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Eppure il determinismo, che non lascia alcun posto alla libertà umana e obbliga a considerare come illusori, tutti i fenomeni della vita, racchiude una reale causa di debolezza: la contraddizione immediata e irrimediabile con i dati più certi della nostra coscienza. [1]

Ettore Majorana, 1942

[1] Yet determinism, which does not leave any place for human freedom and obliges to consider as illusory, all the phenomena of life, contains a real cause of weakness: the immediate and irremediable contradiction with the most certain data of our conscience.

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Related publications

- Interplay between social influence and competitive strategical games in multiplex networks, Scientific reports, 7(1), 7087, Roberta Amato, Albert Díaz-Guilera and Kaj-Kolja Kleineberg
- Opinion competition dynamics on multiplex networks, New Journal of Physics, 19(12), 123019, Roberta Amato, Nikos E Kouvaris, Maxi San Miguel and Albert Díaz-Guilera

Chapter 1

Introduction

1.1 Introduction to Physics of Social Systems

The research of laws able to describe behavioral regularities found in human society has a long history. The pioneering work could be attributed to the mathematician philosopher Thomas Hobbes, who, in the 1651, composed the book *Leviathan*. In the Leviathan, he started to create the theoretical framework of universal features regarding individuals' preference and the nature of interaction between them. Thomas Hobbes theory was the first step in trying to understand the fundamental rules governing social system and to make predictions on collective behaviors [1]. This intuition about humans collective behavior is illustrated in the famous *Leviathan* cover page, reported in Fig 1.1, where the State is represented as a single body made up of individuals

"the people is not in being before the constitution of government as not being any person, but a multitude of single persons" [2].

Hobbes noticed that isolated individuals and interacting individuals exhibit different behaviors, but he was not able to explain how this collective entity emerged from the multitude of unities. This transition remained *"almost magical spontaneous generation, âĂŸlike a creation out of nothing by human wit"* [3, 4]. Since then, we continue to question about this *spontaneous* transaction from heterogeneity to homogeneity .

"However obscure their causes, [permit] us to hope that if we attend to the play of freedom of human will in the large, we may be able to discern a regular movement in it, and that what seems complex and chaotic in the single individual may be seen from the standpoint of the human race as a whole to be a steady and progressive though slow evolution of its original endowment [5]. "

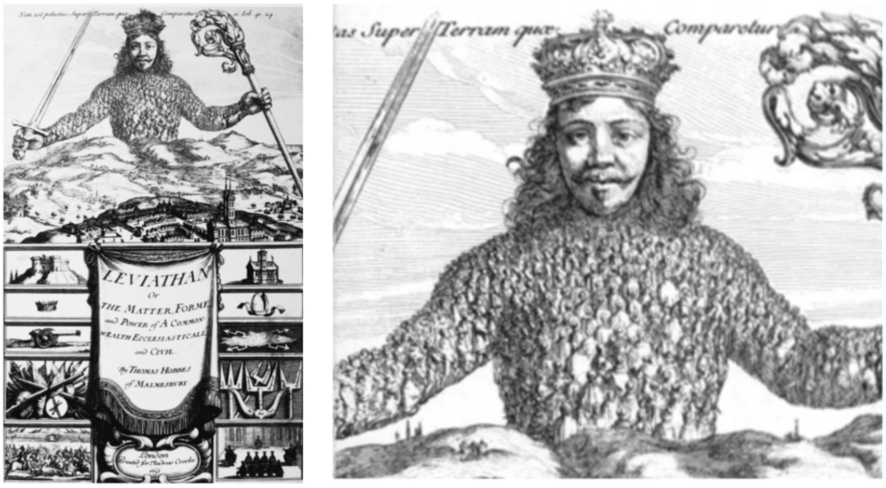

Figure 1.1: Cover of Leviathan of Thomas Hobbes (1651). On the right a zoom of the upper part.

People who interact seem to become more similar, building those social conventions that underlie social and economic relations [6, 7, 8, 9]. Examples range from driving on the right side of the street, to language, rules of politeness or moral judgments. Conventions emerge either thanks to the action of some formal or informal institution, or through a self-organized process in which group level consensus is the unintended consequence of individual efforts to coordinate locally with one another [7, 10].

The order arises spontaneously from disorder: if people were isolated, it is natural to think that each one would form a own different opinion on a specific topic, would exhibit a unique set of cultural features and would invent own language to name the objects [11]. The famous case of *Nicaragua Sign Language* is an evidence of that [12, 13]. The *Nicaragua Sign Language* is a spontaneous language created almost entirely by children between the 1970s and 1980s. In Nicaragua, before the 1970s deaf people were almost isolated from society and there was no language of the signs shared by the community. Each one, then, develops a personal language to communicate with one's own family. Finally at the end of 1970s a school in the area of Managua was opened. However, in this school no sign language was taught, the school served mainly to get people out of the isolation. The young children began to interact, and the interaction brought to life a shared language. This phenomenon of moving from a myriad of different languages, or opinions or...etc. to a state of convergence is often identified as the process of agreement. Agreement in human interactions indicates the existence of regularities at large scale as collective effects. From a statistical mechanic point of view they can be associated to a spontaneous transition from disorder to order. James Clerk Maxwell made the first real jump in this direction, noting his problem about gases consisting of continually moving particles could be associated with the problems of the average behavior of a society formed by individuals [1, 14]:

> "In studying the relations between quantities of this kind, we meet with a new kind of regularity, the regularity of averages, which we can depend upon quite sufficiently for all practical purposes" [15].

The intertwinement between physics and sociology produced the development of important philosophical and mathematical tools for both disciplines. Of course, the forces governing physical systems are different from those governing human interactions. Social driving forces are identify with social influence [16], homophily [17], reciprocity [18, 19] ... etc.

Modeling social systems inevitably leads to ignore a lot of human characteristics. One of the goals is to be able to characterize the interaction with simple rules and be able to reproduce the collective dynamic, understanding

> "whether this approach can shed new light on the process of opinion formation" [11].

Humans becomes agents interacting in a controlled environment through simple dynamic rules. In these Agent-Based (A-B) models, the agents can assume different states representing opinions, language, strategies..etc. The dynamical rules, with which agents can change their states, are expressed defining a probabilistic transition rate between different configurations of the system at successive times: each agent composing the system will make a certain choice with a certain probability influenced by the rest of the system. How an agent is influenced by others plays a crucial role in the macroscopic outcome and is determined by two characteristics: the choice of the social driving forces and the topology of the interaction.

In modeling the process of agreement the typical driving force is the mechanism of imitation : two interacting agents will end up in the same state, one of the two will imitate the other with a certain probability. Subsequent local consensus will result in a global consensus. A shared state can be reached also thanks to the mechanism of *adaptation* where individuals make decisions on some action based on a common knowledge [10]. These kind of models are often called "Game" and the actions, the strategies. Games are adaptive models because agents need to adapt their strategy depending on how *it worked* in relation to the strategies chosen by other agents. The dynamical equilibrium is reached when agents decided not to change their strategy and it is not given that all the agents will consent on the same one. Another important class of A-B models concerns contagion process where a state spreads among agents through an epidemic process and in general an agents need to be exposed several time to a specific state before adopting it [10, 20]. In the context of collective behaviors that can be developed in social systems, I will deal with opinion dynamics, with the phenomenon of human cooperation and with social norms evolution. Opinion dynamic and social norm evolution will often intersect with a description in terms of shared language. Indeed, the phenomenon of the existence of a common language is often taken as a prototype of the process of agreement. Cristina Bicchieri defines norms as the *Grammar of Society* [8]

> "[...] because, like a collection of linguistic rules that are implicit in language and define it, social norm are implicit in the operation of a society and make it what it is " [8].

Another fundamental aspect in the evolution of social consensus is the topology of the interaction that define, broadly speaking, who interacts with whom. In this thesis we will see how, although the dynamical rules are the same, several topologies lead to different results, pointing out how the structure of interaction is fundamental in the creation of specific human collective behaviors. Social systems structure is represented through the use of complex network tools, a mathematical representation of a group of interacting entities in which the interaction among constituents is crucial in the emergence of organized structures and collective behavior.

In the following sections a brief presentation of the theory of networks and of the models that will be used in the main corpus of the thesis is presented. Part of my research consist in extending these models to more complex network structures, called Multiplex, which will be presented at the end of this chapter. It will then move to the main body of the thesis where I will present the models conceived during my research.

The first model describes the influence of different social contexts in the formation of consensus. We consider that people interact with others in many situations or environments like work, family, spare time, etc; and different contexts can therefore produce conflicting social influences. We have proposed to study which conditions favor consensus and which coexistence of opposing opinions. This study was inspired by a model originally proposed to describe the language competition in bilingual societies, the Abrams-Strogatz model [21].

In the second model we analyze the role of social influence in competitive strategical game, situations where personal and common interest are in conflict. Several studies and experiment ([22], [23], [24]) have been proposed to understand the birth and the survival of cooperation in humans societies. Here we will focus on the interplay between opinions and actions, considering, also in this case that, social interaction and game dynamics can take place in different domains.

The last model concerns the study of the evolution of social norms, namely what happens when a new convention replaces an old one. Collective shifts in behavior resulting in the adoption of new norms may seem like a paradox, as consensus seems a one-way process : once reached, it should last indefinitely. But instead we are witnessing an evolution of society, customs, language, etc. And little is known about how a population undergoes behavioral change, due to the difficulties of finding representative data. We investigate the process of norm change by looking at 2,364 linguistic norm shifts occurred in English and Spanish over the last two centuries and propose a model able to explain the data.

1.2 The Role of Topology: Complex Networks

Social systems, as well as many physical and biological systems, are constituted by interrelated components (i.e. individuals, animals, cells...etc) whose interaction produces emerging behavior (i.e. self-organization, adaptation, collective behavior, etc...) that do not exist at the level of individual constituents [25]. An appropriate description of such systems and of the possible dynamics that can be generated, must therefore include a modeling framework that allows a simple representation of the interaction. This need has laid the foundations for the development of the so-called Complex Network Science. A network (or graph, as it is called in mathematics) can be defined as an abstract structure composed by nodes connected by a set of links or edges encoding the interaction [26].

Since their introduction, there has been a growing interest in network science thanks to the multiple applications in a wide range of disciplines, as diverse as physics, sociology, biology, economics, etc... In social sciences, the theory of networks opened the way to a deeper understanding of many phenomena that could not be explained in terms of individual behaviors perspective. One of the first examples of this application dates back to the 1932, when there was a striking phenomenon of runaways at the Hudson School for Girls in New York. In just two weeks, a substantial group of girls, a rate 30 times higher than the norm, left the school. The psychiatrist Jacob Moreno suggested that this *epidemic* phenomena was due more to the flow of social influence and ideas among the girls than to individual factors [27, 28]. Thus, Moreno and her collaborator, Helen Jennings, using the "sociometry" technique, represented graphically individualsâĂŹ subjective feelings among the girls in such a way that the links in the resulting social structure constitute the patterns by which the girls influenced each other, Fig. 1.2.

" In a way that even the girls themselves may not have been conscious of, it was their location in the social network that determined whether and when they ran away. " [28]

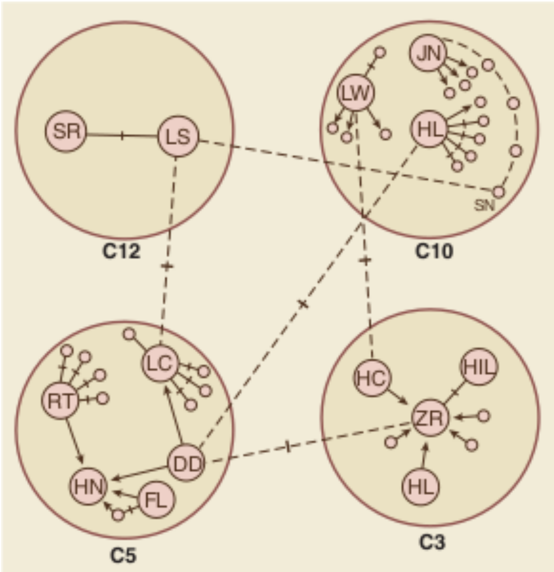

Figure 1.2: MorenoâĂŹs network of runaways. The four largest circles (C12, C10, C5, C3) represent cottages in which the girls lived. Each of the circles within the cottages represents an individual girl. The 14 runaways are identified by initials (e.g., SR). All nondirected lines between a pair of individuals represent feelings of mutual attraction. Directed lines represent one-way feelings of attraction. From "Network Analysis in the Social Sciences", S.P. Borgatti et al. [28].

Networks science began in 1736 as a mathematical theory of graphs, thanks to the Swiss mathematician Leonhard Euler who proposed it as a solution to the Königsberg bridge problem. The problem to solve is whether it is possible to find an optimum path that cross each of the seven bridges only once. For this purpose, Euler conceived a mathematical structure in which the river banks and the bridges were represented by vertices and edge of a *graph*, as shown in Fig. 1.3

This representation allowed him to schematize the problem and find a solution that could be applied to any bridge configuration. Euler found that even if only one node

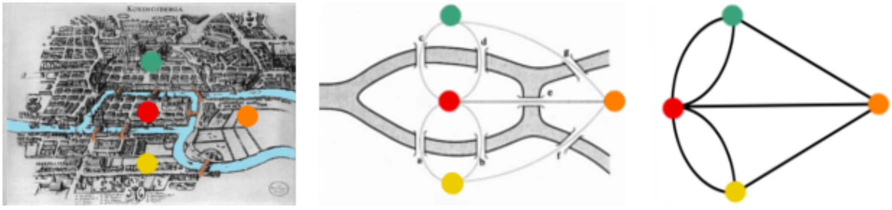

Figure 1.3: The city of Königsberg and the construction of the graph

has an odd number of connections there is no path that contains all the edges only once [29].

Network theory, started as a mathematical theory, began to expand, finding numerous applications in different disciplines. Some networks are real physical networks like

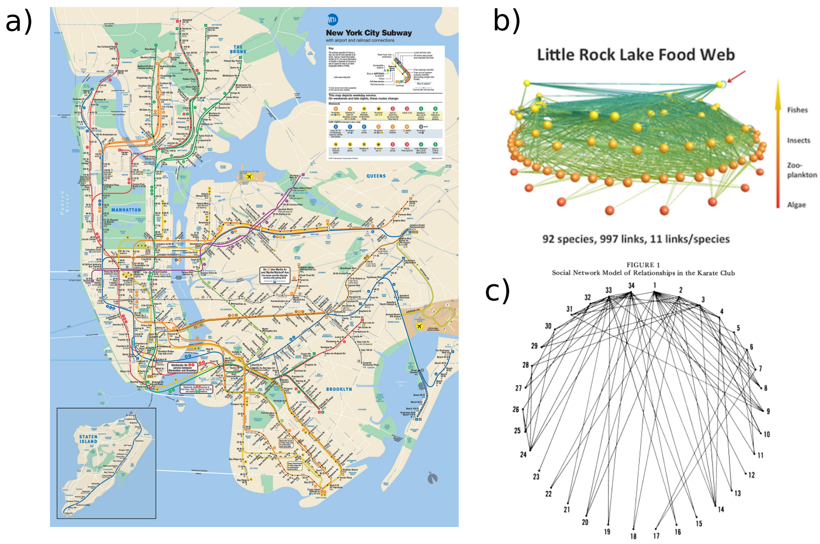

Figure 1.4: Networks Examples.a) New York Subway Network. b) Little rock ecological network. c) Karate club's network.

the neural network that connect the neurons in the brain or public transportation network, Fig. 1.4.(a), but other are more abstract. For example the ecological network "Food web" built in [30], Fig. 1.4 (b). This network captures the predator-prey interaction in the Little Rock Lake in US : a pair of species is connected if one species eats the other. At the beginning the networks representing real systems were quite small, due to the difficulty of finding big data sets. For example, social networks could only be done through direct interview of the people involved, like the famous 'Karate club' network constructed by Wayne Zachary [31] on the basis of empirical observations of interaction between the Karate club's members, Fig. 1.4 (c). The network is constituted by 34 individuals and represent friendship relations between the members. This social network was particularly studied in the contest of community detection. "In network science we call a *community* a group of nodes that have a higher likelihood of connecting to each other than to nodes from other communities" [32]. Zachary reported 78 pairwise links between members who have a regular interaction outside the club. The dataset has attracted attention thanks to a particular event that occurred during the observation period: a disagreement developed between the administrator and the instructor of the club resulted in the instructor's leaving and starting a new group with some of the members. This fact reveled an underlying community structure in the club's network, Fig. 1.5 $a)$. Thus, many community finding algorithms are often tested on the Zachary's club dataset to prove their ability in inferring the two communities from the topology of the network before the split [33, 34, 35]. This proceeding was started by Girvan and Newman in 2002 [33] and since then is exploded, Fig. 1.5 $b)$. The article became so cited in the complex networks science community that an honorific group with annexed prize was invented: "The first scientist at any conference on networks who uses Zachary's karate club as an example is inducted into the Zachary Karate Club Club, and awarded a prize." [36]

With the advent of the digital age, this changed and many of the actual networks studied have hundreds or thousands of nodes and as many links. Over time scientists from different disciplines had to develop a wide range of tools for analyzing, modeling and understanding networks. Furthermore, the study of the different interaction topologies showed that the structure of a network can reveals underline properties of the system it represents.

There are different measures that capture different aspects of the network. The sim-

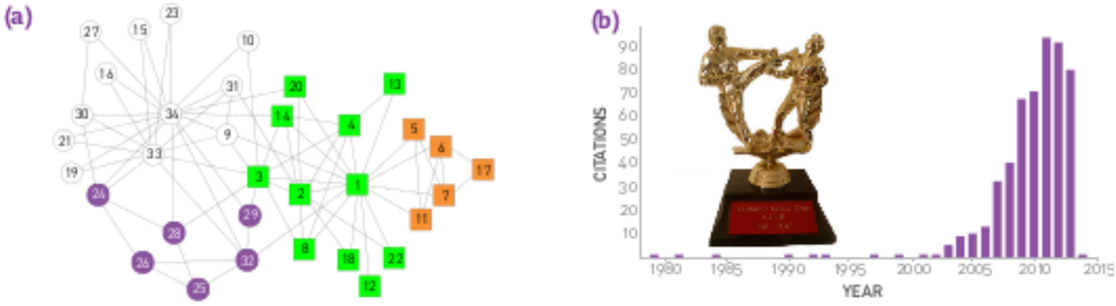

Figure 1.5: Form "Network Science", Barabási, Albert-LÃąszló (2015) [32]. $a)$ The connections between the 34 members of Zachary's Karate Club. Links capture interactions between the club members outside the club. The circles and the squares denote the two fractions that emerged after the club split in two. The colors capture the best community partition predicted. The community boundaries closely follow the split: The white and purple communities capture one fraction and the green-orange communities the other. $b)$ The citation history of the Zachary karate club paper [31] mirrors the history of community detection in network science. Indeed, there was virtually no interest in ZacharyâĂŹs paper until Girvan and Newman used it as a benchmark for community detection in 2002 [33].

plest measure is the *degree* of a node and express its number of connections, namely her number of *neighbor*. Typically the degree of a node i is denoted by k_i. Every link connects a pair of nodes and can be directed and point from one node to the other or undirected and connect the nodes symmetrically. In directed networks the degree of a node is distinguished between *in-degree*, the number of incoming links, and the *out-degree*, the number of outgoing links [37]. An example of directed network could be the network of Web pages where the in-degree of a node represent the number of links a web page gets while the out-degree the number of pages to which it points to. In this work we will consider the only undirected networks.

The degree distribution, $P(k)$, defines the probability that a randomly selected node has k links. Empirical results show that, typically, large real networks exhibit a power-law degree distribution of the form $P(k) \approx k^{\gamma}$, where γ is in the range between 2 and 3 [38]. The characteristic of these networks lies in the fact that the nodes that are at the tails of the degree distribution have a number of connections much higher than the average. These few, but significant nodes are called *hubs*, and play an important role in network structure. Another common property of real networks, in particular of social networks, is an high level of *clustering*, namely the tendency of nodes of having common neighbors [39, 40]. This characteristic is quantified by the *clustering coefficient*, defined as the probability that two extracted neighbors of a given node are connected. Finally, real social networks exhibit another important property, the so called *small word* property. During the 1980, Stanley Milgram [41] perform an experiment to find out the average degree of separation among two random people living in the United States. The experiment consisted in choosing a sample of man and women, who live somewhere in US. To each of these persons would be given the name and the address of the same target person, located in Massachusetts. Each of the participants has to deliver a letter to the target, using only a chain of acquaintances. The letter could move only from person to person who know each other. The median of the distribution of the steps the letter had to do to reach the target was just 5. With this experiment Milgram discover that real social network are characterize by small-word phenomena. This property can also be defined as a *short path length* property. The characteristic path length L is the average shortest path between any two pair of nodes, namely the minimum number of edges that an hypothetical walker has to cross to go from one node to the other.

1.2.1 Random Graphs

The simplest example of complex network is the random graph, or the so called Erdös-Rényi Networks (ER) [42], in which the links are randomly distributed among the nodes. There are two similar ways to construct this type of networks: in the first, the number of nodes and the total number of links are fixed, while in the second one is the number of nodes and the probability that a pair of nodes is connected [29].

Generally the second method is used for its practicality. Starting from a graph of N nodes and no links, each pair of nodes will be connected with probability p. The total number of links,M, is an expected value given by the relationship $M = p\frac{1}{2}N(N-1)$, where $\frac{1}{2}N(N-1)$ indicates the total number of pairs.

