Consequence of reputation in an open-ended naming game

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We study a modified version of the naming game, a recently introduced model which describes how shared vocabulary can emerge spontaneously in a population without any central control. In particular, we introduce a mechanism that allows a continuous interchange with the external inventory of words. A playing strategy, influenced by the hierarchical structure that individuals' reputation defines in the community, is implemented. We analyze how these features influence the convergence times, the cognitive efforts of the agents, and the scaling behavior in memory and time.

I. INTRODUCTION

The origin and spread of languages and the evolution of their differentiation are problems addressed by various theories that cross different philosophical orientations, varying from nativism and evolutionary approaches to behaviorism and conventionalism. A good archetype of this last perspective can be retrieved in the last of Wittgenstein's reflections [1]. There, language is seen as an activity that arbitrarily attributes meanings to words throughout the function they assume in the life of humans. Meaning is defined by the use of language: "the meaning of a word is its use in the language" [1]. In this light, language is a sort of training to react in a specific way in relation to a specific sign: a *language game*.

A particular linguistic problem that can be considered as a grounding test for this hypothesis is the rise of a new linguistic quantity. Linguists tried to characterize in a quantitative way such changes [2], using simple mathematical models to describe the rise (or fall) of a linguistic element [3]. In particular, among the possible different linguistic changes, we will point our attention to learning processes characterized by fast dynamics, such as, for example, the birth of neologisms. Looking at dictionaries, it is possible to see how every year thousands of new words appear or substitute others. Moreover, reading and comparing throughout different periods some newspapers, we can observe how many words or syntactic changes spread out or substitute the old ones. Finally, we can observe the emergence and death of jargon, technical words, or idiomatic expressions. These facts may be considered as a good paradigm for testing more general theories, which can also account for long-term processes, in the same way as, in biology, the study of very fast evolutionary shifts, comparable to the life span of a human being (microevolution), can give insights into the behavior of evolutive processes characterized by geological time scales.

A well-known artificial experiment implemented to simulate these fast learning dynamics, with the aim of testing the general hypothesis concerning the origin of languages which we have sketched above, is the "talking heads experiment"

[4]. There, embodied software agents bootstrap a shared lexicon without any external intervention. Robots concretize a *language game*, developing a vocabulary throughout a self-organized process, a naming game. In resemblance with Wittgenstein's point of view, language can be seen as an autonomous adaptive system shaped and reshaped by the use and the behavior of the local linguistic activity [5].

Recently, these studies have also attracted the interest of the statistical physics community. The dynamics of such naming games is characterized by a period of spread and diffusion of new competing words, followed by a sudden transition [2,3] towards the use of a single word. These facts are quite common to other well-known social dynamics [6], where a population aims to reach a common and shared state, the consensus [7]. One of the novelties of these studies consists in the fact of abandoning any evolutionary approach [8], dealing with the emergence of communication conventions on fast time scales. Moreover, no central control, which can determine a global coordination, is considered, even in the form of some selection force.

A first study in this direction has recently appeared [9], directly inspired by the experiments conducted with the use of robots [5]. In that work the naming game is modeled as simply as possible, with the aim of implementing a lower bound in complexity and processing power. Each player is characterized by an inventory of words associated with an object. At each time step two players, randomly chosen, interact following some simple rules. The speaker retrieves a word from his inventory or, if his inventory is empty, invents a new word and transmits the selected word to the hearer. If the hearer's inventory contains such a word, the communication is a success. The two agents update their inventories so as to keep only the word involved in the interaction. Otherwise, the communication is a failure and the hearer adds an association between the new word and the object. These simple rules put into action three mechanisms: an uploading mechanism that introduces new words from an external inventory of words, an overlapping mechanism that allows the spreading of a particular word among the players, and an agreement mechanism that deletes useless words. With these simple mechanisms the system undergoes a disorder-order transition towards an absorbing state characterized by a single word for all the players. This behavior scales up to very large populations.

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As stated before, Baronchelli *et al.* [9] implemented their naming game inspired by the behavior of the talking heads experiment. In this paper, we are interested in modeling some features more related to a real community of speakers, such as, for example, a classroom of pupils becoming competent in a language, a community of foreigners learning a new language, or the dynamics of jargon creation in a metropolitan tribe or in a group of researchers coping with new objects or concepts. Looking at these situations, the real world language is open ended, with no evident constraints on the possible number of different words. We can evidence a sort of fluidity by which new words can enter or leave the lexicon inventory.

