuously varies as a result of interaction between the agents. These dynamics will be studied by viewing the whole of associations as a dynamical system. The variables of this system are association strengths between a meaning and a word for some agent. The interpretation behavior, governed by estimates of the conditional probabilities $P(\mu|\sigma)$, will not be regarded here. Since the production behavior already determines the optimal interpretation behavior, the extra information these variables would yield is not expected to weigh up to the large increase of the state space this would involve.

There are four situations that need to be distinguished by the agents in the environment, and the environment contains five agents. In the ideal case, each of these four concepts has a single word strongly associated to it, in which case there are at least four words. In the beginning of the experiment most, but not necessarily all words will be used at least a few times in each situation due to exploration and the random initialization. Not all possible associations necessarily exist; in practice however, they do. Therefore, the analysis will consider all combinations of meanings and words for all agents as variables, resulting in a dynamical system with at least 5 agents * 4 meanings * 4 words = 80 dimensions. Depending on the production behavior of the agents, extra words may be introduced, in which case the dimensionality of the system increases.

3. Attractors in the Deterministic Case

It will be investigated whether the deterministic system of communication has attractors that correspond to perfect communication.

3.1 Definition of Point Attractors

A point attractor is a point where all neighboring trajectories are directed towards the fixed point. Such a point 'attracts' the system to itself from some neighborhood. This neighborhood is called the *attractor basin.* Although the informal definition of attractors may seem quite clear, there is no consensus about a formal definition for attractor. In (Strogatz, 1994), an attractor is defined as a closed set A for which the following conditions hold:

1. A is an *invariant set*: any trajectory that starts in A stays in it for all time.
2. A *attracts an open set of initial conditions.* This means that A attracts all trajectories that start sufficiently close to it. The largest neighborhood for

which this is true is called the *basin of attraction* of A.

3. A is *minimal*: there is no proper subset of A that satisfies conditions 1 and 2.

A mathematical proof of the existence of attractors corresponding to perfect communication can be found in my Ph.D. thesis (De Jong, 2000). In the following, an experimental investigation will be presented.

3.2 Location of Hypothesized Attractors

The objective of this investigation is to test the hypothesis that the communication system has attractors that correspond to perfect communication. The above definition provides a way to perform this test; if conditions 1 and 2 hold in points where communication is perfect, then the hypothesis can be confirmed. The third condition is trivial for point attractors since a single point has no proper subsets.

The first step is to identify the points in phase space where communication is perfect. Since all variables take values in the interval [0..1], the phase space has the shape of an n-dimensional hyper-cube, see fig. 1. If communication is perfect, the word that is most strongly associated must be different for each meaning. Also, these associations must be the same for every agent. The most stable configurations satisfying these conditions are corners of the phase space hyper-cube, where the strong associations are maximal (1) and the weak associations minimal (0). These corners are hypothesized to be attractors. Note that these conditions select a very small fraction of the corners; in the vast majority of the corners, the conditions are violated, e.g. because for some agent several words are strongly associated with one meaning or vice versa, or because corresponding meanings of different agents are associated with different words. Concretely, while the number of corners in the 80-dimensional space is astronomical ($2^{80} \approx 1.2 \cdot 10^{24}$), only $4! = 24$ of these combine each of the four meanings with the same unique word for all five agents.

In the experiments, the first two conditions can be tested by examining the distance to the nearest attractor. Given the above analysis of the location of attractors, a straightforward way to obtain this value is to calculate the distance to the nearest corner of the hyper-cube. Since not all corners are hypothesized attractors, combining this distance with an indicator that says whether the corner is a hypothesized attractor allows us to monitor the convergence to hypothesized attractors experimentally.

3.3 Neighborhood of Attractors

The condition that the systems should tend towards attractors has to hold within a neighborhood around these attractors. Thus, it is necessary to define neighborhoods around the attractors within which the movement of the system is to be monitored. Here, these neighborhoods will be determined as a region centered around each attractor where the distance to the attractor is below a threshold, see figure 1. This definition yields an n-dimensional hyper-sphere with a radius equal to this threshold. The figure illustrates this for the three dimensional case.

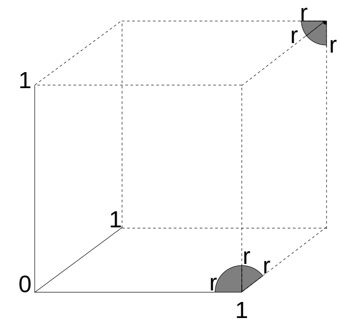

Figure 1: Schematic representation of attractor neighborhoods. Each axis represents the association strength for some word, meaning, and agent. In reality the number of dimensions is much larger (80 or more), and the radius of the hyperspheres much smaller (0.001).