The degree distribution will be a binomial degree distribution and the probability that a randomly extracted node i has degree $k_i = k$ is given by:

$$P(k_i = k) = \binom{N-1}{k} p^k \, (1-p)^{N-1-k} \, . \tag{1.1}$$

The average degree can be directly calculated as $\langle k \rangle = \sum_k kP_k = p\,(N-1)$. In the limit of $N \to \infty$ and fixed $\langle k \rangle$ the binomial distribution converge to a Poisson distribution

$$P(k) \approx \frac{\langle k \rangle^k}{k!} e^{\langle k \rangle} \, . \tag{1.2}$$

As the probability to connect each pair of nodes is identical and equal to p, the probability that two neighbors of a node are connected is still equal to p. This means that the clustering coefficient of an ER network is simply p. Anyway, it can be considered as a small size effect since it just depends on the ratio between M and N and it is not a real characteristic of this kind of networks.

1.2.2 Small word networks

Real social networks exhibit short characteristic path length and high clustering coefficient [26, 43]. These two features are the characteristic sign of the network class named *small-word networks* or alternatively Watts-Strogatz network from the name of those who suggested the model to generate them [43, 44]. Starting from a ring of N nodes and m links per node, each edge is rewiring with probability p. Each node can reconnect one of its m link to an other random selected node with probability p. This model can be view as an interpolation between an ER graph and a regular ring lattice. In fact for $p = 1$ it reduces to a pure random network with $mN/2$ number of links.

1.2.3 Preferential attachment

At the end of the XX century Bárabasi and Albert studied a large subset of the Web networks, the nd.edu domain [45, 46]. The Web network grows continuously in a totally unregulated way, i.e. there is not a central organization that decide the links between pages. Their results show that despite the owner of a Web page can freely choose its connections, the whole Web system obeys scaling laws characteristic of self-organized systems. Similar characteristic were found in the scientific collaboration networks [43, 45]. In a scientific collaboration networks, the node represent authors of scientific paper and are connected if they have written a paper together. In particular they notice that new nodes, such as new web pages or the first article of a new author, tend to link to nodes with more connections, a process called *preferential attachment* [32]. The recognition that many real networks are characterized by growth and preferential attachment inspired the model called the Barabási-Albert model, which can generate, as a consequence of the two mechanism, a scale-free networks [26, 32, 46].

The model develops as follows. At time zero the network is composed by $N_0 = m + 1$ nodes, each of which with m links. At each time step a new node appears and have to connect to m existing nodes with a probability proportional to their degree. The probability to connect to a node i with degree k_i is given by $\frac{k_i}{\sum_j k_j}$. The resulting degree distribution is a power-low degree distribution $P(k) \approx k^{-\gamma}$, with exponent $\gamma = 3$. The preferential attachment triggers a phenomenon often called "rich-get-richer", in which a node with high degree tends to increase its degree because attracts new nodes. In contrast with the ER network the preferential attachment mechanism implies that nodes with high degree have a non vanishing probability to be present.

1.3 Agent based models of social dynamics

In the context of collective behaviors that can be developed in social systems, I will deal with opinion dynamics, the phenomenon of cooperation and social norms evolution. The field of opinion dynamics and social norm includes studies on language dynamics. The language is a prototype phenomena of collective agreement both in the case of language competition within a bilingual society or in the evolution of orthographic norm adoption. In this section I will expose the main results of the voter model and the Abrams Strogatz model respectively for the opinion and language dynamics and evolutionary game theory for the study about cooperation.

1.3.1 Opinion Dynamics : The Voter Model

Various models have been proposed [11, 47, 48, 49] to describe the process of social consensus, with the so-called *voter model* being one of the simplest and most studied [50]. It is a model with two equivalent states, first introduced to describe the competition of biological species [50] and later named as the voter model in ref. [51] for its immediate interpretation in term of voting [11]. The voter model is based on the mechanism of imitation. Individuals are placed on nodes of a networks and are endowed with a binary opinion $\sigma \pm 1$. At each time step a node, randomly selected, will change her state, $\sigma \to -\sigma$, by adopting the state of a randomly selected neighbor [11, 52, 53]. The flipping probability, for a given node i is given by:

$$P(\sigma_i \to -\sigma_i) = \frac{1}{2}\left(1 - \frac{\sigma_i}{k_i}\sum_{j \in n_i} \sigma_j\right) , \tag{1.3}$$

where k_i is the degree of the node i and n_i are its neighbors. The switching probability, defined in eq. (1.3), becomes zero if a node has the same state as its neighbors. This means that the voter dynamics is driven by the presence of *active links*, ρ, namely links between nodes with different option. The density of active links is defined by:

$$\rho = \frac{1}{2L}\sum_{\langle ij \rangle} (1 - \sigma_i \sigma_j) , \tag{1.4}$$

with L being the total number of links. If the system is completely disordered $\rho = 1/2$ while if it is completely ordered $\rho = 0$. The ordering dynamics is then defined by the evolution of ρ, considered as the proper order parameter if the dynamics occur on a complex networks [11, 52]. The state of the system, i.e. its degree of order, is described by the *option polarization* (often called magnetization for its analogy with the magnetic systems), defined as the difference between the fractions σ_+ of nodes in the state 1 and the fractions $\sigma_- = 1 - \sigma_+$ of nodes in the state -1. The option polarization m is:

$$m = \sigma_+ - \sigma_- = 2\sigma_+ - 1 \, . \tag{1.5}$$

In a network of size N, degree distribution P_k and mean degree $\langle k \rangle = \sum_k P_k k$ the density of active links ρ can be expressed in term of option polarization m by the relation [54]:

$$\rho = \frac{1}{2}\psi\left(1 - m^2\right) , \tag{1.6}$$

where $\psi = \frac{\langle k \rangle - 2}{\langle k \rangle - 1}$.

Exact results for the voter model in a complete graph have been derived in [55] and later extended for random uncorrelated graph in [54]. Given the global density of active links ρ, the probability that a node i switch its state is can be written in the form $P(\sigma_i \to -\sigma_i) = \frac{\rho}{2}$. If a node of degree k switch its state then the global option polarization m changes by $\pm\Delta m_k = \pm\frac{2k}{\langle k \rangle N}$. The possible transition probabilities are:

$$\begin{aligned} W_{m \to m+\Delta m_k} &= W_{m \to m-\Delta m_k} = \rho P_k \\ W_{m \to m} &= (1-\rho) P_k \, , \end{aligned} \tag{1.7}$$

where P_k is the probability of selecting a node of degree k. The transition probabilities, Eqs. (1.7), defined a random walk process performed by the variable m in the interval $(-1, 1)$ with absorbing barriers in 1 and -1. The master equation for the probability $Q(m, t)$ of having option polarization m at the time t is given by (For a complete derivation see F. Vazquez et al [54]):

$$\frac{\partial}{\partial t} Q(m,t) = \frac{1}{\tau}\frac{\partial^2}{\partial m^2}\left[\left(1 - m^2\right) Q(m,t)\right] . \tag{1.8}$$

Eq. (1.8) is a diffusive equation with diffusion coefficient $1-m^2$, it means that the voter dynamics can be interpreted as a symmetric random walk in the interval $(-1, 1)$ with two absorbing barriers at the ends. If the system reach the full consensus, namely all nodes consent on one of the two option, $m \pm 1$, the transition probabilities vanish.

The characteristic ordering time τ depends on the topology of the network by the relation:

$$\tau = N\frac{(\langle k \rangle - 1)\langle k \rangle^2}{(\langle k \rangle - 2)\langle k \rangle_2} \tag{1.9}$$

where $\langle k \rangle_2$ is the second moment of the degree distribution, $\langle k \rangle_2 = \sum_k k^2 P(k)$. For a complete graph τ reduces to $\tau = \frac{1}{N}$, giving the general results that in mean field approximation the characteristic time to reach the full consensus scale with the network size [53, 55]. Eq. (1.8) is solvable with standard methods for the Fokker-Planck equations [54, 55]. Given $Q(m,t)$ the expression for the evolution of the average density of active links is derivable by the relation $\langle\rho(t)\rangle = \frac{1}{2}\psi\int_{-1}^{1}(1-m^2)\,Q(m,t)dm$, which solution is:

$$\langle\rho(t)\rangle = \frac{1}{2}\psi\left(1-m_0^2\right)e^{-t/\tau}\,. \tag{1.10}$$

The ordering dynamics is governed by the exponential decay of the density of active link with characteristic time τ. Thus, the role of the topology in the evolution of the system is captured by the characteristic time to reach the full consensus.

1.3.2 Language Dynamics : The Abrams Strogatz Model

The Abrams-Strogatz model (AS model) is a two-state model introduced to describe the decay of minority languages in bilingual societies [21]. It can be considered as a generalization of the voter model in which the two options are here two non-equivalent languages, A and B. The languages are not perceived by the individuals in the same way, they have complementary *prestiges* reflecting the difference in the social status of spoken languages. Language A has a perceived status given by S, while the perceived status of language B is given by $1-S$. An additional parameter, the *volatility*, ν, was introduced in the latter model to indicate the tendency to switch the use of a language. Individuals are capable of speaking both languages and can choose to change it by taking into account the density of speakers of the other language and its prestige. The switching probabilities defined by $P_{A\to B} = (1-S)(1-X)^{\nu}$ and $P_{B\to A} = SX^{\nu}$. At the limit of mean field (MF) approximation, the dynamics of the AS model is well described by the evolution of the A-speakers density X, which reads $\frac{dX}{dt} = (1-X)P_{B\to A} - XP_{A\to B}$. The substitution of the switching probabilities into the latter equation yields the MF equation,

$$\frac{dX}{dt} = (1-X)X\left[X^{\nu-1}S - (1-X)^{\nu-1}(1-S)\right] \tag{1.11}$$

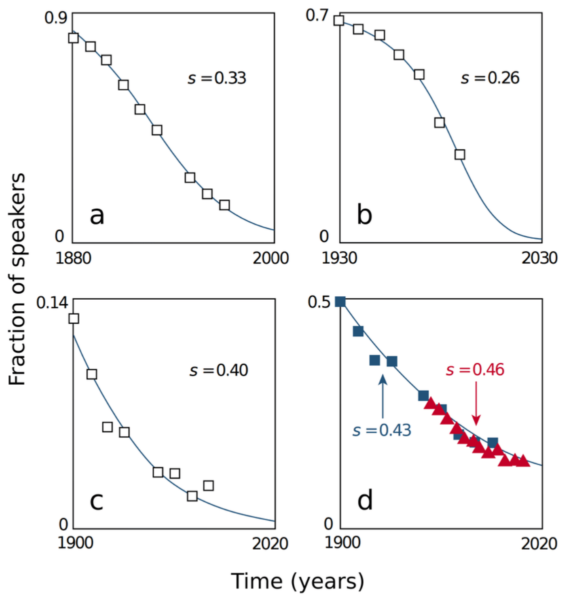

Figure 1.6: From the Article : "Modeling the dynamics of language death", *Abrams and Strogatz, Nature 2003* [21] : "The dynamics of language death. Symbols show the proportions of speakers over time of: **a**, Scottish Gaelic in Sutherland, Scotland; **b**, Quechua in Huanuco, Peru;**c**, Welsh in Monmouthshire, Wales; **d**, Welsh in all of Wales, from historical data (blue) and a single modern census (red)."

The prestige parameter is restricted in the range $0 \leq S \leq 1$, but the case $S < 1/2$ is symmetric to $S > 1/2$. Eq. (1.6) admits three fixed points for $a \neq 1$ and two fixed points for $a = 1$. The fixed point that corresponds to the coexistence state of the two languages is always unstable for $a \neq 1$, while does not exist for $\nu = 1$, i.e. the system always reaches a consensus state, where all individuals speak language A or B. The AS model fits to real aggregated data of endangered languages such as Quechua (in competition with Spanish), Scottish Gaelic and Welsh (both in competition with English) [56, 57] and they find $a \approx 1.3$ [21]. Fig. 1.6 shows data and model predictions of the four cases treated.

Subsequently the model was considered on networks to study the effect of different pattern of connectivity on the language dynamics. For volatility $\nu = 1$, the Abrams-Strogatz model becomes a biased voter model with bias S. On a random network of mean degree $\langle k \rangle$, the interface density of active links between nodes speaking different language, evolve according to:

$$\rho \approx e^{\left[\frac{|v|(\langle k \rangle - 2)}{\langle k \rangle - 1 - |v|}\right]t} \tag{1.12}$$

where $v = 2S - 1$. For $v = 0$ the equation reduces to the voter model.

1.3.3 Social Dilemma and Evolutionary game theory

Social dilemma defines a class of situations in which individuals interests are in conflict, and are often modeled with a two players game. One of the most famous example of these kind of games is the *Prisoner's Dilemma*. The players are two partners in crime, A and B, imprisoned separately. To each of them is offered a penalty discount if one testifies against the other. The benefit of each individual choice is determined by the *payoff matrix* shown in Tab. 1.1. The best individual strategy is to be the only one to testify against the other. However, the best group outcome is realized if both prisoners cooperate and do not testify.

	A confess	A does not confess
B confess	Each serves 1 year	A: 3 years, B: goes free
B does not confess	A: 3 years, B: goes free	Each serves 2 year

Table 1.1: Example of a Prisoner's Dilemma Payoff Matrix

Different payoff matrices can determine different optimal strategies and define distinct games. Anyway at the base of every game there is the conflict between individual and group benefits: cooperation (C) and defection (D) represent the two alternative choices behind social dilemmas [58, 59]. The general payoff matrix is of the form: Where R is the reward if both players cooperate, S is the sucker payoff obtained by a cooperator against a defector, T is the temptation payoff gained by a defector against a cooperator and P is the payoff obtained if both play as defector [58**?**]. Relation among the values of R, P, S and T determine the type of game and its dynamical outcome

$$
\begin{array}{c|cc}
 & \text{C} & \text{D} \\
\hline
\text{C} & \text{R} & \text{S} \\
\text{D} & \text{T} & \text{P}
\end{array}
$$

The game can be iterative and therefore players can change strategy over time. In this thesis we will deal with iterative games on networks where each player will collect her profit playing at each time step with all her neighbors. In an iterative game players can change their strategy over time. Generally, each agent i will copy the strategy of one of her random extracted neighbors j with a probability depending on their payoffs.

Thus, if $R > S > P$ and $R > T > P$ the optimal strategy is to cooperate and the game evolves to a total cooperation regardless of the fraction of the initial cooperators, for this reason this kind of game takes the name of of *Harmony Game*. The opposite situation is the Prisoner's Dilemma, with $T > R > P > S$ and a final outcome of all defection. The third game is defined by the relations $R > T > P > S$. This game, called *Stag-hunt*, is a coordination game and the optimal output is reached when all the players play the same strategy. The last game, defined by $T > R > S > P$, called the *Hawk-Dove* game, is characterized by the coexistence between a fraction of cooperators and defector. Given s_i the strategy of a player i and π_i her payoff, four basic types of update rules are defined:

- Proportional update: $P(s_i \rightarrow s_j)$ with a probability $\approx \pi_j - \pi_i$
- Unconditional imitation : the player imitates the strategy of the neighbor with the larger payoff
- Best response : the player choose that would have yielded the largest payoff given the neighbors's strategies
- Pairwise comparison: $P(s_i \rightarrow s_j) \approx \frac{1}{1+e^{-\beta\left(\pi_j - \pi_i\right)}}$.

The last update rules admits that players can make mistakes, that is, they can choose unfair strategies.

Without loss of generality we can set $R = 1$, $P = 0$ and analyze the outcome of the games in the $S - T$ plane. As en example of dynamical evolution, we can consider the proportional update. In mean field approximation, the density of cooperator $C(t)$ evolve in times according to

$$C'(t) = C(t)\,(1 - C(t))\,(S + C(t)\,(1 - S - T))\ , \tag{1.13}$$

where $(S + C(t)\,(1 - S - T))$ is the difference between the payoff accumulated by cooperators and defector. Fig. 1.7 shows the four games, depending on the values of S and T, and the respective density of cooperators in the mean field approximation. In iterative games where each player is positioned on the node of a network and plays

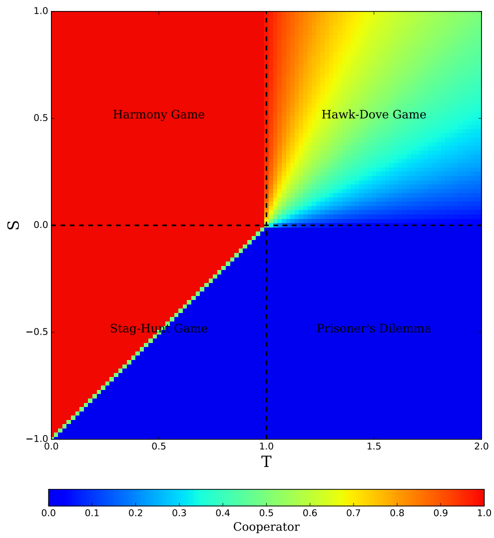

Figure 1.7: Asymptotic density of cooperators in the $S - T$ plane, in the MF approximation

with its neighbors, the network structure can determine changes in the evolution of the density of cooperators. In Fig. 1.8 three different outcomes, depending on the kind of network, are presented. Regular networks and ER networks exhibit similar behavior. Scale-free networks, on the other hand, seems to support cooperation. In [60] the authors explain this result as an interplay of two mechanisms that depend on the topological characteristics of this type of networks. The first is due to the existence of many long-range connections among cooperators promoting the formation of compact clusters of cooperators when the cooperators are able to occupy such highly connected sites, which indeed happens. The second is due to the heterogeneity in the connectivity patterns that, in some way, balances the effect of the temptation to defect ($T > 1$). As a result in the Snowdrift domain, cooperators dominate for all values of $T > 1$ and in the Stag-Hunt domain, cooperators survive even when $S < T - 1$ [60].

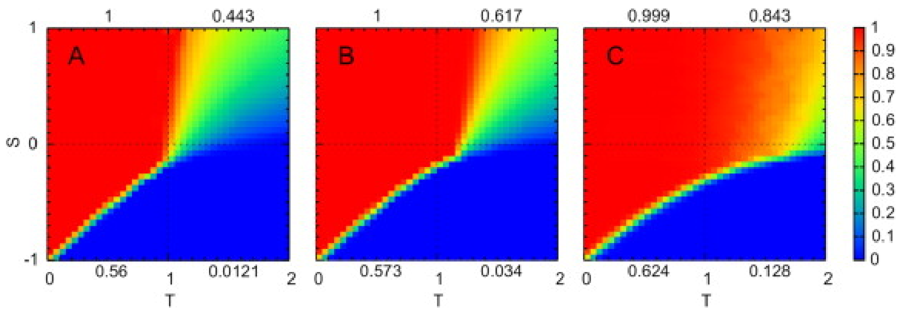

Figure 1.8: Proportional update for different kinds of networks. A: Regular network, B: Erdos-Renyi neywork, C: Barbasi-Albert network. Average degree $< k >= 8$. From "Game on networks and cooperation: Models and experiments", A. Sánchez, CCSS Colloquium. ETH Zurich, 2010 [61].

1.4 From one layer to multilayer social structures

Many real system are often formed by a variety of coexisting topologies[62, 63]. One way to characterize this property is assuming that connections of different nature belong to different layers . This construction generates a structure that takes the name of Multilayer Network [64]. The layers share the same set of nodes and each node has now two types of links: intra-layer links that connect her to the nodes of the same layer and inter-layer links that connect her to the nodes present in other layers. If a node is connected only with its respective counterpart in other layers then the Multilayer networks is called *Multiplex*.

Multilayer networks have been used to analyze public transportation systems [65, 66, 67], spreading of awareness and infection [68, 69], the dynamics of ecological populations [70], cultural dynamics [71], as well as the evolution of social networks [72, 73]. As an example, Fig. 1.9 shows the multilayer system of the different airline company operating in Europe.

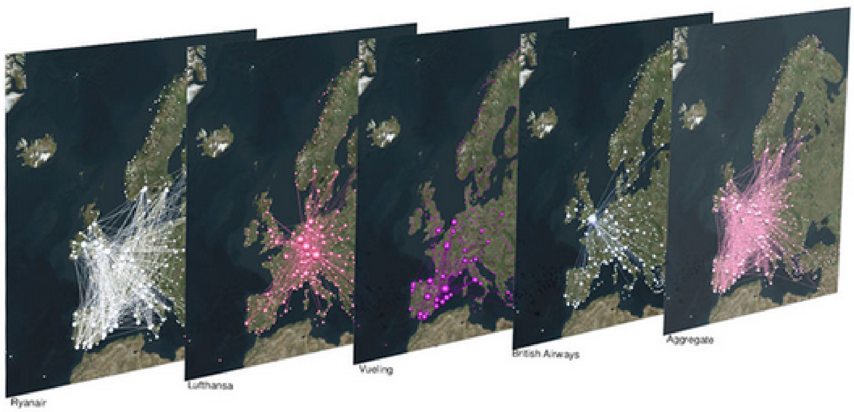

Figure 1.9: From "*Emergence of network features from multiplexity*", A. Cardillo et al. [74]. Multiplex composed by the different airline operating in Europe.

There are many types of multilayer transportation networks, i.e. urban transportation system is one of the main applications since it is the result of the superposition of the bus, subway and railway networks [65, 66, 67]. Biological systems are often represented as a multilayer system in which each layer considers different interaction among the basic constituents (i.e. proteins, cells...etc) like physical chemical, genetic, including regulatory, inhibitory, etc. [75, 76]. Coexisting networks are also observed in social systems [66, 77]. When the multiplex represent humans interaction, the links can be classified according to the type of relationship, i.e. work, friendship, family, etc or to type of action, i.e. taking into account the different social influences such as social media and face to face interaction, or buy and sell, etc... [78, 79, 80].