In contrast, in the original model [9], each agent can store an unlimited number of different names only potentially, and not as a matter of fact. This may be understood if we look with attention to that dynamics. Even if the agents can store an unlimited number of words, the rules of the game allow the introduction of new words only if the agent's inventory is empty, and this happens just in the first Monte Carlo (MC) steps. After this fast transient, when everybody has, at least, one word, the system manifests itself like a closed system and no other new words are included. The game is characterized by a fixed number of different words, which, throughout the overlapping and agreement mechanism, reduces to 1.

A second point that we want to investigate is the limited feedback between speakers in the case of a failed communication and its relation with social structures. In a real situation of failure, the hearer is led to learn the association between the object and the word only if he recognizes the speaker as a sort of teacher. Even the fact that the speaker and the hearer are able to establish, by means of a subsequent action, if the talk was successful or not does not seem a sufficient factor to justify the learning of a new association. In contrast, in the original definition of the naming game, the overlapping mechanism always forced the hearer to learn from the speaker. This dynamics is perhaps the most powerful for reaching the consensus state, but may be considered realistic only in the case where the speaker has the fixed role of monitoring and the hearer of reproducing—in other words, if the speaker acts as a teacher and the hearer like a student. It is realistic to suppose that, in general, these roles are defined by the social structure of the community, and not randomly assigned during each communication.

The previous scenario, described by Baronchelli *et al.* [9], assumed players operating under full anonymity. A general attention for the social structure of the community and for the role of population heterogeneity has already appeared in later works, which defined heterogeneous topologies, where different agents play different roles [10], or noncompletely random interactions [11]. However, in our work, we want to point our attention to more specific facts. In situations relating to humans, players may accumulate information about their environment and specifically about potential future interaction partners. All players carry some sort of reputation reflecting their success in communication and, through observation of third-party interactions and gossip, a player's reputation may become known to others. Finally, we can suppose that all players care for learning from agents known to generally obtain successful interactions.

We were naturally led to explore if the model of Baronchelli *et al.* [9] is robust to changes that contemplate these general assumptions and what effects these elements have on the dynamics and statistical behavior of the system. We are interested in introducing these new elements not only to describe a more realistic situation, but to test the robustness of the mechanisms outlined in the previous model and to investigate if some simple new structure is able to improve our system performance in reaching consensus.

For these reasons, in the remainder of the paper, we will present our version of the model. We will describe an open system where each agent can actually store an unlimited number of different names. This fact is possible thanks to a dynamics that allows the introduction of different words at every MC step. Moreover, we introduce a hierarchical structure between the agents playing our naming game, making it possible to distinguish between the players which act as teachers and the ones which act as learners. This will be obtained through the establishment of the concept of status or reputation, a universal feature of human sociality, which can be generally related to numerous large-scale human collective behaviors [12]. The importance of introducing this concept becomes clear if we look at language as a kind of collective, and not individualistic, problem-solving process.

II. MODEL

The game is played by P agents. An inventory that can contain an arbitrary number of words represents each agent. Moreover, an integer number (R) labels each player and represents its reputation across the community. We introduce reputation as a score [13] which is variable in time. The population starts with a random distribution of the R values, and during the time evolution, the reputation of each player changes according to its performance during the game, following the rules explained below. During each interaction, the agent with greater reputation acts like a teacher and the other one like a learner.

At each time step, the following microscopic rules control our model.

(1) The speaker, with reputation R_S, retrieves a word from its inventory or, if its inventory is empty, invents a new word.

(2) The speaker transmits the selected word to the hearer, characterized by the reputation R_H.

(3a) If the hearer's inventory contains such a word, the communication is a success. The two agents update their inventories so as to keep only the word involved in the interaction. The speaker's reputation increases by 1.

(3b) Otherwise, the communication is a failure. If $R_S > R_H$, the hearer adds the new word to its inventory and the speaker does nothing. If $R_S < R_H$, the speaker invents a new word and the hearer does nothing. The speaker's reputation decreases by 1.

The implementation of these rules defines an open-ended system where an unlimited number of words can be really invented. Players invent new words if their inventory is empty (which happens only in the early stages of the simulation) or if their communication is a failure. In fact, we can

think that, in real life, individuals that are not able to communicate are naturally led to look for new words.