3.4 Experimental Investigation of Attractors

In this section, experiments are reported that investigate the two conditions of the attractor definition: existence of an open set of initial conditions and invariance.

3.4.1 Open Set of Initial Conditions

The second condition for attractorhood states that an attractor must have a neighborhood such that whenever the initial condition of the system falls within this neighborhood, the distance to the attractor must tend to zero (see figure 2). This subsection starts with a description of the experimental procedure to investigate this, which is based on perturbations, and presents the results of the experiments.

The Perturbation Procedure A perturbation here signifies that the system is taken out of its current state and moved to a random point within a distance r of the nearest corner of the hyper-cube that contains the phase space (see fig. 1). That is, its new state is a random point within the hyper-sphere with radius r and the corner as its center. Most of the corners do not represent successful systems of communication. However, if the corner corresponds to good communication and is a hypothesized attractor, the distance of the system's state to this hypothesized attractor is monitored over time. The question that is then investigated is whether this distance tends to zero. As remarked above, interpretation information is not represented in the phase space. All interpretation information stored by the agents is reset during a perturbation, thus rendering the state related to interpretation neutral.

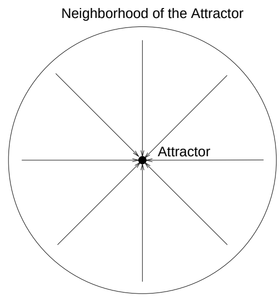

Figure 2: Schematic rendering of an attractor in the deterministic system. The arrows show only one possible configuration of vectors; many vector fields, e.g. fields with spiraling trajectories, satisfy this condition.

Locating Hypothesized Attractors In the experiments, a perturbation is performed every 10,000 time steps, starting at time step 50,000. The first question to be asked is whether the deterministic system moves towards a state of good communication. If this is the case, the second condition of the attractor definition can be tested.

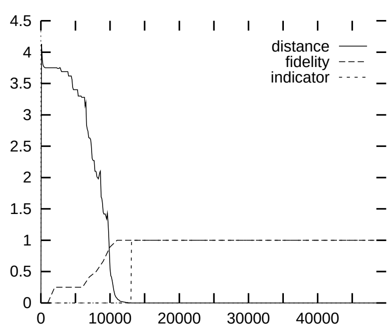

Figure 3: Evolution of the distance to the attractor over time. The system spontaneously converges to an attractor.

Figure 3 at each point in time shows the distance of the state of the communication system to the nearest corner of the aforementioned hyper-cube. The top left graph shows the complete time series. Apart from the distance, two other lines are plotted: *fidelity* and *indicator*. Fidelity measures the quality of communication, and is calculated as the average probability that a concept encoded into a word by one agent will yield that same concept when the word is decoded by another agent. *Indicator* is an indicator function that tells whether the system is in the neighborhood of a hypothesized attractor. It is binary, and takes the value of one when the system is within a hyper-sphere with radius 0.001 of a corner of the hyper-cube that represents an ideal system of communication.

In figure 3, it is seen that during an initial period, until around time step 5,000, there are only small changes in the distance. Note that the corner to which the distance is calculated may change during this period, since the location of the system in phase space keeps changing. Then the system starts moving towards a particular corner.

The decrease of the distance to the corner is steep, and corresponds to a rise in fidelity. After 11,000 time steps, the fidelity measure equals one, which implies that the system is moving towards a corner with ideal communication. The neighborhood of this corner is not entered until just after time step 13,000 (see indicator), when the distance to has dropped below 0.001.

An optical inspection of the graph appears to show that the distance quickly drops to zero. However, since updates move the association strengths *towards* zero or one, these values can never be reached completely, and hence the distance will never actually *reach* zero.

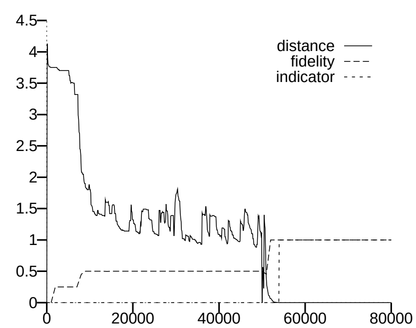

Figure 4: Distance to the attractor over time. Here the deterministic does not approach the attractor spontaneously, but after a perturbation it (time step 50,000) an attractor is found.