If all these systems are modeled by just one layer networks either the various kinds of connections are not considered or all layers are overlapped. In both cases the multiplex effect, that has important implications in the dynamics occurring, is neglected. Recently, thanks also to the possibility of finding new data, many systems could be represented in terms of interdependent or multilayer network and new nontrivial structural properties and relevant physical phenomena have emerged [66, 76, 81].

In this thesis I will focus on multiplex representing humans interaction and I will use this structure to introduce some tensions in the system. In the first model I will consider conflicting social pressures, while in the second model the interplay between social and strategical interest. In both cases the multiplex structure allows to represent more sophisticated behaviors of individuals who may have different behaviors in different layers.

1.5 Multplex social networks and dynamics

Multilayer networks allow for a more realistic approach in the study of individuals interactions which can communicate through different types of channels and/or can participate in concurrent interaction patterns. For example, empirical data prove that more than 50% of internet users is inscribed in two or more of the social network sites among Facebook, Twitter, Instagram, Pinterest and Linkedin, as reported by the Pew Research Center [82]. Fig. 1.10 shows how users are distributed among the various social networks, e.g. how many users of Facebook also use Twitter.

The multiplex representation of society raises the question of what kind of relationship there is among the structure of the various layers. As seen in the case of a single layer, social systems are not totally random networks, they present topological features that depend on the mechanism by which people undertake social relations, like clustering or preferential attachment. It is natural to think that if an individual participates in different social contexts, her multiple layer interactions are not completely independent

Social media matrix

% of users of each particular site who use another particular site (e.g., 29% of Pinterest users also use Twitter)

	Use Twitter	Use Instagram	Use Pinterest	Use LinkedIn	Use Facebook
% of Twitter users who...	N/A	53	34	39	90
% of Instagram users who...	53	N/A	37	30	93
% of Pinterest users who...	29	31	N/A	29	87
% of LinkedIn users who...	31	24	28	N/A	83
% of Facebook users who...	22	23	25	25	N/A

Pew Research Center's Internet Project August Tracking Survey, August 07 –September 16, 2013. Interviews were conducted in English and Spanish and on landline and cell phones.

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Figure 1.10: The "*Social Media Matrix*". Source : Pew Research Center' s Internet Project September Combined Omnibus Survey, September 11-14 & September 18-21, 2014 [82]

of each other. In Fig. 1.11, an example of multiplex composed by online and off-line relations is reported [83]. The different people composing the four networks are not connected randomly within each layer, but the multiplex representation of their interaction highlines interesting social patterns. For example Cici, Mat and Mark are all connected both in friend and work networks. The kinds of *correlations* among layers concern both how many links are repeated in the different networks and the importance of a node (for example its degree or its centrality) [83]. It was shown that the relation between the layers in real multiplex can be characterized by geometric correlations in hidden metric spaces underlying each layer of the system [84, 85, 86]. There are two kind of correlations: Popularity correlations, which are correlations between the degrees the nodes have in the layers, and similarity correlations, which control the probability of links overlap between layers. The model we will use to represent these type of correlations is the Geometrical Multiplex Model (GMM) [84, 87] (For the mathematical formulation of this model see Appendix A.1).

An other fundamental question is understand whether all the possible levels of in-

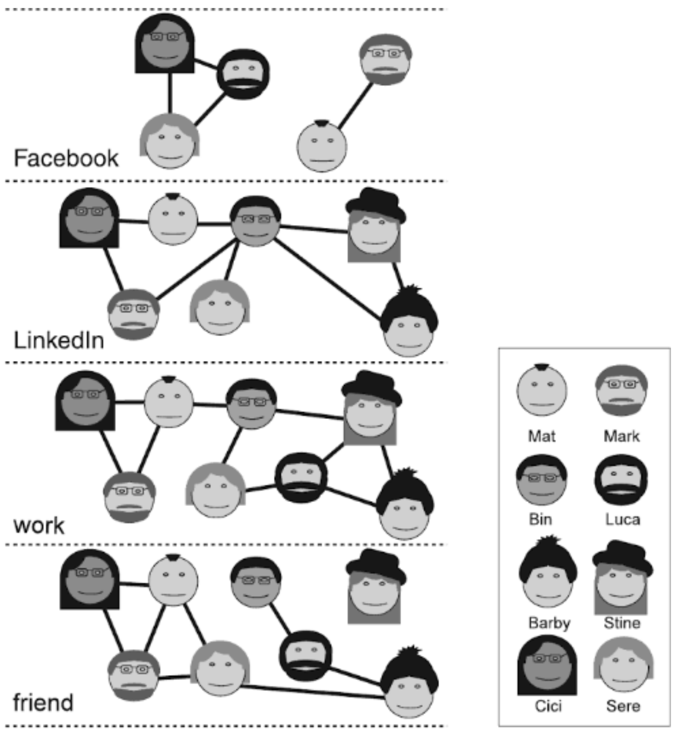

Figure 1.11: From "*Multilayer social networks*", M.E. Dickison et al. [83]

teraction among the constituents of a complex system are important or if some of them might be redundant, irrelevant or uninformative [76, 76, 88]. Recently, Diakonova et al. [88] demonstrated the irreducibility of a multiplex version of the voter model. In that approach a multiplex network was considered, where a fraction of nodes and links can be present in both layers and any change in the state of those nodes in a layer is instantly replicated to the other. This mechanism affects the voter model dynamics and significant differences from the classical single layer case were found. They shown that the single layer approximation for the network of social interaction is accurate only when there is little or no interaction between the layers, namely sufficiently small links or nodes overlap. The modeling of the people interaction through the multiplex structure also allows to consider contemporaneously more dynamics that contribute in the various social phenomena that can arise, as for example in the opinions formation. In [89], Quattrociocchi et al. proposed a model in which media (Tv, newspaper, etc) and social influence are two separated but interdependent processes. They found that an agreement or disagreement among the media can shape the consensus in the opinions' space.

The multiplex structure of an individual's social interactions also has important implications in social dilemma outcome [90]. Gómez-Gardenes et al. [90] shown that in a PrisonerâĂŹs Dilemma game, cooperation is able to resist under extremely adverse conditions, for which the usual one layer model fails. Correlations between layers topologies support the survival of cooperation [90, 91].

As a matter of fact, different studies [92, 92, 93, 94] have shown that social systems are indeed *real* multiplex system meaning that they are characterized by some properties, like i.e. the correlations among layers or coexisting different dynamics, which can not be overlooked in modeling collective humans phenomena.

CHAPTER 2

Consensus dynamics on multiplex networks

A community of people who use language can be interpreted as a complex dynamical system that solves collectively the problem of generating a shared communication system [95]. The choice of a common language is one of the most typical examples of the process of collective agreement in humans society. Here we present a model inspired by the language competition problem but that can describe the more general dynamics of social consensus.

2.1 Introduction to language competition problems

Imagine the history of mankind, not as a history of peoples or nations, but of the languages they speak. A history of 5000 languages, thrown together on this planet, constantly interacting. Imagine the treaty of Versailles not as an event of international diplomacy, but in terms of people putting on their best French to make themselves understood and achieve the greatest advantage. Think of Cortes conquest of Mexico in 1532 not as an outrageous narrative of bravery, cruelty and betrayal, but in terms of the crucial role of his Indian mistress Malinche, interpreter between Aztec and Spanish. Think of the sugar plantations, where the up rooted slaves were thrown together, as meeting places for many African languages. Imagining all this, two things come to

> mind: first, how closely the history of languages is tied up with and is a reflection of the history of peoples and nations. Second, how little we know of languages in contact". What happens in communities where several languages are spoken? How can speakers handle these languages simultaneously? When and why will the different languages actually be used? Which consequences does language contact have for the languages involved? [96]

Interaction for humans is above all communication. Years of immigration, emigration and colonialism led to the formation of multilingual societies, or societies officially monolingual but made of groups of bilingual or plurilingual individuals. This mixture of languages and ethnicity in contact gave rise, over time, to different kind of linguistic organization of society. Appel and Muysken distinguish three fundamental kind of bilingual societies [96]. In the first the two languages are spoken by two different groups and each group is monolingual. This form of societal bilingualism often occurred in former colonial countries where there are just few bilingual individuals taking care of the intergroup communication. In the second society, most of the people are bilingual and the two languages are treated in the same way. In the third form, finally, there is one group monolingual and the other bilingual. In most cases this last group will form a minority, perhaps in the sociological sense: *it is a non-dominant or oppressed group like in Greenland, for example, where the people who speak Greenlandic Inuit must become bilingual, i.e. learn Danish, while the Danish-speaking group, which is sociologically dominant, can remain monolingual.*

When speakers use two languages, they will obviously not use both in all circumstances: the language choice in a fixed conversation may depends on the group which the speakers belong to, the particular social interaction and the topic of the conversation [97, 98]. This general perception has been explored by J. Fishman who has been studying Puerto Ricans in New York, work that has resulted in such famous research reports as 'Bilingualism in the Barrio' [99]. The point of departure for Fishman was the question: *who speaks what language to whom and when?* [100] Fishman studies are focused mainly on the "domain" where the language is used [101, 102]. The domain is defined as *the clustering of characteristic situations or settings around a prototypical theme that structures the speakers' perceptions of these situations* [100].

Bloom and Gumpez [103] argue that the language has not a meaning on its own but the person who use it give it: the social meaning of a particular language is the result of a negotiation between interlocutors. Each social situation is a new negotiation and it is influenced by the previous ones. Repeated negotiations over time can bring to a 'Function specialization' of a language [104], for which special fields of conversation or social event may acquire a preferred language, or a particular language can be representative of the standard of the conversation, for example the use of the dialect that immediately puts the interlocutors in an informal context. When the two languages acquire a definitive different status as, indeed, the dialect and the national language, we talk of Diaglossia [105]. This interaction-negotiation must consider that in a verbal communication also the social and personal identity are in game. Person who are speaking should want to reduce the difference between them and converging in the choice of the language or empathize the difference maintaining their own [104].

Being able to close in a few mathematical parameters the whole range of socio-linguistic reasons pushing two bilingual persons to choose a language rather than another is very difficult. In the model we present in this Chapter, we will focus on the influence of different domains of conversation in the dynamics of language competition. We propose a generalization of the AS model (explained in Sec. **??**) on a multiplex network where each layer represent a different social interaction. Distinct social interaction could meaning different perception of the languages, for which in each layer the languages have different prestiges.

An important novelty of this model is the existence for scenarios of coexistence between languages. The AS model results are matter of discussion since the extinction of one of the competing languages is predicted, although in some cases the coexistence occurs, as the authors remark. The preservation of both languages was explained by Patriarca and Leppanen [47] introducing the existence of two disjoint zones where each language is predominant. However, their results cannot explain the survival of both languages in only one zone of competition. The different scenarios of coexistence we find can be qualitatively interpreted as the different bilingual societies identified by R. Appel and P. Muysken [96]. In the AS model the two languages are fixed in time and compete through the choice of speakers. All this line of models starting from the AS that view the two languages unchangeable in time and not influenced each other ([106], [107], [108]), can be assimilable to the consensus dynamics problems. In fact the AS model with volatility equal to one coincides with a biased voter model in the mean field approximation. Even though our model is inspired by competition between languages, its general aspect also applies to the theme of opinion dynamics. For this reason we will will talk about competition between two generic options from now on.

2.2 Competition of options on Multiplex Networks

Going beyond the AS model for competition of languages, we propose a model for describing the competition between two abstract options, A and B (they can be languages, opinions, voting intention, etc.) on a multiplex network. The model is based on a modification of the AS model with volatility equal to one and keeping the idea that the two options have different perceived status. In this system social interactions occur within distinct layers that may be originated from different contexts like family or business networks, Facebook or Twitter, etc. Nodes and their counterparts across layers correspond to the same individuals participating in different networks. Intra-layer links denote the individuals' connections within each network, while inter-layer links indicate the mutual influence of the individuals' state across layers (see Figure2.1).

We assign a state σ_i^α to each node i ($i = 1, 2, \dots N$) in the layer α ($\alpha = I,\ II$), such that $\sigma_i^\alpha = 1$ (or 0) if the node has the option A (or B) in the layer α. We also endow the prestige S_α to the option A, different in each layers; the corresponding option B has a complementary prestige $1-S_\alpha$. We restrict the values of the prestige to $S_I \in [0.5, 1]$ and $S_{II} \in [0, 0.5]$, in order to guarantee that the two layers do not have the same preferred option. The state of a node in a layer influences its own state in the other layer with

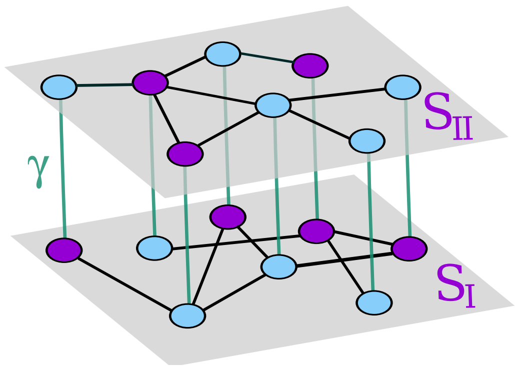

Figure 2.1: Graphical illustration of the multiplex AS model. Different layers $S_I,\ S_{II}$ denote the different networks where individuals participate. Intra-links (black solid lines) correspond to the individuals' connections within each network, while inter-links (green solid lines) indicate the mutual influence of strength γ of the individuals' state across layers.

strength γ, where $0 \leq \gamma \leq 1$. The limited values of γ correspond to opposing situations so, $\gamma = 0$ denotes that the two layers are independent, while $\gamma = 1$ represent the situation where individuals are not influenced by their neighbors.

The dynamical evolution of this multiplex-organized system is described below. A randomly chosen node i in one layer can change its option according to the transition probabilities,

$$
\begin{aligned}
P^{\alpha}_{i,A\to B} &= \Big(1-\gamma\Big)\Big(1-S_{\alpha}\Big)\Big(1-\frac{1}{k^{\alpha}_{i}}\sum_{j=1}^{N} G^{\alpha}_{ij}\sigma^{\alpha}_{ij}\Big) \\
&+ \gamma\Big(1-\sigma^{\bar{\alpha}}_{i}\Big), \\
P^{\alpha}_{i,B\to A} &= \Big(1-\gamma\Big)S_{\alpha}\frac{1}{k^{\alpha}_{i}}\sum_{j=1}^{N} G^{\alpha}_{ij}\sigma^{\alpha}_{ij} + \gamma\sigma^{\bar{\alpha}}_{i}\,, \qquad (2.1)
\end{aligned}
$$

where G^{α}_{ij} is the adjacency matrix of layer α, with elements $G^{\alpha}_{ij} = 1$, if the node i and j are connected in layer α and $G^{\alpha}_{ij} = 0$ otherwise, k^{α}_{i} is the degree of the node i in layer α and with $\sigma^{\bar{\alpha}}_{i}$ we denote the state of the node i in the other layer .

Eq. (2.1) denotes that the likelihood an individuals change his option in a given layer is a linear combination, in terms of γ, of the fraction of its neighbors in the other state and its own state in the other layers.

The novel framework of the multiplex networks we employ here induces a new steady state, never observed in the classical voter model, where the two options coexist. Absorbing states of total consensus, where all individual appear to have a single option A or B are also found. We have to notice that even if one layer instantly reaches a consensus state, the nodes maintain the ability to change thanks to the influence of the other layer.

In the following we present the stability analysis of these steady states starting with a mean field (MF) description of the dynamics. Then, we build a theory for the ordering dynamics to analyze the system's evolution towards the steady states. We also perform numerical simulation on complete (all-to-all) and Erdös-Rényi networks in order to verify our theoretical findings. Finally we will introduce different kinds of correlations between layers to study how the topology influence the dynamics and the distribution of the states among the nodes.

2.3 Mean Field Approach and Master Equation

One of the central problems in the analysis of opinion dynamics is to understand under which conditions a collective agreement occurs. The dynamical evolution of the system (2.1) is analyzed by means of a MF approach, where a previously used order parameter for the single-layer networks [54, 108] is employed. The state of the system is characterized by the *option polarization* (often called magnetization) and is defined as the difference between the fractions X_α of nodes in the state 1 (option A) and the fractions $1-X_\alpha$ of nodes in the state 0 (option B). Therefore, we obtain for the layer α the option polarization

$$m_\alpha = X_\alpha - (1 - X_\alpha) = 2X_\alpha - 1\,, \tag{2.2}$$

which defines the state of the system and lies in the interval $[-1, 1]$, where $m_\alpha = -1$ denotes the winning option B and $m_\alpha = 1$ of option A.

The dynamics of the system is governed by the presence of *active links*, namely of links connecting nodes in different states, because the probability for a node to switch into the other state depends on the density of its active links. Two types of active links are associated with each node: *active intra-layer links* if the node in layer α does not consent with its neighbors in α and *active inter-layer links* if it has different state in the different layers. The density of active intra-layer links of the layer α is given by the expression,

$$\rho_\alpha = \frac{1}{2L^\alpha} \sum_{i=1}^{N} \sum_{j=1}^{N} G_{ij}^\alpha \left(1 - \delta_{\sigma_i^\alpha, \sigma_j^\alpha} \right), \tag{2.3}$$

where $\alpha = I,\ II$, δ is the Kronecker's delta and L^α is the total number of links in the layer α. The density of the active inter-layer links reads

$$\rho_\perp = \frac{1}{N} \sum_{i=1}^{N} \left(1 - \delta_{\sigma_i^I, \sigma_i^{II}} \right). \tag{2.4}$$

In the employed MF approximation, each node in a layer is connected with all other nodes of that layer. Therefore, we can naturally express the densities of intra-layer and inter-layer active links as a function of m_α as,

$$\rho_\alpha = 2X_\alpha \left(1 - X_\alpha \right) = \frac{1}{2} \left(1 - m_\alpha^2 \right) \tag{2.5}$$

and

$$\rho_\perp = X_I + X_{II} - 2X_I X_{II} = \frac{1}{2} \left(1 - m_I m_{II} \right). \tag{2.6}$$

We derive the Master Equation for the probability $Q(m_\alpha, t)$ that the system has option polarization m_α at time t. If at a given time step δt a node change its state, the option polarization changes by $2/N$. The probabilities of the possible changes in m_α are

$$\begin{aligned}
W(m_\alpha \to m_\alpha + \frac{2}{N}) &= \left[(1-\gamma) S_\alpha \rho_\alpha + \frac{\gamma}{4} (1 - m_\alpha)(1 + m_{\bar{\alpha}}) \right], \\
W(m_\alpha \to m_\alpha - \frac{2}{N}) &= \left[(1-\gamma)(1 - S_\alpha) \rho_\alpha + \frac{\gamma}{4} (1 + m_\alpha)(1 - m_{\bar{\alpha}}) \right], \\
W(m_\alpha \to m_\alpha) &= 1 - (1-\gamma)\rho_\alpha - \gamma \rho_\perp .
\end{aligned} \tag{2.7}$$

Then the probability to have option polarization m_α at time t, $Q(m_\alpha, t+\delta t)$ reads

$$\begin{aligned} Q_\alpha = \ & \textstyle\sum_{k_\alpha} \ W(m_\alpha + \frac{2}{N} \to m_\alpha) Q(m_\alpha + \frac{2}{N}, t) \\ & + \ W(m_\alpha - \frac{2}{N} \to m_\alpha) Q(m_\alpha - \frac{2}{N}, t) \\ & + \ W(m_\alpha \to m_\alpha) Q(m_\alpha, t)\,. \end{aligned} \tag{2.8}$$

where Q_α stands for $Q(m_\alpha, t+\delta t)$.

Substituting the transition probability and considering that $\delta t = 1/2N$, we find the diffusion Fokker-Plank equation

$$\begin{aligned} \partial_t Q_\alpha = \partial_{m_\alpha} \Big\{ & \Big[\ \frac{1}{2}(1-\gamma)(1-2S_\alpha)(1-m_\alpha^2) \\ & + \ \frac{\gamma}{2}(m_\alpha - m_{\bar{\alpha}}) \Big] Q_\alpha \Big\} \\ & + \ \frac{1}{N}\partial^2_{m_\alpha} \Big\{ \Big[\frac{1}{2}(1-\gamma)(1-m_\alpha^2) \\ & + \ \frac{\gamma}{2}(1 - m_\alpha m_{\bar{\alpha}}) \Big] Q_\alpha \Big\}\,. \end{aligned} \tag{2.9}$$

We can rewrite the Fokker-Plank equation in the diffusive form

$$\partial_t Q_\alpha = -\partial_{m_\alpha} \Big[\partial_{m_\alpha} V Q_\alpha \Big] + \frac{1}{2N} \partial^2_{m_\alpha} \Big[D_\alpha Q_\alpha \Big]\,, \tag{2.10}$$

where

$$\begin{aligned} \partial_{m_\alpha} V &= (1-\gamma)(2S_\alpha - 1)\rho_\alpha + \gamma(m_{\bar{\alpha}} - m_\alpha)\,, \\ D_\alpha &= (1-\gamma)\rho_\alpha + \gamma \rho_\perp\,. \end{aligned} \tag{2.11}$$

We notice that the influence of each layer on the other appears not only in the potential but also as an additive term in the diffusion coefficient. We can say that the term $\rho_\perp$ controls the diffusion of the two options in the two layers. The two potentials felt by the two layers have an opposite minimum because of the setting in the prestiges; for $\gamma = 0$ each layer would reach the full consensus in opposite options.