The process determines, across the population, a hierarchical structure that allows one to define distinct roles during each communication event. This structure is dynamical and changes throughout the temporal evolution. Every player is continually assessed: reputation is defined as a score, which can be high or low, depending on the previous rounds of the play.

III. NUMERICAL RESULTS

We will describe the time evolution of our system looking at some usual global quantities [9]: the total number of words (N_{tot}) present in the population, the number of different words (N_{dif}), and the success rate (S), which measures an average rate of success of communications.

An initial transient exists where agents have an empty inventory. In this early stage, in each interaction, each speaker invents at least a new word and each hearer can possibly learn one. Already in this phase, the behavior of our model differs from the one of the original naming game. In fact, in those simulations, this phase corresponds to the rise of N_{dif} and finishes when the total number of different words reaches its maximum, equal to $N/2$, which is maintained along a plateau. With our model, the curves $N_{tot}(t)$ and $N_{dif}(t)$ behave in the same manner and constantly increase until a maximum is reached at t_{max}. During this long learning phase, the total number of distinct words does not display any plateau (see Fig. 1).

When the redundancy of words reaches a sufficiently high level, the number of successful plays increases. The curves $N_{tot}(t)$ and $N_{dif}(t)$ begin a decay towards the consensus state, corresponding to one common word for all the players, reached at time T_{con}.

In this dynamics it is possible to distinguish between two phases. The first one is where the system reorganizes itself, building correlations as a consequence of a collective behavior. It starts when the time evolution reaches T_{max} and $S(t)$ maintains a linear increase in time. The second one is when the disorder-order transition takes place and a very fast convergence process toward the absorbing state occurs. Also in our model, the system passes through a quick reorganization before entering the fast transition dynamics. In Fig. 1 we report the temporal evolution for $N_{tot}(t)$, $N_{dif}(t)$, and $S(t)$. In these simulations the initial values of the agent's reputation follow a Gaussian distribution centered in 0 with standard deviation $\sigma=5$.

In general, the system evolution is strongly dependent on the initial condition of the reputations' distribution. The evolution is obviously not dependent on the mean value of the R distribution, but depends on the value of its spread σ. For example, if we start with every player characterized by the same value of R, for the same parameters, the maximum of the total number of words reaches the largest value. In other words, in this situation the system needs the greatest memory size. Increasing the spread value, the necessary memory size decreases, until reaching a minimal value. For instance, if we chose the R values from a Gaussian distribution and we in-
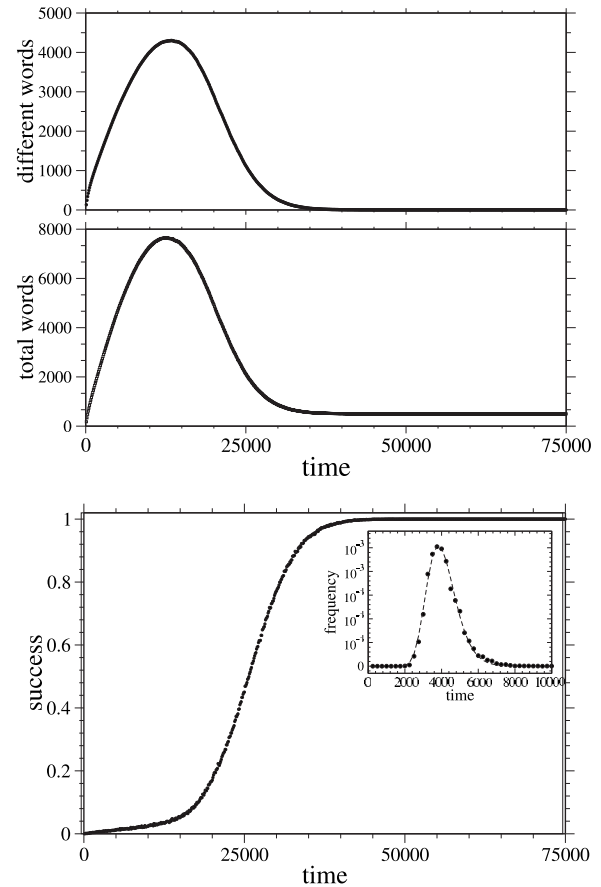