Figure 4 shows a typical example of a run where accurate communication did not develop spontaneously. The distance to a corner also drops substantially in the beginning, but not as far as in the previous case. Then, instead of converging exponentially to the corner, it keeps varying at an intermediate level. As the graph shows, the fidelity of communication during this period is only 0.5. At time step 50,000, the first perturbation takes place. The system is moved to a random location within 0.001 distance of the closest corner, visible as a quick drop in the distance value. However, this closest corner does not correspond to good communication. This can be seen from the fact that the fidelity measure does not immediately rise to one. Inspection revealed that words did not always uniquely identify a meaning, but were sometimes associated with several meanings. This is the result of the deterministic selection mechanism that governs word production; if one word happens to be slightly stronger associated to several meanings at one point by many agents, the agents continue to use this word for these meanings, resulting in a sort of deadlock that can only be resolved by perturbing the system to other regions of the state space.

Course of the Distance to Hypothesized Attractors The experiments above illustrated two ways for the system to reach corners of the hyper-cube where communication is ideal: spontaneously, or as the result of a perturbation. In all of the ten runs, the system arrived at such a corner after one or two perturbations or, in four of the cases, without requiring any perturbation at all. This situation enables us to investigate the second condition of the attractor definition.

In the following experiment, the hypothesized attractors found using the procedure described above are used as starting points near to which the perturbations will place the system. The evolution of the distance to this hypothesized attractor is then monitored over time. For the hypothesized attractor to be a real attractor, this distance must tend to zero. In this experimental investigation, this criterion is examined in two ways.

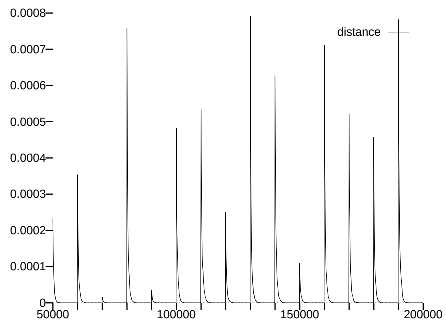

Figure 5: Results of one of the perturbation experiments. After every single perturbation, the system reacts with a steep descent towards the hypothesized attractor.

First, the evolution of the distance to the hypothesized attractor after the perturbations is inspected visually. Figure 5 shows this evolution for the 15 perturbations of the first run of the experiment. Every 10,000 time steps, the system is perturbed. These events are seen as sudden increases in the distance in the graph, with values that can vary between zero and 0.001, the boundary of the hyper-sphere that has been selected as a neighborhood within which convergence is tested. After every single perturbation, in all of the runs, the system reacted with a steep descent towards the hypothesized attractor.

Second, it has been investigated whether the descents towards the attractors are monotonic by calculating the differences between every pair of subsequent distances. A descent is monotonic if at every time step the distance to the attractor either decreases or remains at the same level. If all descents are monotonic, this is an *indication* that the distance to the attractor tends to zero. This examination showed that after every single perturbation, in all of the runs, the distance to the attractor decreased monotonically.

In summary, all of the hypothesized attractors that were encountered had the property that the system, when having its initial condition in a neighborhood of this attractor, tends to move towards the attractor, which concludes the experimental investigation of the second condition of the attractor definition.

3.4.2 Invariance

The final part of the experimental evidence required to investigate the presence of attractors is the condition that attractors must be invariant sets; any trajectory that starts in A stays in A for all time. This question has been examined for all of the attractors that were found in the perturbation experiment. The procedure was as follows. At the end of each of the perturbation experiments described in the previous subsection, at time step 200,000, the system is always at close distance to a hypothesized attractor. Since the stochastic update rule never really converges to equal the goal value, this distance will never be zero, save the extremely unlikely case where the initial state of the system happens to be an attractor. At time step 200,000 then, the system is moved to the location of the hypothesized attractor. Subsequently, the experiment is continued as usual, without any perturbations. If the location of the system in phase space remains exactly equal for a substantial number of steps, this is experimental evidence that the hypothesized attractor is an invariant set. In all runs of the experiment, the system maintained a zero distance to the hypothesized attractor after time step 200,000. Thus, the experimental evidence also satisfies the requirement of invariance in the definition of an attractor.

Figure 6 shows the distance to the attractor over time, in units of 10^{-14}. Apart from the evolution of this distance before time step 200000, the only event that is visible is the perturbation onto the attractor. From time step 200000 on, the system has a zero distance to the hypothesized attractor. Inspection of the data showed that the distances do indeed equal zero, i.e. there are no fluctuations at a scale that escapes visual detection. This result was identical for all of the runs.