In the thermodynamic limit the diffusive term is canceled and the potential $V(m_I, m_{II})$ defined in (2.11) has three extrema:

1. Two corresponding to the states of full consensus:

   a) $(m_I, m_{II}) = (1, 1)$ (consensus to A)

   b) $(m_I, m_{II}) = (-1, -1)$ (consensus to B)

2. One that stands for the non-consensus steady state where options A and B coexist and given by,

$$\begin{aligned} m_I^* &= \frac{-2 + \sqrt{\frac{a}{b}}\sqrt{ab-4}}{a}, \\ m_{II}^* &= \frac{+2 - \sqrt{\frac{b}{a}}\sqrt{ab-4}}{b}, \end{aligned} \tag{2.12}$$

where $a = (1-\gamma)(2S_I - 1)/\gamma$ and $b = (1-\gamma)(2S_{II} - 1)/\gamma$. The stability of the fixed points is analyzed by imposing $\partial_{m_\alpha} V = 0$ and by studying the eigenvalues of the corresponding Jacobian matrix. We find that in the range of parameters defined by the relation:

$$\frac{a}{a-1} \leq b \leq -\frac{a}{a+1}, \tag{2.13}$$

the steady state of coexistence, given by eqs. (2.12), is linearly stable while the state of full consensus is unstable. Out of this region, instead, the state of coexistence vanishes and the states of full consensus become stable; for $b > -\frac{a}{a+1}$ the system consents to the option A, while for $b < \frac{a}{a-1}$ it consents to B. By substituting $b = -\frac{a}{a+1}$ in eq. (2.12) we have $m_I = m_{II} = 1$, while by substituting $b = \frac{a}{a-1}$, $m_I = m_{II} = -1$. This means that out of the region expressed by eq. (2.13), the two solutions of full consensus and coexistence coincide. The coexistence solution varies continuously from -1 to 1 generating a second order absorbing phase transition. The steady states of full consensus are absorbing, frozen states and the switch probabilities vanish. The steady state of coexistence, instead, is an active dynamical state (c.f. [109]), where individuals continue switching and the system visits a set of configurations which are macroscopically equivalent in terms of ordering. Therefore, by varying parameters according to eq. (2.13) we find an absorbing transition in which the system goes from an active state to a frozen configuration state.

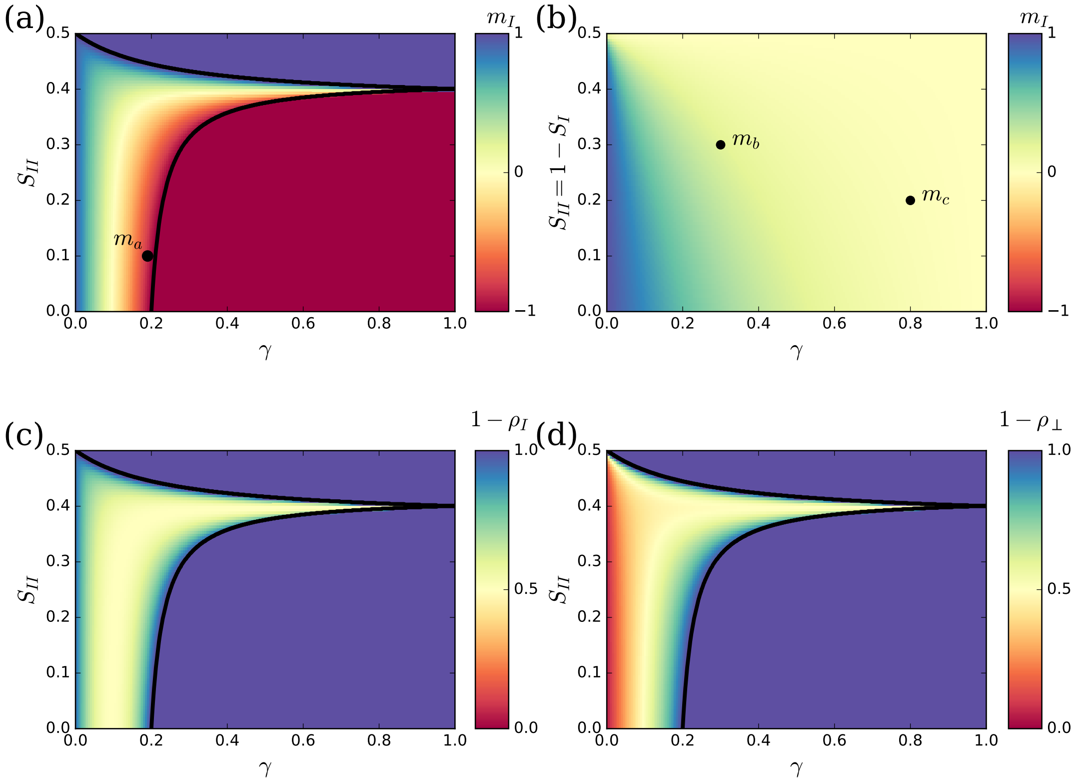

Figure 2.2: Density plots of the MF stable solution in the parameter space $S_{II} - \gamma$. The option polarization m_I of layer I is shown for a) $S_I = 0.6$ and b) $S_I = 1 - S_{II}$. c) shows the fraction of nodes in layer I that are in the same state, $1 - \rho_I$; $S_I = 0.6$. d) show the fraction of nodes that are in the same state in both layers, $1 - \rho_\perp$; $S_I = 0.6$. Black curves enclose the parameter area where two options coexist and correspond to inequalities (2.13), when crossing black lines, inside out, the system undergoes an absorbing transition. The point m_a refers to Fig. 2.4, while the points m_b and m_c to Fig. 2.3.

The complementary of the density of active links, $1 - \rho_\alpha$, gives what is called the *coherent domain* [110], namely the density of links between nodes in the same state. Figure 2.2 shows the landscape of steady states of system (2.12) in the parameter space $\gamma - S^{II}$ for fixed S^I. For $\gamma = 0$ the two layers do not communicate and the multiplex is reduced to two independent AS systems (see (2.1)). Each layer reaches a steady state of consensus, however, the states are complementary, namely one layer consents to the option A while the other layer to the option B. For $\gamma = 1$ both a and b are equal to zero and each solution of the form $m_I = m_{II}$ is a potential stable solution. The two layers communicate with the stronger possible coupling but the individuals are not affected by their neighbors (see (2.1)). A randomly chosen node replicates its state from the other layer, resulting in a frozen steady state where every node has the same state across the layers, while none of the layers reaches consensus. For other values of γ the previously found condition (2.13) determines three different dynamical regimes displayed in figures 2.2 (for $0 \leq \gamma < 1$). Figure 2.2(a) shows the option polarization m_I of layer I. For $b > -\frac{a}{a+1}$ the system consents to the option A (the violet area) while the option B dominates for $b < \frac{a}{a-1}$ (the red area). For the other parameter values the two options coexist (area enclosed by the black curves). The resulting density of the connected nodes lying in the same states is presented in figure 2.2(c), while the density of coherent nodes between the layers, namely nodes in the same state in both layers is depicted in figure 2.2(d).

Using the terminology of language, the densities $\rho_\perp$ measures the density of bilingual individuals. The novelty of this model compared to single-layer models of language competition is that the presence of bilinguals naturally comes out from the dynamic assumptions.

We verify the results obtained from our MF approximation by constructing multiplex networks of different sizes composed by complete networks in their layers. In the coexistence regime the options are distributed between the nodes in two different ways presented in figure 2.3. In figure 2.3(a) the parameter values allows the existence of both, nodes in different states in different layers and nodes with the same state across layers. The densities ρ_α and $\rho_\perp$ of active links are different from zero. In figure 2.3(b),

γ is strong enough to drive the system in a steady state where the nodes have the same state in both layers. However, small fluctuation from this steady state are observed. The intra-layer densities $\rho_{I,\,II}$ of active links are different from zero, while the inter-layer density $\rho_{\perp}$ fluctuates with very small amplitude around zero. If states represent the languages spoken, their different distribution among individuals can be interpreted as the different kinds of bilingual society described by Appel and Muysken [96]. In fact we find a scenario where most of the people is monolingual and a scenario where most of the people is bilingual. This interpretation will be deepened when we will introduce correlations between the layers.

2.4 Finite size effects in multiplex networks with complete layers

As mentioned above, the coexistence state is an active state, namely the probabilities to switch the state do not vanish and the system keeps fluctuating around the fixed point. In finite size networks, however, these fluctuation can drive the system to an absorbing state of full consensus. This is a finite size effect and has been observed in our stochastic simulations. In our settings the two layers have opposite prestiges, therefore for $\gamma = 0$ the option polarization m_I would go to 1 while m_{II} would go to -1.

Figure 2.4 shows the evolution of m_I for different realizations in the case of complete networks of (a) $N = 500$ nodes and (b) $N = 1000$ nodes. In this particular setting of parameters ($\gamma = 0.2$, $S_I = 0.6$ and $S_{II} = 0.1$) the MF stable solution (the solid horizontal line) denotes a coexistence steady state but the fluctuations due to the finite size effect brings it to a full consensus in option B, with both $m_\alpha = -1$. The time the system remains around the MF solution depends on the size of the system and on the values of the parameters. A different case is the unbiased model where both the prestiges are equal, $S_\alpha = 1/2$. The potential reduces to

$$V_{\frac{1}{2}} = \frac{1}{2}\gamma(m_I - m_{II})^2 \tag{2.14}$$

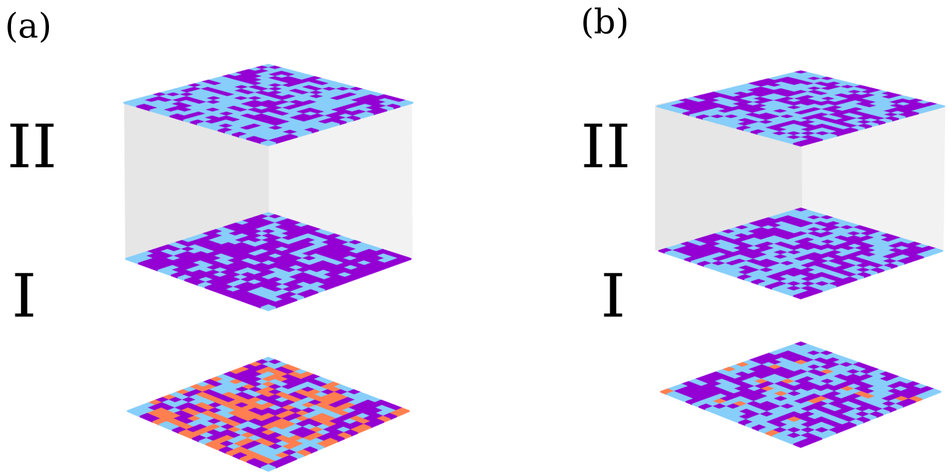

Figure 2.3: Two different scenarios of coexistence are shown in a multilayer representation for single realization at the time $t/N = 10^4$ with complete graph of $N = 1000$ nodes. The tree levels refer to : Layer II on the top, layer I in the middle and on the bottom the aggregate layer which counts the nodes in the same state in both layers and the nodes with different states. The colors violet and blue stand for the option A and B, respectively, and the orange indicates nodes with different options in different layers. In a) the parameters are $\gamma = 0.3$, $S_I = 0.7$, $S_{II} = 0.3$. In b) the parameters are $\gamma = 0.8$, $S_I = 0.8$ and $S_{II} = 0.2$. These two scenarios refer, respectively, to the points m_b and m_c of Fig. 2.2.

and each solution of the form $m_I = m_{II}$ is a solution. In this particular setting we can study the effects purely induced from the multiplex structure because we can avoid the effect generated by the competition between the two options in the two layers. From the related theory for single-layer networks [54] we know that in this case the system will reach an absorbing state with a characteristic time that scales with N. In the multiplex

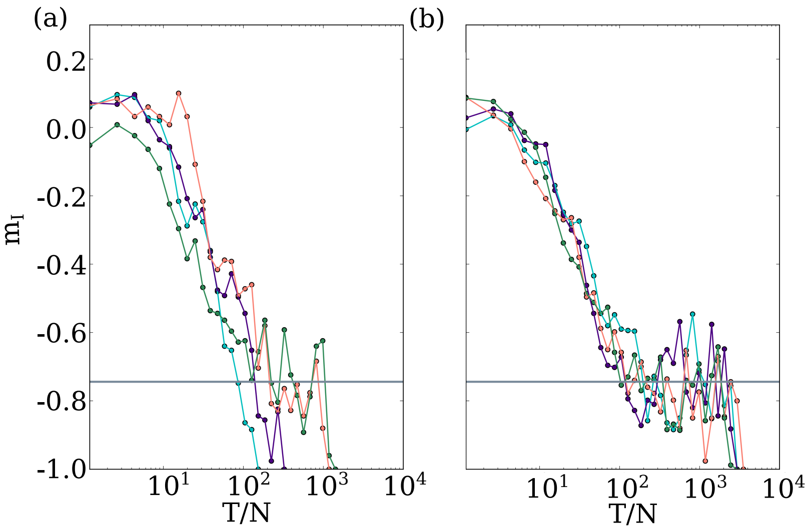

Figure 2.4: Evolution of the option polarization m_I is shown for different realizations in complete graphs of (a) $N = 500$ nodes and (b) $N = 1000$ nodes. The solid horizontal line represent the MF solution corresponding to the point m_a of the Fig. 2.2 .The other parameters are $\gamma = 0.2$, $S_I = 0.6$ and $S_{II} = 0.1$.

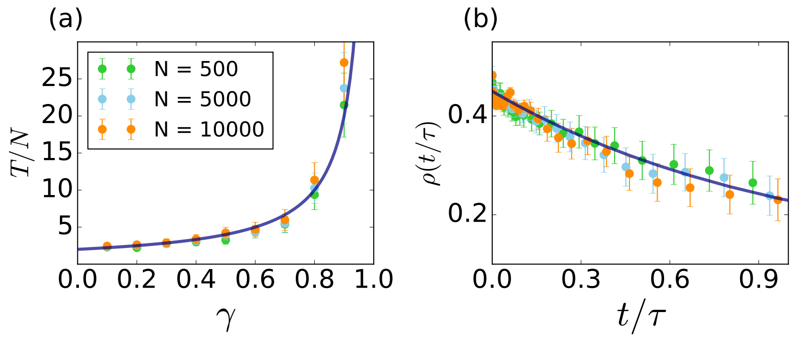

Figure 2.5: (a) The characteristic time to reach the full consensus is shown as a function of γ. The solid line represents the relation of (2.15). (b) Exponential decay is shown as a function of rescaled time τ (solid line $\rho_{\perp}(t/\tau)$). In both figures the points represent an average over 50 realizations for different complete graphs of $N = 500$, 5000 and 10000 nodes. The prestiges are $S_{\alpha} = 0.5$.

networks considered here we find by fitting (see Fig. 2.5) a factor arising from the inter-layer interaction, $c(\gamma) = \frac{1}{1-\gamma}$, for which the characteristic time to reach consensus takes the form

$$\tau = 2N\frac{1}{1-\gamma} = 2Nc(\gamma)\,. \tag{2.15}$$

This relation is consistent with the studies presented in [54] where the scaling factor is the inverse of the prefactor of the active links in the diffusion coefficient. The factor $2N$ results from the total number of nodes in the whole two layers system.

From the time evolution expressed in [54], we can approximate $\rho_{\perp}(t) \approx e^{-t/\tau}$. Figure 2.5(a) shows the characteristic time (2.15) for different size and different γ, using complete networks. Figure 2.5(b) shows the evolution of the density $\rho_{\perp}$ of active inter-layer links. We can thus conclude that in the case of equal prestiges the multiplex effect translates into an extension $c(\gamma)$ in the life time of the coexistence options state.

2.5 Erdös-Rényi Networks

Here we extend the ansatz of [54] by considering Erdös-Rényi networks. In [54] it was found for the density of the active links that the relation $\rho = \frac{1}{2}\psi\left(1 - m^2\right)$ is valid for a complex network of mean degree $\langle k_\alpha \rangle$, where $\psi = \frac{\langle k_\alpha \rangle - 2}{\langle k_\alpha \rangle - 1}$. We test this assumption for multiplex networks consisting of different Erdös-Rényi layers of various size and mean degree discovering that also in our case it is valid (see Fig. 2.6).

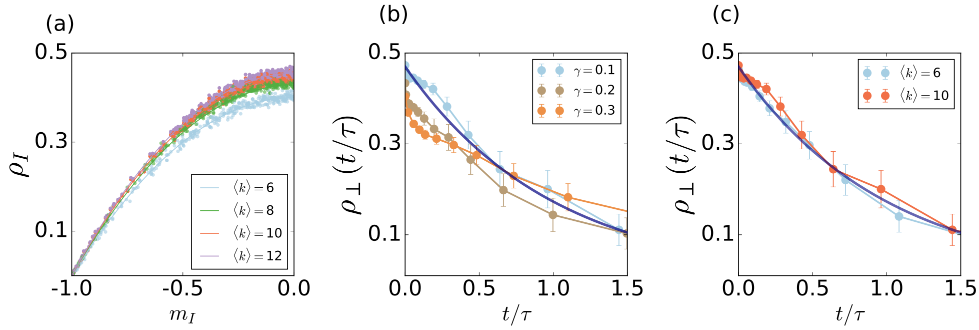

Figure 2.6: (a) The density of active links in layer I is shown as a function of the option polarization. Solid lines correspond to (2.16), while the dots are the average over 50 simulations for two Erdös-Rényi networks of $N = 10000$ and different mean degrees. (b) Shows the time evolution of the average density of inter layers active links for fixed mean degree, $\langle k_\alpha \rangle = 10$ and different γ. (c) shows the time evolution of the average density of inter layers active links for fixed $\gamma = 0.1$ and different mean degree. The time is rescaled by the factor expressed in (2.19) and the solid line correspond to an exponential decay. The dots represent the average of 50 realizations for Erdös-Rényi networks of $N = 10000$.

Then, the density of intra-layer active links reads

$$\rho_\alpha = \frac{1}{2}\psi_\alpha\left(1 - m_\alpha^2\right). \tag{2.16}$$

The inter-layer density, instead, does not depend on the topology of the network. In order to extend the Fokker-Planck (2.10), for the case of two Erdös-Rényi networks we consider that if at a time step a node with k_α changes its state, the option polarization changes by $\pm\delta_{k,\alpha}$, with $\delta_{k,\alpha} = \frac{2k_\alpha}{\langle k_\alpha \rangle N}$. Since each node has inter-layer degree equal to one, the inter-layer change remains $\frac{2}{N}$. Substituting the relation 2.16 in the transition probabilities expressed in (2.7) and the right expression for $\pm\delta_{k,\alpha}$, we obtain for the Fokker-Plank equation $\partial_t Q_\alpha = -\partial_{m_\alpha}\Bigg[\partial_{m_\alpha} V Q_\alpha\Bigg] + \frac{1}{2N}\partial^2_{m_\alpha}\Bigg[D_\alpha Q_\alpha\Bigg]$, the following terms

$$\begin{aligned}
\partial_{m_\alpha} V &= (1-\gamma)(2S-1)\rho_\alpha + \gamma(m_{\bar{\alpha}} - m_\alpha)\,, \\
D_\alpha &= \frac{\langle k^2_\alpha \rangle}{\langle k_\alpha \rangle^2}(1-\gamma)\rho_\alpha + \gamma\rho_\perp\,.
\end{aligned} \tag{2.17}$$

Previous studies of the voter model (prestige equal to $1/2$) on complex networks [54], have revealed the relation between the characteristic time to reach the full consensus and the topology of the network. Previously [54] it was found for an Erdös-Rényi network with mean degree $\langle k \rangle$ a scaling factor of the form

$$T_{ER} = \frac{\langle k \rangle \Big(\langle k \rangle - 1\Big)}{\Big(\langle k \rangle + 1\Big)\Big(\langle k \rangle - 2\Big)} N\,. \tag{2.18}$$

By imposing $\gamma = 0$, the diffusion coefficient D_α of eq. (2.17) reduce to the one layer case $D = \frac{\langle k^2 \rangle}{\langle k \rangle^2}\rho_\alpha = \frac{\langle k^2 \rangle}{\langle k \rangle^2}\psi\Bigg(1-\rho^2\Bigg)$. Notice that the expression of T_{ER} is the inverse of the prefactor of the density of active links term. In our multiplex extension, by setting $S_a = \frac{1}{2}$ we check the same relation for the scaling factor by considering that the prefactor of the active links term results to be the one layer term multiplied by $1-\gamma$,

$$\tau = 2c(\gamma)T_{ER}\,. \tag{2.19}$$

As in the case of eq. (2.15), the factor 2 accounts for the total number of nodes $2N$. In figures 2.6(a) and (b) we observe the time evolution of the average density $\rho_\perp$ of inter-layer active links for fixed mean degree $\langle k_\alpha \rangle$ and different values of γ, showing the validity of the assumption expressed by eq. (2.19).

2.6 Impact of correlation

With a fully connected population we have shown how the coupling of two layers with two different preferred options can generate various kind of options coexistence. Here we are interested on how the topology of the two networks and correlations between layers can influence the distribution of the states among the nodes. Previously it was shown that the relation between the layers in real multiplex can be characterized by geometric correlations in hidden metric spaces underlying each layer of the system [84, 85, 86]. There are two kind of correlations: *Popularity correlations*, which are correlations between the degrees the nodes have in the two layers, and *similarity correlations*, which control the probability of links overlap between layers.