FIG. 1. Top: temporal evolution for the total number of words $[N_{tot}(t)]$ and for the number of different words $[N_{dif}(t)]$. Bottom: the success rate $[S(t)]$. Data are averaged over 100 simulations with $P=500$. The agents' initial reputations follow a Gaussian distribution with standard deviation 5. In the inset we present the distribution of the convergence time ($P=100$, $\sigma=5$). Data are fitted by a log-normal distribution.

crease the standard deviation, the maximal memory necessary to begin producing successful plays decreases, reaching a minimal value for a standard deviation equal to 5 (an optimal value that is preserved for different population size). In this interval of σ values, the convergence time towards consensus seems to be not very different. In contrast, if we further increase the spread, T_{con} considerably increases. Long-standing quasi stationary states appear, characterized by the presence of a fixed small number of words designating the same object. Figure 2 clearly shows this behavior.

We can understand this fact noting that, if an agent has a relatively high reputation, in general, it will not learn words from other players. On the other side, it will have a greater chance to propagate its own words. Agents with the highest R value will act as nucleation points, causing the spreading of the words that generally survive in the final state. For this reason, in a population with a broad distribution in the reputation, a higher number of prestigious agents form different clusters of agents with the same words and a coarsening dynamics generates a very slow evolution towards consensus.

Furthermore, we explore how initial asymmetric distributions of R could affect the system evolution (see Fig. 3). We
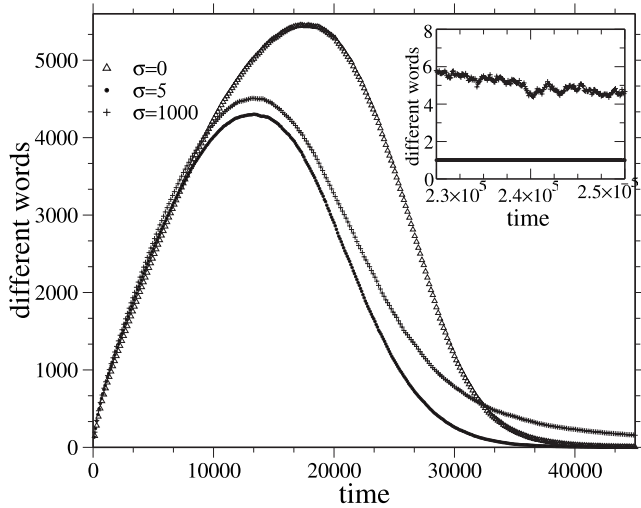
FIG. 2. Temporal evolution for the number of different words for a population with a reputation obtained from Gaussian distributions with different standard deviations (σ). The inset shows the very slow convergence towards the absorbing state, characterized by the presence of long-standing quasistationary states. Data averaged over 100 simulations with $P=500$.

run simulations with two classes of agents: one with $R=5$ and the other with $R=-5$. When the subpopulation of $R=5$ players is a majority, the system performance, in terms of memory cost, gets worst. In contrast, if the number of players with a high reputation is smaller, the memory cost necessary for reaching consensus is sensibly reduced. These results have a simple interpretation. It is easier to reach consensus in an authoritarian community, where the few individuals with a high reputation can easily and efficiently spread their words among population.

IV. SCALING LAWS

From the viewpoint of applications and for better understanding the model behavior, we investigate how the macro-
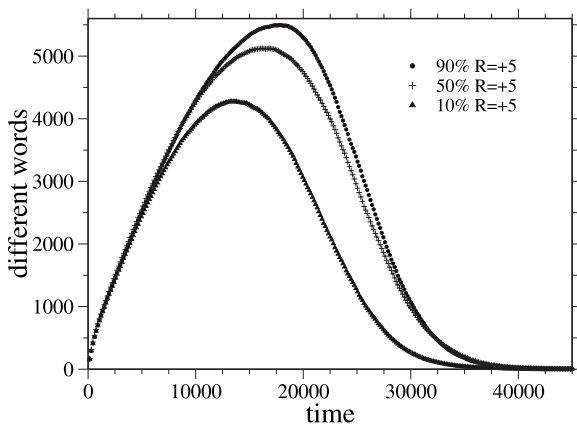