4. The Stochastic System

So far, it has been demonstrated experimentally that the hypothesized attractors of the deterministic system are indeed attractors. In itself, this result is of limited interest; since the selection of words takes the word with the

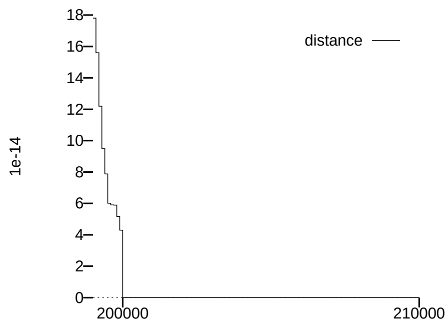

Figure 6: Distance to the attractor over time. Once the system is placed inside an attractor (time step 200000), it remains there.

highest association value, the communicative behavior does not change once the system is within an attractor neighborhood. The value of this analysis is in its potential to shed light on the behavior of the *stochastic* system. This system is almost identical to the deterministic system, with the important difference that association strengths are not used in absolute comparison during word selection, but as weighted relative probabilities (see eq. 1). In the stochastic system, communication develops more consistently and for a wider range of parameters. Due to the stochastic nature of the system, the definition of attractors cannot be used. However, it will be shown here that the stochastic system does have points that play a very similar role to that of the point attractors in the deterministic system. In section 4.1, I define a modified concept of attractors that can be used in the stochastic system and, using the same experimental procedure as before, demonstrate that the stochastic system contains such attractors.

The improved development of communication in the stochastic system can be seen from fig. 7. The graph shows that for the stochastic system (right), the development of communication is consequently observed over the whole range of parameter values that has been examined. Only for the very lowest value of 0.005 is the average fidelity substantially lower than one. For this parameter setting, one of the ten runs reached a fidelity of 0.75, while the fidelity of the other nine runs all exceeded 0.99. For the deterministic system on the other hand (left), the average fidelity exceeded 0.8 in only one of the experiments. In summary, these experiments show that in the system of communication under study, stochasticity is a useful characteristic of the system that has a positive influence on the development of communication.

4.1 Attractors in the Stochastic System

The variables of the deterministic system were association strengths between meanings and words. Since communicative behavior does not change as long as the word

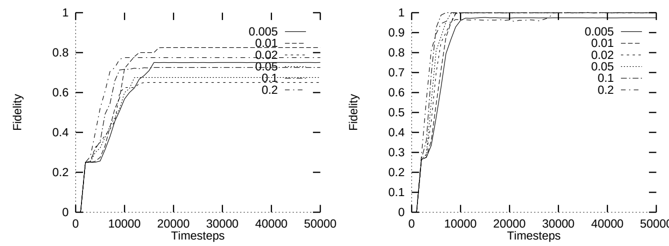

Figure 7: Fidelity averaged over ten runs for six different values of the **max-last-error-for-signals** parameter for the deterministic (left) and stochastic (right) system.

which has the highest association strength remains the same for each situation, this yields more information about the system than the production probabilities. In the stochastic system however, changes in the magnitudes of the association strengths do influence communicative behavior when the system is in the neighborhood of an attractor. Therefore, the production probabilities will be used as the variables determining the state of the stochastic system.

For the stochastic system, the distance to an attractor will always continue to fluctuate due to the stochastic word selection mechanism, and does not necessarily converge to zero from some neighborhood of initial conditions. However, it may well be the case that such a system is attracted by a point, quickly moves towards it, and stays within a small *neighborhood* of the point. Here, the condition that the system should move towards the point will be replaced by the condition that the system should move towards a hyper-sphere around the point. Analogously, instead of requiring the system to remain inside the point attractor when it is placed there, the system is required to remain inside the hyper-sphere around the point. The conditions are not strict; rather, the degree to which they are satisfied determines to what extent such points can be compared to point attractors in a deterministic system. Thus, the definition is introduced to analyze the behavior of the system under investigation, but provides no formal rigidity.

To test whether a point satisfies these new attractor conditions, tests analogous to that of section 3. may be carried out; the system should, when placed within a larger neighborhood of the attractor, with high probability move into the smaller neighborhood determining the attractor, and it should, once within this neighborhood, with high probability remain within it; see figure 8. If these two conditions hold, the small neighborhood may be viewed as an attractor, and will be called such in the following.