To understand the impact of correlations, we perform numerical simulation using the geometric multiplex model (GMM) developed in [84] (See Appendix A.1). We compare GMM with different correlations setting and GMM with ER networks. In all the cases the multiplex networks are composed by layers of $N = 2000$ nodes and mean degree $\langle k \rangle = 6$. The most significant effect is observed in the distribution of the states among the nodes. Similarity correlation, increasing the probability of link overlap between layers, promotes inter-layer group of nodes in the same state and connected in both layers, namely coherent islands. If the whole system has a favourite state ($S_I \neq 1 - S_{II}$), finite size effects bring the system to the absorbing state of full consensus for which the coherent islands of that state increase at the expense of the other.

To appreciate the correlation effects we consider the case of symmetric prestige. Figure 2.7 shows the evolution of the option polarization m_{II} (top row) and the inter-layer active links $\rho_\perp$ (bottom row) for $S_I = 1 - S_{II} = 0.55$ and $\gamma = 0.3$. Figure 2.7(a) and (d) refer to GMM with uncorrelated layers, Fig. 2.7(b) and (e) to GMM with fully correlated layers while Fig. 2.7(c) and (f) to ER networks. The behaviour of $\rho_\perp$ is significantly different for the correlated and uncorrelated case. For ER and uncorrelated GMM, $\rho_\perp$ fluctuates around the MF solution. Instead, in the strong correlated system, the size of the coherent islands grows generating a slower decay of $\rho_\perp$. The top row of Fig. 2.7 shows that none of the three systems has reached the absorbing state for which the basic difference lies in the distribution of the states between nodes. We can conclude that the strongly correlated system is in a state of coexistence of different coherent islands. This feature becomes more evident for high values of the coupling as showed in Fig. 2.8 where $\gamma = 0.8$. In that case, for some realizations, the system reaches the full consensus in two different ways: In the uncorrelated case thanks to a single fluctuation due to finite size effects, while in the full correlated case one of the coherent island grows and incorporates the entire system.

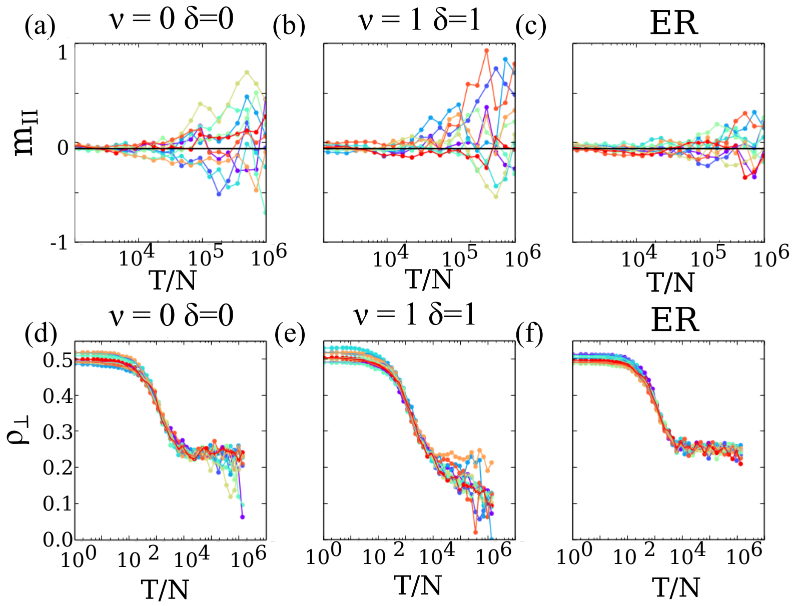

Figure 2.7: Option polarization and inter layer active links. We compare ER networks ((c) and (f)) with GMM ((a), (b) and (d), (e)) of $N = 2000$ nodes and $\langle k \rangle = 6$ mean degree. The power law degree distribution of GMM has an exponent of 2.9. (a) and (d) show uncorrelated networks whereas (b) and (e) show fully correlated networks. The parameters of the model are $S_I = 1 - S_{II} = 0.55$ and $\gamma = 0.3$. The top row show the evolution of option polarization in layer II while the bottom row the evolution of inter layer active links. The different colors stand for different realization and the solid black line denotes the MF solution.

An other important measure reveals how the similarity correlation acts on the distribution of the states between nodes. In Fig. 2.9 we set $S_\alpha = 0.5$, for which neither the layers nor the whole system has a favored state (the coupling is $\gamma = 0.3$). In Fig. 2.9(a) the system is uncorrelated (i.e. $\delta = 0$ and $\nu = 0$), in Fig. 2.9(b) $\delta = 1$ and $\nu = 0$, in Fig. 2.9(c) $\delta = 0$ and $\nu = 1$ and in Fig. 2.9(d) $\delta = 1$ and $\nu = 1$. C_B defines the coherent island in the option B, namely the density of links overlaps between nodes in the state B in both layers. We notice that C_B increases considerably when we set the similarity correlation ν equal to one. By comparing the figures 2.9(a) with Fig. 2.9(b) and (c) we notice that the action of the popularity correlation alone does not produce significant effects while the similarity correlation increases the coherent islands.

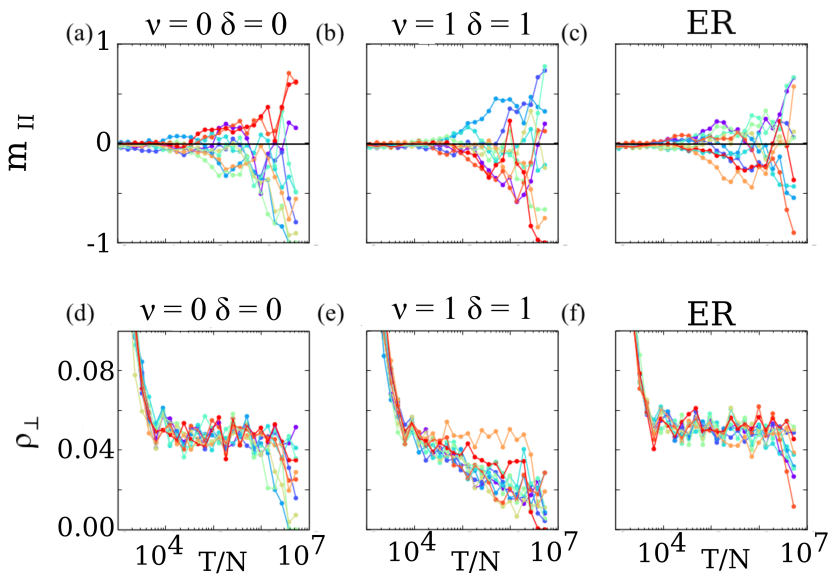

Figure 2.8: Option polarization and inter layer active links. We compare ER networks ((c) and (f)) with GMM ((a), (b) and (d), (e)) of $N = 2000$ nodes and $\langle k \rangle = 6$ mean degree.The power low degree distribution of GMM has an exponent of 2.9. (a) and (d) uncorrelated and (b) and (e) fully correlated networks are shown. The parameters of the model are $S_I = 1 - S_{II} = 0.55$ and $\gamma = 0.8$. The top row show the evolution of option polarization in layer II while the bottom row the evolution of inter layer active links. The different colors stand for different single realization and the solid black line for the MF solution.

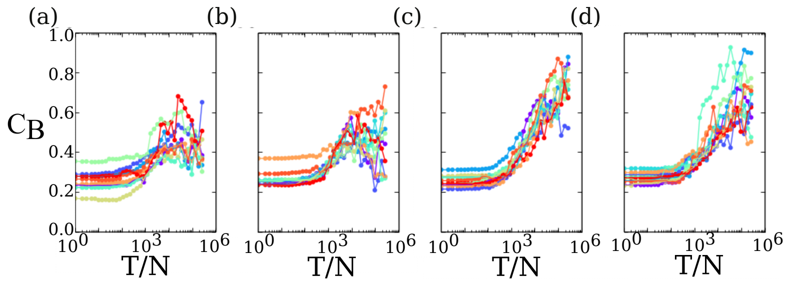

Figure 2.9: Density of links between coherent nodes in the state B, C_B. (a) Uncorrelated, (b) popularity correlation, (c) similarity correlation and (d) fully correlated connections are shown. The parameter of the system are $S_I = S_{II} = 0.5$ and $\gamma = 0.3$. The networks have $N = 2000$ nodes and mean degree $\langle k \rangle = 6$. The power law degree distribution has an exponent of 2.9. Different colors refer to different realizations.

Recalling the concept of language coexistence we can interpret the predominance of coherent islands as a society made up of several monolingual groups. This scenario is thus favored by a high similarity correlation between layers, even if the two languages have the same prestige in both layers. In terms of opinions, instead, it can be interpreted as the phenomena of the polarization of society, occurring when the population is clearly divided into two opposing views [111]. Instead, an independent distribution of links in the two layers generates a more heterogeneous situation.

2.7 Concluding Remarks

This Chapter presents a model of consensus and coexistence of two opposing options in a discrete system organized on multiplex networks.

Though similar models [49, 88] have already been performed in multiplex networks, the main novelty of our study lies to the fact that individuals can have different options in the distinct layers. This naturally reflects that an individual can consent with its connections in a given social context but have different opinion in an other context. Since the two options can represent both opinions and language, it can also represent bilingual people speaking different language in distinct situations.

Our analysis shows that the latter property enriches the system dynamics and allow not only for consensus but also for simultaneous existence of both options. This can be described by two layers having opposite preferred options that generate potentials with opposite minimum (there is no state that satisfy both layers).

Generally the ordering processes are characterized by the *coarsening*: the formation and growth of a coherent domain in one of the two states. In the classical AS model, this coarsening is responsible for the achievement of full consensus in finite systems. In the multiplex system discussed here, indeed, the coarsening of each layers to its preferred option is countered by the presence of the other. Each layer "feels" the other as an additive noise, so that, even if individuals instantly consent in one layer, they preserve the chance to switch due to the influence of the other.

In MF approximation we found a whole range of parameters where the coexistence of the two options is the stable state and the finite size effect is to reduce this region. However, the fluctuations induced by the finite size fluctuations can drive the system from the active dynamical state of coexistence to an absorbing state of consensus.

In particular we have considered the case of equally prestigious options, as in the voter model. The voter dynamics always reaches the absorbing state of full consensus. The ordering process in this case is not due to the coarsening, the system remain disordered until a random fluctuation drive it to consensus. For a single decoupled layer the characteristic time to reach the absorbing state is proportional to the size of the system [53]. In our case, indeed, because we have an additive noise induced by the mutual influence between the layers, this characteristic time depends also on the coupling parameter γ. Therefore, in the presence of finite size fluctuations, the multiplex structure of our system can affect and lengthen the life time of the transient state of dynamical coexistence.

In the case of symmetrical but not equal prestige, however, the system may remain trapped in the coexistence regime even for finite sizes, as as shown by Fig. 2.7.

Mean field results are verified by numerical simulations in multiplex networks consisting of complete graphs, Erdös-Rényi networks and Geometrical Multiplex networks. For the Erdös-Rényi networks we find the same qualitative findings, but local effects modify transition lines for the absorbing transition and lifetimes of active states depending on the degree distribution of the network. With the Geometrical Multiplex networks we examined both the impact of networks' topology and correlations between layers on the dynamics. We find that high correlations between layers promote the coexistence of different inter-layer islands of nodes in the same state for small value of the coupling, while high values of the coupling facilitate the achievement of a full consensus state.

CHAPTER 3

Mixed dynamics on multiplex

3.1 Introduction

In this work we analyze the interplay between social influence and competitive strategical games on multiplex networks where social influence and game dynamics take place in different domains. Several theoretical and experiment studies [22, 23, 24] have been proposed to understand the origin and prevalence of cooperation in systems where individual interests are in conflict. In numerous experiments with human subjects interacting through competitive games within fixed neighborhoods [23, 112, 113], e.g. the classical Prisoner's Dilemma [114] or the Public Goods Game [23, 115], it has been consistently reported that the fraction of cooperators decays in time. This stands in contrast to observations in everyday life where, indeed, we find cooperation to be quite stable and common.

Modification to the standards models have therefore been proposed to explain the emergence of cooperation in these scenarios, like for example direct and indirect reciprocity (image scoring/reputation) [116], kin and group selection [117, 118], success-driven migration [119], or punishment [120].

Another mechanism responsible for the emergence of cooperation in social dilemmas could be the fact that strategic interactions between individuals or institutions do not occur in isolation. In particular, individuals that engage in strategic interactions are simultaneously exposed to social influence and, consequently, the spread of opinions.

Following this line of reasoning, we assume that social influence impacts the decisions of the players [121, 122], and vice versa, that the decisions of the players impact the opinions that are propagated in the system.

We present a model where game theoretical decisions and social influence take place in different layers of a multiplex network [80, 90, 123, 124, 125, 126, 127]. In such systems, each layer contains the same set of nodes, but links are usually different in different layers. However, the layers comprising real multiplex systems are not entirely independent, but exhibit certain relations [84]. As we will show, these relations can lead to interesting behaviors, and hence have to be taken into account when modeling such systems [128]. On top of this topology, we model the dynamics taking place in the game layer by a replication dynamics, where individuals imitate the strategy of successful neighbors [129, 130, 131]. Furthermore, we use a biased voter model [132] in the social influence layer as a proxy for the spread of opinions. These opinions can be seen as a proclamation of the intend of individuals regarding their choice of strategy in the game layer. We assume that there is a tendency for individuals to act in agreement with their proclamations, but allow, in general, that individuals deviate from them. The importance for individuals to be congruent in both domains constitutes the coupling strength between the different dynamical processes. Finally, the aforementioned bias of the voter model represents a general tendency towards the proclamation of cooperative intentions, which could be induced by appropriate media campaigns or similar measures.

3.2 Coupling between game dynamics and social influence

In strategical games, individuals choose a strategy and then obtain a payoff that depends on their own and other players' strategies. Here, we consider the two possible strategies

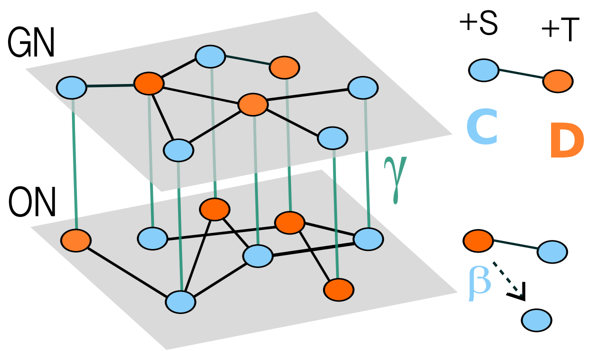

Figure 3.1: Graphical illustration of the multiplex model. Different layers denote the different networks that individuals participate: on the top the Game Network (GN) and on the bottom the Opinion Network (ON). Intra-links (black solid lines) correspond to the individuals' connections within each network, while inter-links (green solid lines) indicate the coupling between layers. On the right, the pictures show a simplification of the dynamic occurring on each network.

cooperate (C) or defect (D). The interactions are then governed by the payoff matrix

$$M = \begin{array}{c|cc} & \text{C} & \text{D} \\ \hline \text{C} & 1 & S \\ \text{D} & T & 0 \end{array} . \tag{3.1}$$

meaning that if player 1 chooses to cooperate and player 2 cooperates as well, both collect a payoff of 1. However, if player 2 defects, player 1 only collects the payoff S and player 2 collects the payoff T, and vice versa. Finally, if both players defect, both obtain no payoff.

In reality, individuals make many successive strategic decisions and adapt their strategies over time. This behavior is commonly modeled by a replicator dynamics [129, 130, 131], in which individuals copy the strategy of a randomly selected neighbor with a probability that depends on the difference of the payoff of the two involved players. In general, individuals tend to copy the strategy of players who have earned a higher payoff compared to themselves. Here, we use synchronized updates, meaning that after each round of the game, in which each node plays one game with each of her neighbors and payoffs are distributed according to the aforementioned payoff matrix, every node i chooses a neighbor j at random and copies her strategy with the probability [133]

$$P_{i \leftarrow j} = \frac{1}{2} \left(1 - \tanh \left[\pi_i - \pi_j \right] \right) \ . \tag{3.2}$$

where where π_i and π_j denote the payoffs of node i and j.

Depending on the values of $S \in [-1, 1]$ and $T \in [0, 2]$, there are different stable choices of strategies. In the Stag Hunt game, for which we have $S < 0$ and $T < 1$, we have bistability: both full cooperation as well as full defection are stable stationary solutions. In the Prisoner's Dilemma, i.e. for $S < 0$ and $T > 1$, full defection is the only stable strategy. In the Snowdrift game, $S > 0$ and $T > 1$, the only stable solution is an intermediate density of cooperators. Finally, in the Harmony game, $S > 0$ and $T < 1$, only full cooperation is a stable solution.

However, in reality, individuals do not make decisions exclusively based on the payoffs of their neighbors. Instead, individuals are simultaneously exposed to social influence and hence the opinion of the peer group of an individual cannot be neglected in understanding what drives cooperation in strategic games. Opinions of individuals propagate through a contact network. This behavior is widely described by the voter modelc [132], where individuals adopt the opinion of a randomly selected neighbor. We assume that propagating an opinion that is considered anti-social, like defection in our model, is less likely than propagating opinions which are socially accepted, like cooperation in our model. This could be the result of media campaigns or similar measures. We take this effect into account by introducing a bias, $\beta \in [0, 1]$, into the voter model. Individuals then adopt the opinion of a randomly selected neighbor with probability β if this opinion is cooperation and $1 - \beta$ if it is defection. Values of $\beta > 0.5$ hence reflect a positive bias towards cooperation in the opinion dynamics. The opinion of an individual can be understood as her proclamation of intend regarding her choice of strategy in the game layer. As mentioned before, social influence has an impact on the decision of individuals. To mimic this fact, we couple the opinion propagation and game dynamics. In particular, we define a parameter γ, which represents the tendency of individuals to act in agreement with their proclamations, and hence constitutes the coupling strength between social influence and game dynamics. In particular, at each update step, with probability γ a node copies her state from one layer to the other. The copying process can be understood as the tendency of individuals to pursue congruence between their actions and proclaimed opinions. Finally, with the complementary probability $1-\gamma$ each node updates the strategy in the game layer according to the game dynamics and in the opinion layer according to the biased voter model (see Fig. 3.1).

3.3 Mixed populations

In reality, individuals or institutions interact in strategic games via a contact network, like a network between firms or countries. We will discuss the influence of the structure of the contact network and of the correlations between different networks later. For now, we study the model on a mixed population, in other words, we assume a homogeneous and infinite population in the absence of dynamical correlations and noise.

The mixed population (meanfield) assumption allows us to derive differential equations for the evolution of the density of cooperators c_I in the game layer and c_{II} in the opinion layer. For the N nodes of the system, let us assign a state σ_i^α to each node i ($i = 1, 2, \ldots, N$) in the layer α ($\alpha = I, II$) such that $\sigma_i^\alpha = 1(0)$ if the node has the strategy, respectively opinion, $C(D)$ in the given layer α. In the GN, the probability for a node i to copy the strategy of one of her random selected neighbor j is given by eq. (3.2).

Each player's payoff can be calculated from the payoff matrix given by equation 3.1. In particular, each player plays $\langle k\rangle$ games with randomly chosen opponents, where $\langle k\rangle$ is the mean degree in layer I. The average payoff for cooperators, c_I, and defectors, d_I, is then given by

$$\begin{aligned}\pi_{c_I} &= \langle k\rangle\, c_I + \langle k\rangle\, S\,(1-c_I)\\ \pi_{d_I} &= \langle k\rangle\, T c_I\end{aligned} \tag{3.3}$$

The transition probabilities corresponding to the game dynamics are expressed by

$$\begin{aligned} w_I(D \to C) &= \frac{1}{2} c_I \left(1 - \tanh\left[\pi_{d_I} - \pi_{c_I}\right]\right) \\ w_I(C \to D) &= \frac{1}{2} \left(1 - c_I\right) \left(1 - \tanh\left[\pi_{c_I} - \pi_{d_I}\right]\right) , \end{aligned} \tag{3.4}$$

for which the term regarding the game dynamics ($\gamma = 0$) in the evolution of cooperators of layer I is

$$\begin{aligned} \partial_t c_I &= (1 - c_I)\, w(D \to C) - c_I w(C \to D) \\ &= c_I \left(1 - c_I\right) \tanh\left[\langle k \rangle \left(c_I(1 - T) + S(1 - c_I)\right)\right] , \end{aligned} \tag{3.5}$$

The dynamics in the opinion layer, II, is described as a biased voter model for the diffusion of opinions with a bias $\beta \in (0, 1)$ towards the "cooperate" opinion. A node i will then adopt the state of one of its randomly selected neighbors j with probability β if $\sigma_j^{II} = 1$ and $1 - \beta$ if $\sigma_j^{II} = 0$. The voter model dynamic is given by

$$\partial_t c_{II} = (2\beta - 1) c_{II} (1 - c_{II}) \,. \tag{3.6}$$

The interaction terms are given by $(c_{II} - c_I)$ for layer I and $(c_I - c_{II})$ for layer II.

Combining the equations through a linear combination in terms of γ, yields

$$\begin{aligned} \partial_t c_I =& (1 - \gamma) c_I (1 - c_I) \tanh\left[\langle k \rangle \left(c_I(1 - T) + S(1 - c_I)\right)\right] + \gamma(c_{II} - c_I) \,, \\ \partial_t c_{II} =& (1 - \gamma)(2\beta - 1) c_{II}(1 - c_{II}) + \gamma(c_I - c_{II}) \,, \end{aligned} \tag{3.7}$$

where S and T denote the parameters from the payoff matrix, equation (3.1), $\gamma \in [0, 1]$ controls the strength of the coupling between the opinion and game dynamics and $\beta \in [0, 1]$ is the bias of the opinion dynamics. Finally, $\langle k \rangle$ denotes the mean degree of the contact network.