FIG. 3. Temporal evolution for the number of different words for populations with an initial asymmetric distributions in R. The population with 10% of agents having $R=+5$ (teachers) and the others having $R=-5$ shows the minor memory requirement. The population with 90% of teachers needs the largest amount of memory. Data averaged over 100 simulations with $P=500$.
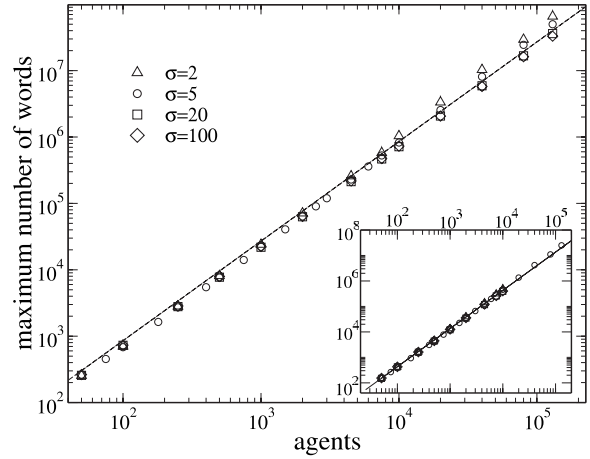

FIG. 4. Maximum number of total words and, in the inset, maximum number of different words for different population sizes. The lines have slope $3/2$. In all simulations the agents' initial reputations follow a Gaussian distribution. These results are robust with respect to the choice of different σ values.

scopic observables scale with the size P of the population.

At first, we look at the scaling behavior of the system memory size. The maximum number of total words ($\max[N_{tot}]$) and the maximum number of different words scale according to the same power law: $P^{3/2}$ (see Fig. 4). The behavior of $\max[N_{tot}]$ is the same founded by Baronchelli *et al.* [9]. In contrast, in their work, the maximum number of different words scales as P, because it is governed by a different dynamics that does not allow the introduction of an unlimited number of new different words.

In our model, the scaling of the time position of the peak that corresponds to the maximum number of total words is $T_{max} \propto P^{3/2}$ (see Fig. 5). To sum up, the dynamics of accumulation and spread of words of our model is very similar to the one given by the model described in [9], which required a large agents' memory ($P^{3/2}$) and a long period of words exchanging between players ($P^{3/2}$). A way of improving this behavior, experimented for the model in [9], is its implementation on a low-dimensional regular lattice, where a minor memory requirement is found, at the cost of a very slow convergence towards consensus [14].

The time necessary to reach convergence to the global consensus (T_{con}) is the other fundamental quantity that characterizes our system. T_{con} displays an interesting behavior: for small communities ($P<10\,000$) the hierarchical structure defined by the reputation parameter has a strong influence over convergence and

$$T_{con} \propto P^{1.2}.$$

This scaling behavior is slower than the one found by Baronchelli *et al.* [9], where $T_{con} \propto P^{3/2}$. It is also slower of the same model when embedded in a small-world topology, where the convergence process has a $P^{1.4}$ dependence [15] (we remind the reader that slower scaling behavior in P means faster convergence time). Unfortunately, the positive effect of reputation breaks up as the dimension of the community grows. For a population larger than 10 000 individu-
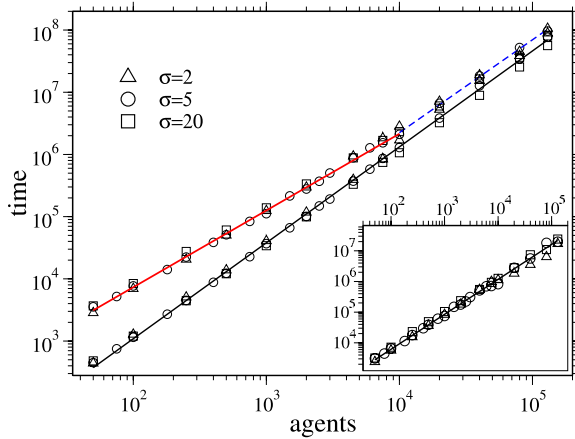
FIG. 5. (Color online) Convergence time (T_{con}, data on the top) and time position of the peak that corresponds to the maximum number of total words (T_{max}, data on the bottom) for different population sizes P. T_{con} displays a crossover: for $P < 10\,000$, $T_{con} \propto P^{1.23 \pm 0.01}$; for $P > 10\,000$, $T_{con} \propto P^{1.49 \pm 0.03}$. T_{max} is well described by a power law with exponent 1.54 ± 0.01. The inset shows the power-law dependence of the alignment time $T_{con} - T_{max}$ (the fitted exponent is 1.14 ± 0.02). We present simulations where the agents' initial reputations follow a Gaussian distribution with different σ values.

als there is a sudden change (see Fig. 5) and the new dependence becomes $P^{3/2}$. This large-scale behavior of T_{con} is an expected outcome: in all the implementations of these models, the P dependence of T_{con} must be higher or equal to the T_{max} dependence. From this perspective, the temporal scaling behavior of the dynamics of accumulation and spread of words (T_{max}), which usually scales as the system memory size, directly influences the P dependence of T_{con}.