For parameter values 0.1 and 0.2, the distances of the individual runs stay well below 0.02 once converged. Therefore, this value will be used as the radius defining the attractors of the stochastic system. In the fol-

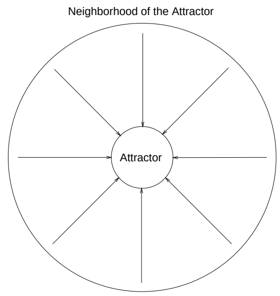

Figure 8: Schematic rendering of an 'attractor' in the stochastic system. When the initial state of the system is within a neighborhood of the attractor (outer sphere) moves into the attractor (inner sphere).

lowing experiments, a value of 0.1 will be selected for the **max-last-error-for-signals** parameter, since the system reaches low distances earliest for this parameter value. A perturbation is performed by putting the system outside the neighborhood determined by the attractor, but within a larger neighborhood of the center of the attractor. For the radius of this larger hyper-sphere, a value of 0.1 will be used. Thus, the experiments should test whether the system, when within a radius of 0.1 of the center of the attractor, moves into the attractor (radius 0.02), and whether the system, once within this radius of 0.02, remains there. These two conditions are the counterparts for the stochastic system of conditions one and two of the deterministic attractor definition.

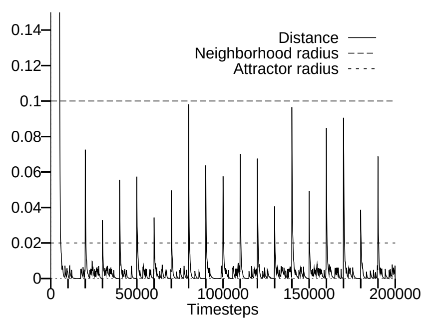

Figure 9: Distance of the stochastic system's state to an attractor over time for one of the runs. After every perturbation, the system quickly recovers and re-enters the attractor.

Figure 9 shows a run of the first perturbation experiment. At time step 5,900, the distance of the system first drops below 0.02. Until time step 20,000, the distance to the center of the attractor continues to vary, but does not exceed 0.02, i.e. system remains within the attractor. Starting with time step 20,000, and repeated at every multiple of 10,000 time steps, a perturbation is performed. The system is moved to a location outside the attractor, but within a neighborhood with radius 0.1. As the graph shows for one of the runs, the system quickly recovers from each perturbation, and re-enters the attractor within several hundreds of time steps. The graph is typical for the ten runs.

As has been explained, the stochastic nature of the system implies that sometimes the behavior will differ from what is generally observed. This was indeed observed on one occasion during the ten runs. In that case, the system did not immediately re-enter the attractor after a perturbation, but first moved outside the neighborhood of the attractor. However, before the next perturbation, the system had re-entered the neighborhood of the attractor again.

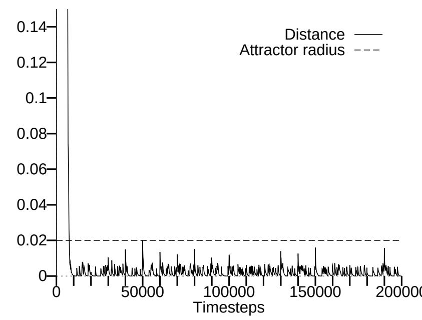

Figure 10: Distance of the stochastic system's state to an attractor over time for one of the runs. At each perturbation, where the system is moved to a random position within the attractor (indicated by the ticks), it remains within the attractor.

Figure 10 shows a run of the second perturbation experiment, where the system is moved to a random location *within* the attractor. After every perturbation in every run the system remained in the attractor, with (again) one exception where the system temporarily escaped from the attractor, but after 1,000 time steps, long before the next perturbation, it had re-entered the attractor again.

Together, the two perturbation experiments that were described are an experimental demonstration of the hypothesis that the stochastic system has attractors that correspond to perfect systems of communication.

5. Conclusions

The development of communication has been analyzed from a dynamical systems perspective. It was demonstrated that a deterministic communication system has point attractors that correspond to perfect communication. The determinism of that system impedes the spontaneous development of communication however. A very similar stochastic system is preferable in this respect, but point attractors require determinism and could not be present. However, the presence of points with a very similar role was suspected. This hypothesis has been tested experimentally by performing perturbation experiments similar to those used for demonstrating the presence of point attractors in the deterministic system. The outcome of the experiments was that the stochastic system has attractors, defined analogous to point attractors but suited for the stochastic system, that correspond to perfect communication.

The significance of this work is twofold. First, it establishes that the association update algorithm for individual agents results in attractors for the population that correspond to perfect communication. At the same time however, the results have a much broader scope, even if speculative; they suggest an explanation of how large populations of animals and humans may come to use the same words in similar situations, an astonishing feat given the huge space of possibilities and lack of central control. That explanation is that once certain regions of this immense association space are entered, interactions between individuals are such that the population as a whole is drawn towards using words the same way.

6. Acknowledgements

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