The parameter γ controls the strength of the coupling. For $\gamma = 0$, the dynamics in the different layers are independent, and for $\gamma = 1$ there are only copying events between the layers such that the individual layer dynamics become inexistent. Therefore, the most interesting behavior is observed for intermediate values of γ. The parameter

β controls the bias in the voter model. For $\beta = 0.5$, one recovers the classical voter model, which has no stable fixpoint, but only two absorbent states (full consensus on opinion 1 or 0). In the meanfield approximation, in this case the behavior of the coupled system ($\gamma > 0$) is equivalent to the isolated game dynamics, and the game dynamics enslaves the opinion layer. This means that in the final state of the system both layers have the same cooperation density which is equivalent to the isolated game dynamics.

The fundamental role of β and γ is to change the value of T and S at which one of the two absorbing state $C = 1, 0$ became a stable solution. For $\gamma = 0$, $C = 1$ is stable for $T < 1$ and $C = 0$ for $S < 0$. If $\gamma > 0$ the stability range becomes:

$$\begin{aligned} S &< \frac{1}{2}\log\left[\frac{2\beta - 1 - \gamma}{(2\beta - 1)(1 - 2\gamma) - \gamma}\right] \\ T &< 1 - \frac{1}{2}\log\left[\frac{(2\beta - 1)(1 - 2\gamma) + \gamma}{2\beta - 1 + \gamma}\right] \end{aligned} \tag{3.8}$$

We find that for example, setting $\beta = 0.6$ and $\gamma = 0.2$, the stability of $c_I = 1$ shifts to $T < 1.1$. This means that the value of temptation to be a defector has to be bigger than in the case of decoupled dynamics.

Figure 3.5 shows the asymptotic density of cooperator in the $S - T$ plane for the game layer in mean field approximation and with Erd́os-Rényi networks. We compare the case of $\gamma = 0$ and $\gamma = 0.2$ and $\beta = 0.7$.

We anticipate that while the mean field behavior is confirmed in Erd́os-Rényi networks, complex layer topologies and multiplex organization, in particular geometric correlations intertwining their layers, increase cooperation. For $\beta > 0.5$ ($\beta < 0.5$), the opinion dynamics in isolation has one stable fixpoint, which corresponds to full consensus of opinion 1 (opinion 0). Therefore, the mean field behavior of the system is different to the isolated game dynamics, which we discuss in the following section.

3.4 Dynamical properties of the system

Let us now consider the dynamical properties of the system described by equation(3.7) for fixed values of $\beta = 0.7$ and $\gamma = 0.2$. We find three regions of different qualitative behavior, depending on the values of parameters T and S. In particular, we find a region in which the system effectively behaves like the harmony game (red region in Fig. 3.2a), which means that only full cooperation in both layers is a stable solution (see Fig. 3.2d). Furthermore, we find a region where the system effectively behaves like the snowdrift game (blue region in Fig. 3.2a). In this region, the only stable solution is a mixed state, where a finite fraction of the population cooperates (see Fig. 3.2f). In this region, in general, the density of cooperators in the game dynamics and those who proclaim cooperation are not the same. Finally, there is a region which can be described as a mixture of the two above cases (green region in Fig. 3.2a). In this region, the system exhibits a bistable behavior. Full cooperation in both layers is a stable solution as well as a mixed state as described above (see Fig. 3.2e)). The bistable region emerges as the system undergoes a saddle-node bifurcation. Let us fix $S = -0.2$ and increase the value of T. At $T = T_{c,1} \approx 0.77$ the system undergoes a saddle-node bifurcation as a pair of fixed points, one stable and one unstable, appear (see Fig. 3.2b) and c)). Increasing T further, at $T = T_{c,2} \approx 1.03$ the system undergoes a transcritical bifurcation and the solution which corresponds to full cooperation becomes unstable. In the supercritical regime, only the mixed state is stable. To sum up, we have shown that the coupling to the biased opinion dynamics shifts the effective behavior of the game dynamics compared to the isolated case. The coupled system exhibits effectively a harmony-like behavior, a Snowdrift-like behavior, or a mixture of both. Interestingly, the coupling to the biased opinion dynamics successfully avoids the situation of complete defection. So far, we have considered a fully mixed, homogeneous population. In the following, we discuss the impact of the topology of the underlying contact networks as well as the relationship between the two layers of the system.

3.5 Impact of the structural organization of the multiplex

Using the assumption of a fully mixed, homogeneous population we have shown how the coupling to the biased opinion dynamics can effectively transform the behavior of the system. However, in reality, networks are heterogeneous and highly clustered, which can have a significant effect on the outcome of dynamics taking place on the network [134].

Furthermore, in reality, the social influence layer and the strategic game layer are neither independent nor identical. In other words, real multiplex networks are not random combinations of their constituent layer's topologies [84]. Hence, the contexts—or domains—in which individuals make strategic decisions and by whom they are influenced are related. In [84] the authors have shown that these relations are given by geometric correlations in hidden metric spaces underlying each layer of the system. These correlations come in two flavors: popularity correlations, which are correlations between the degrees of nodes, and similarity correlations, which determine how likely an individual is to connect to the same nodes in different layers. In simple terms, these correlations control how "similar" the different contexts represented by the layers of the system are. For further details on geometric correlations between layers of real multiplex networks we refer the reader to [84]. Here, we focus on the impact of these structural properties on the dynamics of our model. What is, in general, the impact of geometric correlations on the behavior of the system? In particular, do stronger correlations favor or hinder cooperation? To answer these questions, we perform numerical simulation using the geometric multiplex model (GMM) developed in [84] (see Appendix A.1). The model generates networks with a power-law degree distribution and a tunable level of mean local clustering. Furthermore, we can control the popularity correlations (by tuning parameter $\nu \in [0, 1]$) as well as the similarity correlations (by tuning parameter $g \in [0, 1]$) independently from the individual layer topologies, which allows us to study their impact in isolation. We calculate approximated phase diagrams similar to Fig. 3.2a) using the generated networks by performing numerical simulations. In particular, to capture the bistable region of the system, we perform simulations starting from different initial conditions, in particular $C_{I,II} = 0.01$ and $C_{I,II} = 0.99$ respectively. The regions are separated by critical lines, above (below) which the harmony (full defection) state is reached with a probability of more than 50% (dashed black lines in Fig. A.3 in Appendix A.2). The difference between the critical lines for the different initial conditions is an approximation of the bistable region.

Let us first consider the unbiased voter model, hence $\beta = 0.5$, and $\gamma = 0.2$. The system either reaches full cooperation and consensus ("harmony state"), i.e. $C_I^{\text{final}}, C_{II}^{\text{final}} = 1$, a state where a mixed strategy prevails and full consensus is not reached ("snowdrift state"), or full defection ("PD"). Furthermore, by comparing the outcomes from the different initial conditions, we are able to approximate the bistable region ("SH"), where both full cooperation and full defection are possible solutions. Indeed, for Erd́os-Rényi networks the approximated phase diagram, see Fig. 3.3a), closely resembles the mean field prediction for the isolated game dynamics. Whereas it is well known that heterogeneous topologies favor cooperation [135, 136, 137, 138, 139, 140], here we show that the structural organization of the constituent layers to the multiplex plays an important role. Specifically, the existence of geometric correlations intertwining the layers of realistic multiplexes [84] significantly increases cooperation (compare Figs. 3.3b,c). In particular, in Fig. 3.3d) we compare the area of the harmony region, which is largest if geometric correlations are present. To conclude, the interplay between the dynamics, the complex layer topologies, and—last but not least—the structural organization of these layers into the multiplex leads to an increased cooperation.

Let us now consider $\beta = 0.7$ and $\gamma = 0.2$. The bias in the voter model shifts the system towards cooperation and, for these parameters, the region of full defection disappears. Furthermore, the behavior of the system in the bistable region is now different from the aforementioned case. Full cooperation still is a possible solution, but instead of full defection the other stable solution is given by a finite cooperation density ("mixed region"). This region has no classical analogue.

We observe that heterogeneous and clustered topologies in single layers increase cooperation (compare Figs. 3.3e) and f)). The presence of correlations between the layers increases the region in the parameter space where the harmony solution is approached, and hence further increases cooperation (compare Figs. 3.3f and g). To facilitate this comparison, in Fig. 3.3h) we show the size of the harmony region in the $T-S$ phasespace. Interestingly, the harmony area is larger for the heterogeneous multiplexes with geometric correlations and the unbiased voter model compared to the biased voter model on Erd́os-Rényi random multiplexes. We take this as further evidence for the importance to consider the structural organization of the individual layers into the multiplex.

The impact of correlations in the bistable region is especially interesting. We find that in this region angular correlations lead to a metastable state in which nodes that adopt the same strategy self-organize into local clusters. These clusters are sets of nodes that are located at small angular distances in the underlying metric space. They emerge spontaneously and are metastable in the sense that they can exist for very long times despite the noise present in the system. This behavior is shown in Fig. 3.4. The emergence of these clusters can be interpreted as a polarization of society into defecting and cooperating groups. Finally, the amount and size of the clusters is highly random and, as a consequence, we observe a broad range of final cooperation densities in this parameter region (see Fig. 3.6).

3.6 Concluding Remarks

Cooperation is common in reality in social dilemmas where many theories predict the prevalence of defection. This contradiction could be resolved by taking into account further domains of interactions between individuals, in particular social influence.

We have presented a model based on multiplex networks with two layers. One layer represents the domain in which individuals engage in repetitive strategical games. The second layer corresponds to the domain of social influence, which we model using a biased opinion dynamics. The opinions can be understood as the proclamations of individuals regarding their strategy.

The coupling between the game and opinion dynamics mimics the tendency of individuals to be congruent with respect to their actions and opinions. Even in the absence of a bias in the opinion dynamics, the coupling of the different dynamics combined with complex layer topologies and geometric correlations governing their organization into the multiplex increases cooperation. In reality, a positive bias towards cooperative attitudes could be achieved by media campaigns that promote pro-social behavior. Such a bias shifts the system towards cooperation and can avoid full defection. Furthermore, we have shown that the coupling of these dynamics in combination with geometric correlations between the layers of the system can lead to a metastable state of high polarization, in which nodes that adopt the same strategy self-organize into local clusters. These findings could explain the emergence and prevalence of polarization observed in many social dilemmas.

Our findings show that taking into account the multiplex nature of human interactions and the structural organization of these system is important to understand the emergence of cooperation in social dilemmas and that the interplay between different processes can significantly alter the behavior of the system.

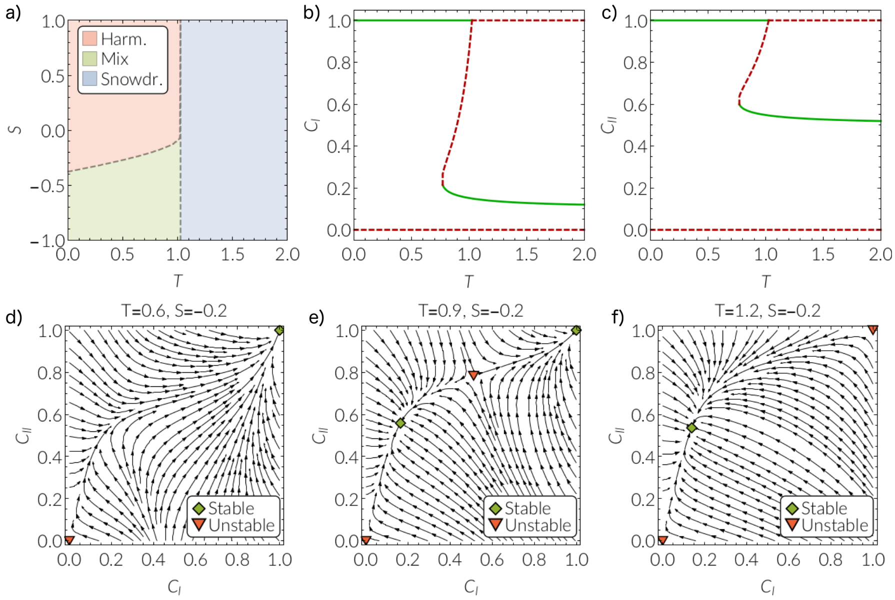

Figure 3.2: Behavior of the system for $\gamma = 0.2$ and $\beta = 0.7$ and $\langle k \rangle = 6$. **(a)** Shows the phase diagram. In the red area, full cooperation is the only stable solution and in the opinion layer, "cooperate" is the only prevailing opinion. This behavior is illustrated in the stream plot shown in $d)$ for $T = 0.6$ and $S = -0.2$. In the blue area in $a)$, the only stable solution is a mixed state with $0 < C_I < 1$ and $0 < C_{II} < 1$, and in general we have $C_I \neq C_{II}$. This region corresponds to $T > T_{c,2} \approx 1.03$. $f)$ shows this behavior as a stream plot for $T = 1.2$ and $S = -0.2$. In the green area in $a)$, we have a bistable behavior, where either full cooperation is approached in both layers, but the mixed state is stable as well. $e)$ shows this behavior as a stream plot for $T = 0.9$ and $S = -0.2$. In the bifurcation diagrams in $b)$ and $c)$ this region corresponds to $T_{c,1} < T < T_{c,2}$, where $T_{c,1} \approx 0.77$. Green solid lines represent stable fixed points and dashed red lines unstable fixed points.

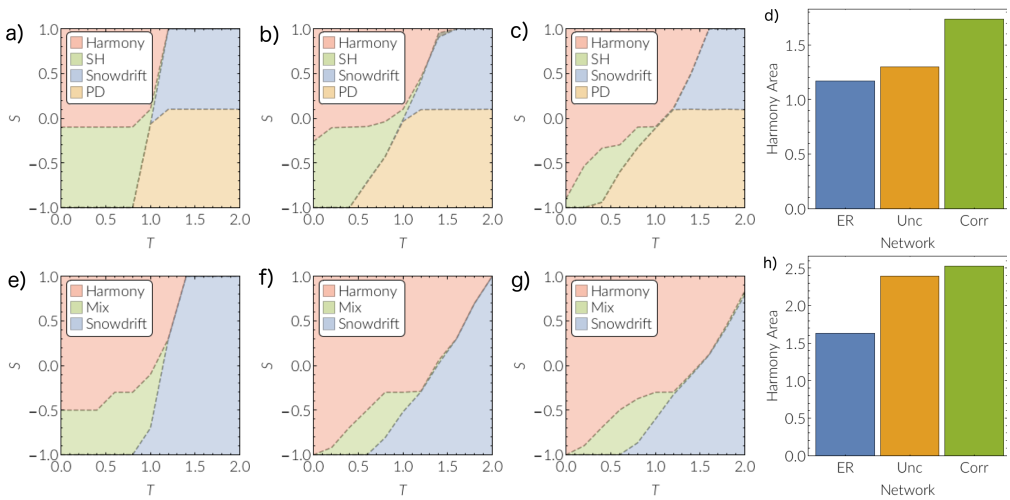

Figure 3.3: Approximated phase diagrams from numerical simulations for $\gamma = 0.2$ and $\beta = 0.5$ $(a - d)$) as well as $\beta = 0.7$ $(e - h)$. $a)$ Erd´os Rényi network. $b)$ Using GMM multiplexes uncorrelated, i.e. $g = \nu = 0$. $c)$ same as $b)$ but with geometric correlation, in particular $g = \nu = 1$. $d)$ shows the size of the "harmony" area in the phasespace for different parameters, i.e. the size of the red area in (a-c). All networks have mean degree $\langle k \rangle = 6$ and $N = 10000$ and the GMMs have power-law exponent 2.9 and temperature $T_{\text{GMM}} = 0.4$. The blue bar is for the Erd´os Rényi networks as shown in (a), the yellow bar represent the GMM model without correlations as shown in (b), and the green bar denotes the GMM model with correlations as presented in (c). $(e - h)$ shows the same as (a)-(d) but for $\beta = 0.7$.

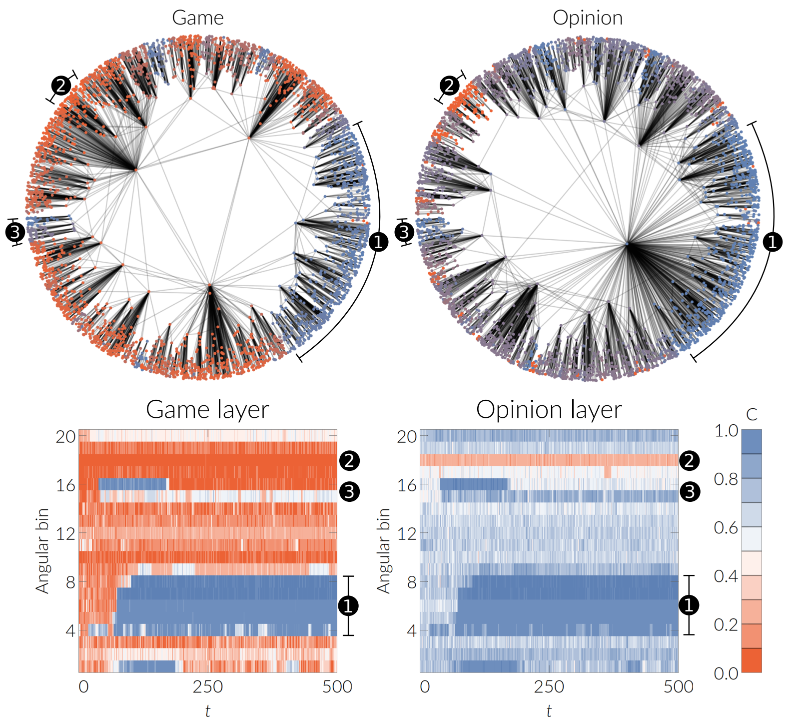

Figure 3.4: Polarization of the system in the presence of angular correlations between the layers ($g = 1, \nu = 0$) for a multiplex with $N = 5000$ nodes, a power-law exponent 2.9, temperature 0.2, and mean degree 6 in both layers. Parameters of the game are $T = 0.8$ and $S = -0.4$, the bias β is 0.7 and the coupling strength is 0.2. Results are for a single realization of our model starting with a density of cooperators of 0.1 in each layer. The top row shows visualizations of the network layers. Color coded is the mean state of the each node, averaged over time. Each time step denotes 1000 update steps of each node. The bottom row shows the evolution of the density of cooperators in each angular bin. Numbers indicate selected clusters of nodes that tend to adopt the same strategy. Each time step t denotes 10^3 rounds.

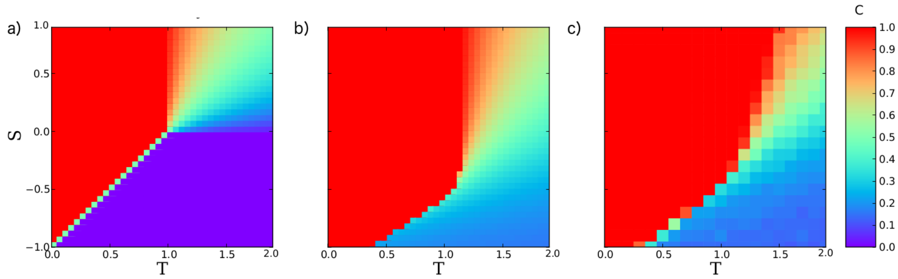

Figure 3.5: **Asymptotic density of cooperators in the $T - S$ parameter space**. a), theoretical MF asymptotic solution in one layer ($\gamma = 0$). b), asymptotic MF solution of eqs 3.7 for $\gamma = 0.2$ and $\beta = 0.7$. c), Average asymptotic density of cooperators for two Erd́os-Rényi networks of $N = 1000$ nodes and mean degree $\langle k \rangle = 5$, with $\gamma = 0.2$ and $\beta = 0.7$.

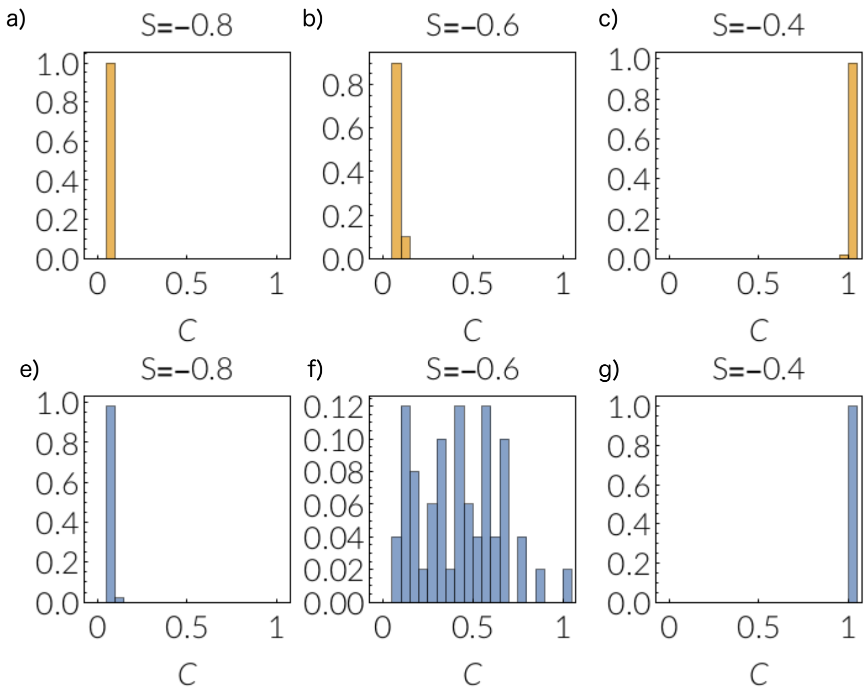

Figure 3.6: Distribution of final cooperation (after 5×10^5 rounds) in the game layer among 50 realizations of our model. The parameters are $\gamma = 0.2$ and $\beta = 0.7$, $N = 10000$ nodes, and mean degree $\langle k \rangle \approx 6$. Network layers have a power-law exponent of 2.9 and temperature $T_{\text{GMM}} = 0.4$. Here, we have fixed $T = 0.6$. Plots **(a)-(c)** show the uncorrelated case ($g = \nu = 0$) and **(d)-(f)** show the case of angular correlations ($g = 1, \nu = 0$). The value of S is shown in the respective plot title.