We can better understand our results with the help of some simples considerations. The convergence time encompasses the time necessary for the conclusion of two different dynamical processes which can have distinct scaling behaviors. The first process, which occurs from the starting of the simulation up to T_{max}, is characterized by the accumulation of new words in the agents' memory and the establishment of correlations between them. In this time span, the introduction of the reputation structure does not have any relevant role in the scaling behavior, which shows the same exponent ($\alpha = 1.5$) as in the classical model. The second process, which extends from T_{max} to T_{con}, is characterized by the fast alignment among all memories. This second time period ($T_{con} - T_{max}$), in contrast, is strongly influenced by the presence of the hierarchical structure introduced by the players' reputation. In fact, if we look at its scaling behavior (see Fig. 5), we find a power law with an exponent β slightly smaller than 1.2. These facts suggest that $T_{con}(P)$ can be alternatively fitted by a linear combination of these two power laws: $T_{con} = aP^\alpha + bP^\beta$.

From these considerations we can interpret the origin of the crossover pointed out by our previous fitting procedure. A characteristic P_{cha} value exists that defines two different scaling behaviors. For $P < P_{cha}$ the slower power law P^β is the relevant one; for $P > P_{cha}$, the relevant is the faster one (P^α). These facts introduce the notion of a small system,

which can be interpreted as the characteristic scale for which the social structure is relevant in defining the overall convergence time of the ordering transition. This happens when the second process of fast alignment results in being more relevant than the dynamics of memory accumulation.

Finally, this analysis suggests to us an interesting supposition. If we arrange the agent plays in a small-world topology where, for the original definition of the naming game, the memory costs and, in particular, the time to reach its maximum are reduced (P dependence [15]), the scaling law $T_{con} \propto P^{1.2}$ may be preserved for any population size.

The scaling behavior of the convergence time that we obtained for relative small communities is quite promising. As far as our knowledge goes, for mean-field-like interactions, only a complex playing strategy which introduces an ordering in each agent inventory (play smart strategy [16]) is able to obtain faster convergence times (a scaling behavior slower than $P^{1.5}$). Such an algorithm implies a cognitive effort for each individual. In contrast, our model introduces a light structure at the collective level of the community which is able to obtain similar behaviors for small populations.

V. CONCLUSIONS

We presented results regarding an implementation of the naming game. Our model describes an open-ended system and embodies a hierarchical structure introduced by players' reputation, which reflects their success in communication.

We showed that convergence towards the use of a single word is possible. The analysis of the scaling behavior, in dependence on the population size, evidences that our model, in the limit of very large populations, belongs to the same universality class of the model of Baronchelli *et al.* [9] for the behavior of the maximum number of total words, its time position, and the convergence time. In contrast, for small communities a slower P dependence in convergence is found. These results assess the robustness of the disorder-order transition for real open-ended systems. They propose an innovative collective structure able to improve the system performance in reaching consensus, suggesting a way for optimizing artificial semiotic dynamics. Moreover, we found how a dependence on the initial distribution of the agents' reputation exists, which can lead to the appearance of long-standing quasistationary states characterized by a low numbers of words. From a more general point of view, we tested the possibility of reaching consensus through a more realistic overlapping mechanism, implemented by introducing the concept of reputation.

Finally, we want to recall how many interesting problems remain open and could be subjects for future works. First of all, it will be interesting exploring the role of the system topology. Different complex topologies could be studied for agents embedded on more realistic networks. As mentioned above, the exploration of the effects of a small-world topology on the two P-scaling regimes could be particularly relevant. In fact, if it could represent a trade-off between the memory peak time and the convergence time, a really faster convergence for large populations would appear. Second, it could be interesting to elucidate if, for large σ values, it is

possible to define an order-disorder transition, with a critical σ value for which consensus is not attainable. More generally, this study should show a thorough investigation of the role of different R distributions.

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