Chapter 4

Collective behavioral changes

The apparent paradox of shifting self-enforcing norms has attracted the attention of researchers for a long time and a quantitative understanding of the processes of norm change has remained elusive so far, probably hindered by the difficulty of accessing adequate empirical data [141]. Here, we address this issue by focusing on shifts in orthographic and linguistic norms through the lenses of about 5 million written texts covering the period from 1800 to 2008 from the digitized corpus of Google Ngram [142] dataset. Following the same approach that has allowed to quantify processes such as the regularization of English verbs [143] or the role of random drift in language evolution [144], we analyze the statistics of word occurrences for a set of specific linguistic forms that have been historically modified either by language authorities or spontaneously by language speakers in English or Spanish. These include words that changed their spelling in time and competition between variants of the same word or expression. To explore the mechanisms of norm change we consider three separate cases:

1. **Regulation by a formal institution.** We analyze the effect of the deliberations of the Royal Spanish Academy, *Real Academia Española* (RAE), the official royal institution responsible for overseeing the Spanish language, on the spelling of 23 Spanish words [145, 146, 147, 148, 149, 150, 151].

2. **Intervention of informal institutions.** We investigate the effect of dictionary publishing in the US, and focus on the updating of American spelling for 724

words [152, 153].

3. **Unregulated (or 'spontaneous') evolution.** We consider the alternation between forms that are either unregulated or described as equivalent by an institution but have nonetheless exhibited a clear evolutionary trajectory in time (i.e., we do not consider the case of random drift as primary evolutionary force [144]). In particular, we examine (i) the evolution over time of the use of two equivalent forms for the construction of imperfect subjunctive verbal time in Spanish, for $1,571$ verbs [154], (ii) the alternation of two written forms of the Spanish adverb *solo/sólo* ('only' or 'alone')[155], and (iii) 46 cases of substitution of British forms (e.g., words) with American ones in the US [156].

We show that these mechanisms leave robust and radically different stylized signatures in the data, and we propose a simple evolutionary model able to reproduce quantitatively all of the empirical observations. When a formal institution drives the norm change, the old convention is rapidly abandoned in favor of the new one [157, 158, 159, 160, 161]. This determines a universal process of norm adoption which is independent of both word frequency and corpus size. A qualitatively similar pattern is observed also for the norm adoption driven by an informal institution, although in this case the adoption of the new form is smoother and word dependent. In the case of unregulated norm changes, finally, the transition from the old to the new norm is slower, potentially occurring over the course of decades, and is often driven by some asymmetry between the two forms, such as the presence of a small fraction of individuals committed to one of the two alternatives [162, 163, 164].

4.1 Data and historical background

4.1.1 Spanish

Founded in 1713, the *Real Academia Española* (Royal Spanish Academy, RAE) is the official institution responsible for overseeing the Spanish language. Its mission is to

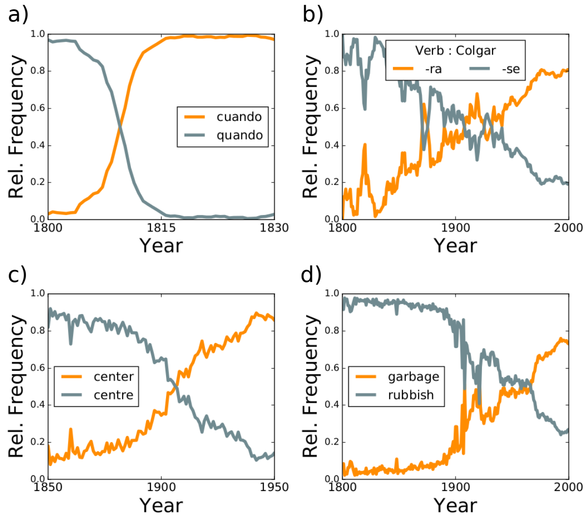

Figure 4.1: Illustrative examples of competing conventions in our dataset (relative frequencies). $a)$ Formal institution: the spelling of the Spanish word "quando" (when) was changed into "cuando" by a RAE reform occurred in 1811. $b)$ Unregulated evolution of two equivalent forms for the past subjunctive, *-ra* and *-se*, for the verb “colgar" (to hang). $c)$ Informal institution: the American Spelling "center" versus the British spelling "centre". $d)$ Unregulated evolution of “garbage", the American variant of the British “rubbish".

plan language by applying linguistic prescription in order to promote linguistic unity within and across Spanish-speaking territories, to ensure a common standard in accordance with Article 1 of its founding charter: “... to ensure the changes that the Spanish language undergoes [...] do not break the essential unity it enjoys throughout the Spanish-speaking world.” [165, 166, 167]. Its main publications are the *Dictionary of Spanish Language* (23 editions between 1780 and today) and its *Grammar*, last edited in 2014. Particularly interesting for our study is the standardization process that the RAE carried on during the 19th century, which enforced the official spelling of a number of linguistic forms [146, 168].

Our data set contains 23 spelling changes occurred in four different reforms, in 1815, 1884, 1911 and 1954 (The complete list of words examined is reported in the Appendix A.3.3) [145, 146, 147, 148, 149, 150, 151]. To illustrate this, Fig. 4.1a) shows the temporal evolution of the spelling change of the word *quando* ('when') into *cuando* –regulated in 1815's reform– in the Spanish corpus, showing a sharp transition (or "S-shaped" behavior [157, 159, 160]). Different is the case of the adverb *solo* ('only' or 'alone'), whose spelling variant *sólo* was added in the RAE dictionary in 1956 after a long unofficial existence supported by a number of academics [155, 164, 169]. We will consider the coexistence of these latter two forms as an example of unregulated evolution.

A major example of unregulated norm change is offered by the Spanish past subjunctive, which can be built in two - equivalent [170, 171] - ways by modifying the verbal root with the (conjugated) ending *-ra* or *-se* (additional details are in Appendix A.3.2). For example, the first person of the past subjunctive of the verb *colgar* ('to hang') could be indistinctly *colga-ra* or *colga-se*. Figure 4.1b) shows the growth of the *-ra* variant, for all verbal persons, over two centuries. A similar behavior is found in most Spanish verbs, the form *-se* being the most used at the beginning of XIX century (preferred $\approx$ 80% of the times) to the less used at the beginning of the XXI century (chosen $\approx$ 20% of the times). This peculiar phenomenon has attracted the attention of researchers for the last 150 years and has not been entirely clarified [170].

Recent results suggest that, whereas individuals typically use only one of the two forms, the alternation between the two variants tends to be found only in speakers who prefer the *-se* form [171, 172], as confirmed also by a recent analysis of written texts [173]. Thus, the users of $-ra$ appear to be effectively committed to this unique form. As we will see below, we will include the possibility of such asymmetries of behavior into our model.

4.1.2 British English vs American English

The emergence of American English was encouraged by the initiative of academics, newspapers and politicians –such as US President Theodore Roosevelt [162]– who over time introduced and supported new reforms [174]. The process gained momentum in the XIXth century, when a debate on how to simplify the English spelling was opened in the United States [163, 175, 176, 177], influenced also by the development of phonetics as a science [178]. As a result, in 1828 Noah Webster published the first *American Dictionary of the English Language*, beginning the Merriam-Webster series of Dictionaries that is still in use nowadays [163, 179]. Some changes, such as *color* instead of *colour* or *center* for *centre*, would become the distinctive features of American English. Figure 4.1c) shows the transition from the British spelling *centre* to the American *center*. The complete list of the 724 words examined is provided in [153] and the double spelling verified by the Merriam-Webster dictionary[152] (The complete list of words examined is reported in [180]).

The phenomenon of 'Americanization' of English [156] is not limited to spelling but includes also the introduction of different words or expression which over time replaced the British ones. Recent works [156, 181] report how the globalization of the American culture might be favoring the affirmation of their specific form of English. We will consider a list of 46 American-specific expressions [156] (The complete list of forms examined is reported in [182]) in relation to their British counterpart, such as *garbage* vs *rubbish* reported in Fig. 4.1d), or *biscuit* vs *cookie*. In both cases, we will consider only books listed in the American English Corpus of Google Ngram.

4.2 Model

We introduce a model that describes the evolution over time of two alternative forms of a word (i.e., two alternative conventions). For example, the two norms might represent each of the different cases described above, such as two spelling alternatives (*-or* vs *-our* as in *color/colour*), two ways to form a verbal tense (*-ra* vs *-se*) or two different words to refer to the same concept (*biscuit* vs *cookie*).

The model describes a system of books where instances of the two conventions are continuously added by authors, as new books are published. Authors select which convention to use (i.e., which form to introduce in the system) either by following the indications of an institution or considering previously published books. In the first case, authors simply adopt the recommended norm (or 'new norm', for simplicity, as we focus on cases of norm change). In the latter case, the convention to be used is selected with a probability proportional to its frequency in the previous history. Additionally, some authors can be committed to one specific form, thus being impermeable to any external influence, as suggested by the literature on the study of orthographic norm change in both English [162, 163] and Spanish [164].

All these elements are implemented in a modified Polya-Urn type model [183], where the urn contains the number of previous occurrences of both forms (conventions). The two different conventions are labeled as 'new' and 'old', and their number is $\mathcal{N}$ and $\mathcal{O}$ respectively. The total number of conventions in the urn, $\mathcal{W}(t) = \mathcal{N}(t) + \mathcal{O}(t)$, evolves in time as $\mathcal{W}(t) = w_0 e^{\frac{\alpha}{\beta} t} = w_0 e^{rt}$. Parameter α quantifies the system's growth rate (growth of the total number of words over time, a parameter that can be experimentally measured), whereas β denotes the response time and accounts for the typical publication time. The evolution in time of $\mathcal{N}$ and $\mathcal{O}$ is described by the following equations

$$
\begin{aligned}
\frac{1}{r}\mathcal{N}'(t) &= (1-c)\left[(1-\gamma)\,\mathcal{N}(t) + \mathcal{W}(t)\gamma E_{\mathcal{N}}\right] + c\mathcal{W}(t)\,, \\
\frac{1}{r}\mathcal{O}'(t) &= (1-c)\left[(1-\gamma)\,\mathcal{O}(t) + \mathcal{W}(t)\gamma E_{\mathcal{O}}\right]\,. \qquad (4.1)
\end{aligned}
$$

New words are inserted by writers (authors). A writer is *committed* to the use of one specific convention, with probability c), or neutral, with probability $1-c$. As discussed above, all committed writers privilege the same convention [162, 163, 164], which is the new norm in the above equations. Neutral writers in turn follow the institutional enforcement, with probability γ, or extract a convention from the urn, with probability $1-\gamma$. For simplicity, we assume that each writer inserts just one convention and that the probabilities c) and γ are constant. When an institution promotes the norm $\mathcal{N}$, it makes an effort $E_{\mathcal{N}} = 1$ and $E_{\mathcal{O}} = 0$ otherwise (see Appendix A.3.1, for the symmetric case of the institution promoting $\mathcal{O}$), if the institution is impartial, both forms are a priori equivalent and $E_{\mathcal{N}} = E_{\mathcal{O}} = \frac{1}{2}$.

By imposing $A = (1-c)(1-\gamma)$ and $B = (1-c)\,\gamma E_{\mathcal{N}} + c$ the equations for $\mathcal{N}'(t)$ takes the form:

$$\frac{1}{r}\mathcal{N}'(t) = A\mathcal{N}(t) + B\mathcal{W}_0 e^{rt}\,, \tag{4.2}$$

which solution is:

$$\mathcal{N}(t) = \frac{B}{1-A}\mathcal{W}(t) + Ke^{Art}\,. \tag{4.3}$$

Setting $\mathcal{N}(t=0) = \mathcal{N}_0$ we have:

$$\mathcal{N}(t) = \frac{B}{1-A}\left(\mathcal{W}(t) - \mathcal{W}_0 e^{Art}\right) + \mathcal{N}_0 e^{Art}\,. \tag{4.4}$$

Using densities $n(t) = \mathcal{N}(t)/\mathcal{W}(t)$ and $o(t) = \mathcal{O}(t)/\mathcal{W}(t) = 1 - n(t)$, the general solution of the system of equations (4.1) is:

$$n(t) = \frac{B}{1-A}\left(1 - e^{-r(1-A)t}\right) + n_0 e^{-r(1-A)t}\,, \tag{4.5}$$

where $n_0 = \mathcal{N}_0/\mathcal{W}_0$.

In the following sections we show that, by appropriately varying the parameter values, this analytic solution is able to reproduce in a quantitative way all the empirical observations described below.

4.3 Results

4.3.1 Regulation by a formal institution

In the main panel of Fig. 4.2 we consider the relative frequency, $n(t)$, of appearance of the new spelling for the 23 words in our dataset affected by RAE reforms [145, 146, 147, 148, 149, 150, 151] (The complete list of words examined is reported in the Appendix A.3.3). By a simple rescaling (translation) of the time axis as $t^* = t - t_r$ (where t_r is the regulation year for each specific pair of conventions), we find that all the experimental curves collapse. A change in the behavior before ($t^* < 0$) and after

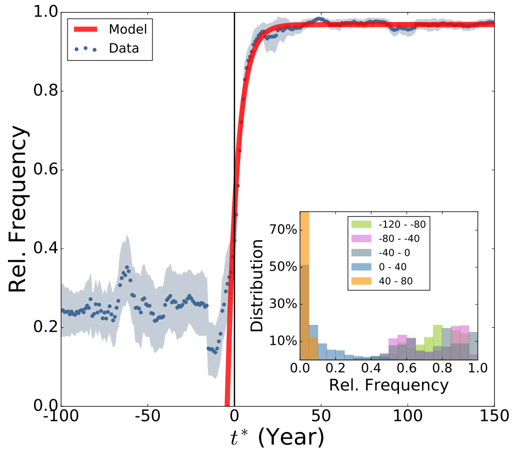

Figure 4.2: Regulation by a formal institution (Spanish, RAE). Main panel: Relative frequency of the new spelling form as a function of the rescaled time t^*. Blue points represent the average over all the considered pairs of words and the gray area the standard deviation of the data. The solid line is the prediction of the model outcome (eq. (4.6)) after parameter fit ($\chi^2 = 8 \cdot 10^{-5}$, $p = 0.99$). The black vertical line denotes the rescaled regulation year $t^* = 0$. Inset: Frequency histogram of the old spelling form for all pair of word forms, for different time periods (negative time refers to periods before the regulation).

($t^* > 0$) the norm regulation is evident. This discontinuity is captured by the distribution of the old spelling among the words before and after the regulation in the inset panel of Fig. 4.2. Importantly, such rescaling indicates the transition is size-independent. For example, for the 1815 regulation, our dataset consists of $S_{1815} = 59$ books and $S_{1815} = 4,149,151$ words, whereas for the regulation enforced in 1954 we have $S_{1954} = 2774$ books and $S_{1954} = 244,138,299$ words, but transition between the old and new form occurs over approximately the same amount of time in the two cases. Thus, regulation by a formal institution yields a norm adoption that follows an abrupt, first-order-like phase transition. For the case of formal regulation model parameters are $\gamma = 1$ and $E_{\mathcal{N}} = 1$, so that eq. (4.5) reduces to

$$n(t) = (1-c)\left(1-e^{-rt}\right) + n_0 e^{-rt}\,, \tag{4.6}$$

The main panel of Fig. 4.2 shows that the fit of eq. (4.6) matches the empirical data ($c = 0.03$, $r = 0.21/\text{year}$ and $n_0 = 0.39$).

4.3.2 Intervention of informal institutions

We now focus on the dynamics occurring between American and British spelling through the analysis of 724 words as they appear in our US corpus (See Appendix A.3.5 for further information and [180] for the complete list). We set $E_{\mathcal{N}} = 1$ (presence of institutions) and take into account that the institution is informal by considering $\gamma < 1$. For each pair of conventions we identify the year τ in which the American form surpassed in popularity the British one (the inset panel of Fig. 4.3 shows the empirical frequency distribution of these surpassing times $P(\tau)$). The main panel of Fig. 4.3 shows that by rescaling time via simple translation $t^* = t - \tau$ all experimental curves collapse, similarly to the above case of formal institution. The model eq. (4.5) reproduces notably well the data collapse (parameters $r\,(1-A) = 0.01$, $\frac{B}{1-A} = 0.4$, and $n_0 = n(t^* = 0) = 0.5$ by construction).

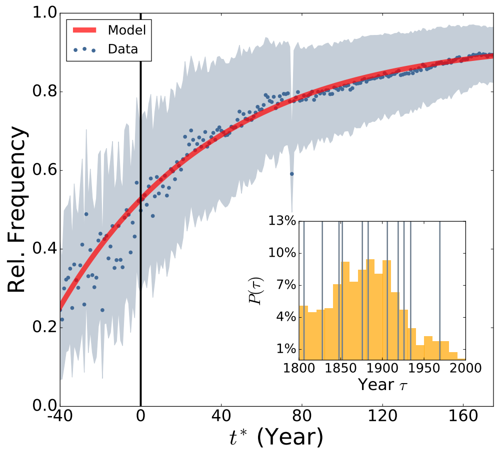

Figure 4.3: Regulation by informal institutions. Main panel: Relative frequency of the American spelling for 724 English words as a function of the rescaled time t^* ($t^* = 0$, denotes the surpassing year, for all pairs of words considered). Blue dots represent the average over all the pairs of words and the gray area the standard deviation of the data. The solid line is the model outcome, eq. (4.5), after parameter fit ($\chi^2 = 6 \cdot 10^{-4}$, $p = 0.98$). Inset: Distribution $P(\tau)$ of the years τ in which the American form overcame the British variant for each word. Vertical lines denote important moments of informal regulations of the US spelling, like dictionary editions or spelling updates (additional details are in Appendix A.3.4)

4.3.3 Unregulated evolution

As a third case we explore the process of unregulated norm change by considering the relative frequency of appearance of the form *sólo* (vs *solo*) (Spanish for 'only')[155] in the Spanish corpus, the relative frequency of appearance in the Spanish corpus of the past subjunctive form ending in $-ra$ and the one ending in $-se$ for $1,571$ verbs (See [184, 185] for the complete list of verbs), and the relative frequency of appearance in the US corpus of 46 cases, among words and expressions, of substitution of British forms with American ones (See [182] for the complete list of cases). As institutions do not play a role we impose $\gamma = 0$ in eq. (4.5).This yields $A = 1 - c$, $B = c$ and a prediction

$$n(t) = \left(1 - e^{-rct}\right) + n_0 e^{-rct}\,. \tag{4.7}$$

Figure 4.4a) and b) show that the growing of the form *sólo* as well as the growing of the $-ra$ form for the subjunctive of Spanish verbs are well captured by eq. (4.7) (solid lines correspond to the model predictions after parameter fitting). The fit returns the values $rc = 0.02$/year for the case of 'solo' and $rc = 0.006$/year for the case of the subjunctive. The parameter r is the growing rate of the system for which is common to all cases related to the same corpus. We can use the value calculated in the fit for the spelling change of Fig. 4.2, $r = 0.21$/year. We obtain $c \approx 0.09$ for Fig. 4.4a) and $c \approx 0.03$ Fig. 4.4b). Finally, Fig. 4.4c) displays the model fitting ($rc = 0.002$) to the growth of American forms, showing again a good agreement between empirical observations and eq. (4.7). It is worth noting that *solo* (without accent) can be used also as an adjective. However, while the competition *solo/sólo* concerns only the adverb, the data do not allow us to distinguish between the adverb or adjective case. Our analysis shows that the adverb is dominant, as the adverb-specific *sólo* is nowadays the most used form, but the non-saturation of the curve in Fig. 4.4a) can be interpreted as a signature of the presence of a percentage of adjectives in our dataset

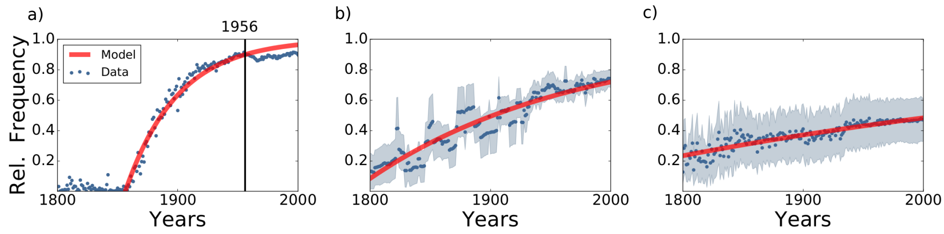

Figure 4.4: Unregulated norm change. a) Case 'sólo' versus 'solo'. Blue dots represent the relative frequency of the Spanish adverb 'sólo' (increasing in detriment of the alternative form 'solo'). Solid line is the prediction of eq. (4.7) for this case, after parameter fit ($\chi^2 = 0.002$, $p = 0.96$). The vertical line signs the year 1956 when RAE intervened explicitly on the case [155, 164, 169]. The curve saturates to a value smaller than 1 probably due the presence of a percentage of adjectives, indistinguishable from the adverb in the data. b) Case of '*-ra*' versus '*-se*'. Blue dots represent the relative frequency of the form *-ra* (increasing in detriment of the alternative but equivalent form *-se*) in Spanish past subjunctive conjugation of verbs, averaged over all verbs considered. Solid line is the specific prediction of eq. (4.7) for this case, after parameter fit ($\chi^2 = 0.01$, $p = 0.91$). c) Case of Americanization of English in US. Blue dots represent the relative frequency of the American variant (with respect to the British variant) in US corpus, averaged over all the expressions examined. Solid line is the specific prediction of eq. (4.7) for this case, after parameter fit ($\chi^2 = 0.004$, $p = 0.95$). For all the cases the grey area identifies the standard deviation of the data.

4.3.4 Microscopic dynamics

As a further assessment, we run stochastic simulations of the urn model to reproduce the microscopic evolution of each pair of conventions for the case of spontaneous transition and for the case of the intervention of informal institution. The simulations are performed as follow. At the begging the urn is composed by $\mathcal{W}_0$ conventions in the state $\mathcal{O}$. At each time $\mathcal{W}_{t+1} = \alpha \mathcal{W}_t$ new books are introduced, whose authors select which convention to use with the following rule. With probability c) the author is committed

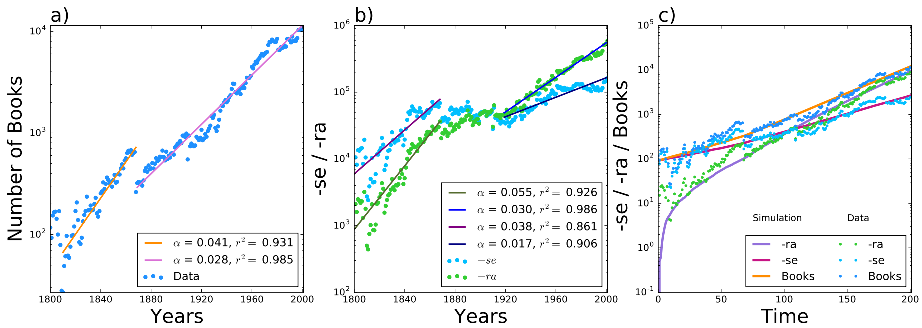

Figure 4.5: Empirical and simulated growing rate for the Spanish unregulated case of alternation of $-ra$ and $-se$. $a)$ shows the growing rate of the number of books for the Spanish corpus. $b)$ shows the growing rate of the total number of occurrence of the form $-ra$ and $-se$. $c)$ shows the comparison of data and simulations

.

and a convention in the state $\mathcal{N}$ is added; with probability $(1-c)(1-\gamma)$ the author selects one convention from the urn and reproduces it; and with probability $(1-c)\gamma$ the author follows the institution effort: with probability $(1-c)\gamma E_{\mathcal{N}}$ a convention in the state $\mathcal{N}$ is added while with probability $(1-c)\gamma E_{\mathcal{O}}$ a convention in the state $\mathcal{O}$ is added. We impose the values recovered by the fitting procedure to set the parameters $c)$ and γ in the simulations. The growing rate α was experimentally measured from the dataset. Figure 4.5.$a)$ shows the growing rate of the number of books for the Spanish corpus and Fig. 4.5.$b)$ the growing rate of the total number of occurrence of the form $-ra$ and $-se$. At last Fig. 4.5.$c)$ shows the comparison of data and simulations.

In each numerical experiment we impose the parameters recovered through the fitting procedure described above. The microscopic dynamics is performed as follow. At the begging the urn is composed by $\mathcal{W}_0$ conventions in the state $\mathcal{O}$. At each time $\mathcal{W}_{t+1} = \alpha \mathcal{W}_t$ new books are introduced, whose authors select which convention to use with the following rule. With probability $c)$ the author is committed and a convention in the state $\mathcal{N}$ is added; with probability $(1-c)(1-\gamma)$ the author selects one convention from the urn and reproduces it; and with probability $(1-c)\gamma$ the author follows the institution effort: with probability $(1-c)\,\gamma E_{\mathcal{N}}$ a convention in the state $\mathcal{N}$ is added while with probability $(1-c)\,\gamma E_{\mathcal{O}}$ a convention in the state $\mathcal{O}$ is added. We impose the values recovered by the fitting procedure to set the parameters $c)$ and γ in the simulations. The growing rate α was experimentally measured from the dataset. We initially

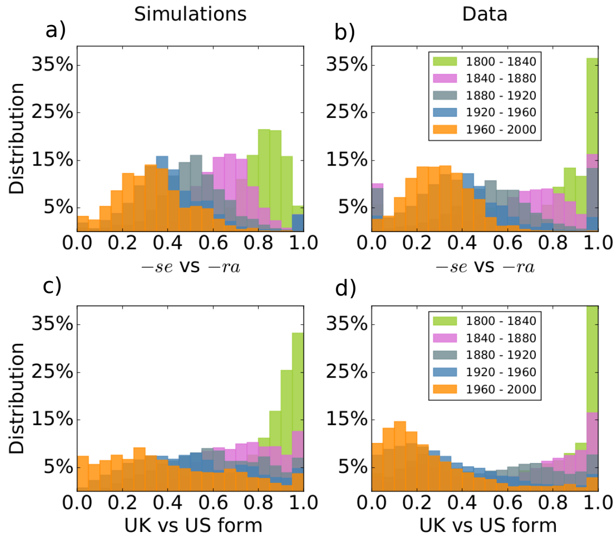

Figure 4.6: Empirical and simulated distributions for the relative frequencies of a given form. Top: Spanish subjunctive case, *-se* variant. $a)$ reproduced the empirical observation of the equivalent distributions $b)$. Bottom: Intervention of informal institution case, UK spelling variant in US corpus. $c)$ can be compared with the actual empirical distribution $d)$. The simulated distributions refer to 200 simulations run with the model parameters recovered through the fitting procedure of the precedent sections.

consider the case of unregulated (spontaneous) norm adoption. In Fig. 4.6$a)$ and $b)$ we report probability distribution of observing a relative frequency $n(t)$ for the verbal form *-se*, estimated by simulating the evolution of all verbs for which we have empirical record. In the numerical experiment, initial conditions were informed by the empirical distribution of verbs frequencies $D(\nu)$ in written texts (a power law distribution of the form $D(\nu) \approx \nu^{-1.75}$). The simulation results suggest that our model captures well the ensemble evolution over time of the whole empirical distributions. Similarly, Fig.

4.6c) and d) show empirical and numerical results for the class of norm adoption via informal authority, in the case of American Spelling change. To account for multiple interventions of informal institutions, numerical experiments were run by 'switching on' the parameter γ at different, randomly chosen, times. Moreover, the American case consists of conventions that manifest themselves through specific set of words, i.e. the spelling of *or* instead of *our* in *behavio(u)r* or *colo(u)r*or *-ize* instead of *-ise* in verbs. Thus, we let γ and c) vary randomly in each simulation to reproduce the fact that, in this case, the transition from the old to the new convention is word-dependent.

By visually comparing empirical distributions of conventions over time for each norm adoption class (inset panel of Fig. 4.2, Fig. 4.6b), and Fig. 4.6d) for formal authority, spontaneous and informal authority respectively) it is evident that, microscopically, the transition from the old to the new convention is governed by different dynamics. For enforcements by formal authorities (inset panel of Fig. 4.2), when the norm is regulated the system simply switches to the new convention. On the other hand, for unregulated (spontaneous) norm change (Fig. 4.6b)) the distribution essentially remains unaltered but for a translation of its mean value which gradually shifts from 1 to 0. Finally, for the interplay between imitation of past history and alignment with the informal institution (Fig. 4.6b)) yields a broadening of the shape of the distribution over time. This difference is captured by the role of γ as evidenced by the simulated distribution of Fig. 4.6a) for $\gamma = 0$ and Fig. 4.6c) for $0 < \gamma < 1$.

4.4 Concluding Remark

In this work we have capitalized on a recently digitized corpus to explore the complex process of norm change in the context of orthographic shifts. Through the analysis of $2,364$ cases of convention shifts occurred over the past two centuries, we identified three distinct mechanisms of norm change corresponding to the presence of an authority enforcing the adoption of a new norm, an informal institution recommending the normative update and a bottom-up self-organized process by which language users reshape the norm. Each of these norm adoption mechanisms displayed different stylized patterns in the data, which we were able to accurately describe and recover in both a qualitative and quantitative way thanks to a single analytical model. By further simulating the model we were also able to accurately describe the microscopic dynamics of norm adoption found empirically. We found that when an institution is present the transition is sharp, and in the case of a formal institution it takes the form of a first-order-like phase transition, whose characteristic time appears to be system-independent and does not depend on the relative importance of the word which is suffering the orthographic change. Conversely, the bottom-up process of spontaneous change is a purely collective phenomenon. The mechanisms of imitation and reproduction are keys to bring about the onset of the new norm, catalyzed by to the presence of committed activists.

The interplay between these two mechanism is represent by a transition driven by an informal institution. The transition is significantly faster than the spontaneous case, therefore associable with a first-order-like phase transition, but at the same time dependent on the word involved.

This work advances the current understanding of norm shifts in language change, most often limited to qualitative illustrations (e.g., the observation that adoption curve of the new norm follows an âĂŸS-shapedâĂŹ behavior)[143, 144].

CHAPTER 5

General Conclusions

When we approach the study of collective human behaviors we find ourselves facing a vast field both in terms of themes to be treated and in terms of existing models. The topics dealt with in this thesis are all part of the general problem of social consensus, namely how a convention flourish and decay and what motivates people to conform to it. Some conventions arise directly from the need to coordinate or conform, such as fashion or speaking the same language, others, instead, apply to situations where there is a tension between individual and collective interest, such as cooperation, reciprocity, etc.. [8].

The simplest and probably the most famous model describing the mechanism leading to the formation of the first type of conventions is the Voter Model, in which the final shared state is the result of direct imitation among individuals. The dynamics of the second type is captured by the evolutionary game theory where individuals, balancing costs and benefits, may decide to conform or to transgress (to cooperate or defect). These models are able to capture some microscopic variables that are essential for understanding average population behavior but, at the same time, they leave open issues such as how coexistence of concurrent conventions is possible, why cooperation in real systems is more common than predicted and how a population undergoes collective behavioral change, namely how an initially minority norm can supplant a majority ones. These are the main questions that have motivated and directed my research over these years. Thanks to different collaborations I was able to deepen the three topics and develop three distinct works.

In the first we focus on formulate a model able to contemplate the coexistence of opposing options as a stable dynamical solution. The model was inspired by a model of languages competition, the Abrams-Strogatz model, in which authors introduce the important observation about how languages in a bilingual society can have a different social status [21]. The perception that a language has a greater prestige than the other leads eventually to the death of the latter. Anyway, while many minority languages are dying, others continue to coexist with the majority ones. Our proposal is that one of the reasons for this coexistence may lie in the fact that different languages are spoken in different situations and that the perception of the status of a language may depend on the context in which it is spoken. In practice, each individual receives competing social pressures. Our model can naturally be applied to the description of the competition between two generic options (i.e. opinion, language, conventions...). We consider that the non-consensus states can be the result of the participation of individuals in distinct networks represented as distinct layer of a multiplex network. Social interactions within a given social context (a layer) are denoted by intra-layer links, while inter-layer links represent the tendency to maintain the same option across the domains. Though similar models [49, 73, 88] have already been performed in multiplex networks, the main novelty of our study lies to the fact that individuals can have different options in different layers. This naturally reflects that an individual can consent with its connections in a given social context but in other context may have different opinion or language. Our analysis shows that the latter property enriches the system's dynamics and allows not only for a global consensus on the same option for both layers, but also for active dynamical states of coexistence: a new mean field solution where both options coexist has been found. These states have also been found in numerical simulations, where, however, finite size effects can finally drive the system to consensus. Moreover, we examined both the impact of networks' topology and correlations between layers on the dynamics by numerical simulations with Geometrical Multiplex networks. We find that high correlations between layers promote the coexistence of different inter-layer islands of nodes in the same state for small values of the coupling, while high values of the coupling facilitate the achievement of a full consensus state.

With the first model we find that multiplex networks have proved to be an excellent tool as it allow the representation of different dynamics simultaneously and more sophisticated behavior of the individuals. We have therefore decided to apply the same *philosophy* to the study of social dilemma.

In the second model we analyze the influence of opinion dynamic in competitive strategical games. Cooperation between humans is quite common and stable behavior even in situations where both game theory and experiments predict defection prevalence. One of the reasons could be just the fact that individuals engaging in strategic interactions are also exposed to social influence and, consequently, to the spread of opinions. To account for this interplay we present a new evolutionary game model where game and opinions dynamics take place in different layers of a multiplex network. We assume that social influence impacts the players actions and, vice versa, that the actions in the game layer impact the opinions propagated in the system. We show that the coupling between the two dynamical processes can lead to cooperation in scenarios where the pure game dynamics predicts defection. In addiction, we consider that the layers comprising real multiplex systems are not entirely independent, but exhibit certain relations between layers structure. These relations affect the dynamics by increasing the level of cooperation and, in some particular setting, by giving rise to a metastable state in which nodes that adopt the same strategy self-organize into local groups. Naturally, real social and strategic interaction networks evolve in time, and their evolution could depend on the strategic choices of individuals[186, 187]. Hence, the inclusion of an evolving and adaptive topology constitutes and interesting task for future work.

A relevant mechanism supporting stable cooperation is the possibility for cooperators to avoid defectors [188]. This can be achieved by providing the opportunity for players to change their neighbors. It was experimentally shown [189] that reputation drives the way individuals change their interaction and, as a consequence, that the reconfiguration of the network supports cooperation. However, in many real situations it is often hard or even impossible for a person to determine who has cooperated or defected [188]. With our multiplex setting we could insert a sort of *expectation of co-operation* based on the information an agent collects from the network of opinions. In fact, Kreps et al. [190] argue that a small belief on an opponent's next cooperation is enough to support a cooperative play. Rather, players could choose not to continue interaction in groups where the outcome is unsatisfying. Furthermore, one could include the competition[86, 124, 191] between different strategic networks, or incorporate external noise[119].

The two models exposed have in common the interaction of concurrent dynamics and the possibility for individuals to take different states in different contexts, namely to be inconsistent. These novelty enrich the dynamics with properties that do not exist in the corresponding single-layer models like for example different types of coexistence between states. Moreover, the consistency of individuals in different contexts is not an imposed parameter but emerge spontaneously from dynamics only under certain conditions. In both models we found that high correlations between the networks composing the multiplex promote individuals consistent behavior and the formation of groups sharing the same state across the layers. These findings suggest that hidden geometric correlations between different layers of multiplex networks can alter the behavior of the dynamics taking place on the top of them significantly, and hence such correlations should be taken into account when modeling dynamical processes on multiplex networks [128]. A recent study [192], always in the field of evolutionary game dynamics, shows that if the degree of nodes is correlated among different layers, which is very common in real multiplex networks [63, 84, 193], the dynamics depends strictly on the topology instead of the payoffs, *topological enslavement* [192]. In particular, the authors show that if the degrees correlations are strong enough, the topological enslavement makes the outcomes of the two opposite games, Prisoners' Dilemma and Harmony Games, indistinguishable.

In the last work of my research we present the first (to the best of our knowledge) extensive quantitative analysis of the phenomenon of norm change, namely what happens when a new social norm replaces an old one. While the possible forces favoring norm change - such as institutions or committed activists - have been identified since a long time, little is known about how a population adopts a new convention, due to the difficulties of finding representative data. We address this issue by looking at changes occurred to 2,365 orthographic and lexical norms in English and Spanish through the analysis of a large corpora of books published between the years 1800 and 2008. We detect three markedly distinct patterns in the data, depending on whether the behavioral change results from the action of a formal institution, an informal authority or a spontaneous process of unregulated evolution. We propose a single evolutionary model able to capture all the observed behaviors and we show that it reproduces quantitatively the empirical data. Our results shed new light on the dynamics leading to the adoption of a new linguistic conventions and may have implications on the more general process of norm change. Today's technology, and in particular online social networks, are reportedly speeding up the process of collective behavioral change [194, 195] through the adoption of new norms, with positive [196] and negative effects [197, 198]. Understanding the microscopic mechanisms driving this process and the signature that it may leave in the data will lead to a better understanding of our society as well as to possible interventions aimed at contrasting undesired effects. In this perspective, our work will be of interest to researchers investigating the emergence of new political, social, and economic behaviors [199].

It is important to delimit the scope of our findings in order to lay the foundations for a future extension. First, we only considered cases for which historical records show that a norm change did occur and we did not attempt to predict whether a specific form is at risk of being substituted or not [143]. Second, we considered that the new convention had an advantage over the old one, represented either by the intervention of an institution or by the presence of committed users [162, 163, 164], and we did not consider examples where random drift is the dominant evolutionary force [144]. In a random drift process, variations within a language may occur without any social mechanisms operating at all: "is change that results from the random fluctuations in replicator frequencies in a finite population" [159]. Over time, these random fluctuations can eventually lead to replacement of a linguistic form with an alternative one [144]. Random drift is identified as an essential null hypothesis in population genetics [144, 200] and cultural evolution [144, 201].

The symmetry breaking between the two alternative forms with respect to the random drift can concern both sociological issues and the word itself. In our model we focused on the first one, analyzing the effect of regulation (formal or informal) and of a committed minority. In this respect, it is worth mentioning that the role of a committed minority has been investigated in the context of various multi agent models where it has been shown to play an important role on the final consensus provided its size exceeds a certain threshold [10, 202, 203, 204, 205]. Other important social mechanisms responsible for the evolution of language can, instead, concern individuals' motivations. A possible extension of our model could be considering costs and benefits of reproducing a norm [8]. In this perspective, the committed minority can be interpreted as defectors who do not intend to change their status to adapt to the majority [206]. Costs and benefits may depend on numerous social and individual factors [7, 8] and generally refer to language innovations [207] or language changes due to conflict between multiple languages as in the contact between two linguistically independent populations [158] or in the loanword phenomena (foreign words incorporated into a language without translation)[208]. In any case, defining the individual factors that drive a person to conform is "one of the major challenges for those interested in the evolution of norms" [7].

The mechanisms regarding the evolution of language find their counterpart and are an expression of the more general phenomena of cultural evolution. The recent digitization of millions of books in different languages opens the way for a new quantitative investigation of cultural trends [142]. Taking advantage of this new possibility, our massive study improves the comprehension and the analytical description of how a population undergoes behavioral change, with implications ranging from study of collective dynamics in large groups to Economics and the Social and Political Sciences.

Summarizing, the results achieved in these works show that in modeling human behaviors an important role is played by the fact that individuals participate simultaneously in different social contexts. This implies that individuals are subject to both the influence of different social dynamics and different, but not independent, connecting structures. We have also shown that, in the complex process of collective shift in norm adoption, the nature of the norm shift leaves distinct patterns in the data represented by three different types of dynamical transition. This last work advances the current understanding of norm shifts in language change, most often limited to qualitative illustrations (e.g., the observation that adoption curve of the new norm follows an âĂŸS-shapedâĂŹ behavior [159]).

Work in the field of collective human behavior can never actually be concluded. Everyone adds a piece, focusing on the role of some specific aspects and each time the deeper understanding of some phenomenon opens the way for new questions. Even if these are formally the conclusions of my thesis, I consider this research the basis for directing my work in the future. After studying separately the role of the interaction between topologies and dynamics, and the the role of different driving forces (i.e. formal or informal authority and committed minority) in collective behavioral change, I naturally developed the curiosity to find out what happens if we combine these ingredients. For example, what is the role of topology in norms transitions? or in which way authorities (formal or informal) or committed activist can alter the affirmation of cooperation or the coexistence of opposing conventions?

Recent studies [141, 209, 210] have actually revealed the crucial role of each of these mechanisms when they work in interaction.

An experiment [209] conducted in 56 US schools shows that a *critical mass* of connected individuals adopting a new conventional behavior can spread the change through a social network [141, 210]. Encouraging a small set of students to take a public distance against bullying reduced student conflicts by 30% in a year. In particular "network analyses reveal certain kinds of students (called "social referents") have an outsized influence over social norms and behavior at the school" [209]. Moreover, local cluster of committed adopters of a new behavior may also emerge due to a perceived recognition of an individual benefit [210]. Many theories [211, 212, 213, 214] suggest that members of a group often infer which is the typical behavior by observing the behavior of the, so called, "social referents", individuals considered important as a source of normative information [209]. It comes out that these social referents have also a central role in the networks, i.e. by having many connections [209].

It is increasingly evident that the topology of the interaction and social dynamics can not be treated separately. Paraphrasing the famous phrase of John Wheeler [1] we could say that *People tell topology how to shape; topology tells people how to behave*.

[1] "Spacetime tells matter how to move; matter tells spacetime how to curve ", John Archibald Wheeler

The Heart of Gold *fled on silently through the night of space, now on conventional photon drive. Its crew of four were ill at ease knowing that they had been brought together not of their own volition or by simple coincidence, but by some curious perversion of physics - as if relationships between people were susceptible to the same laws that governed the relationships between atoms and molecules.*

"The Hitchhiker's Guide to the Galaxy", Douglas Adams.

APPENDIX A

Appendix

A.1 The Geometrical Multiplex Model

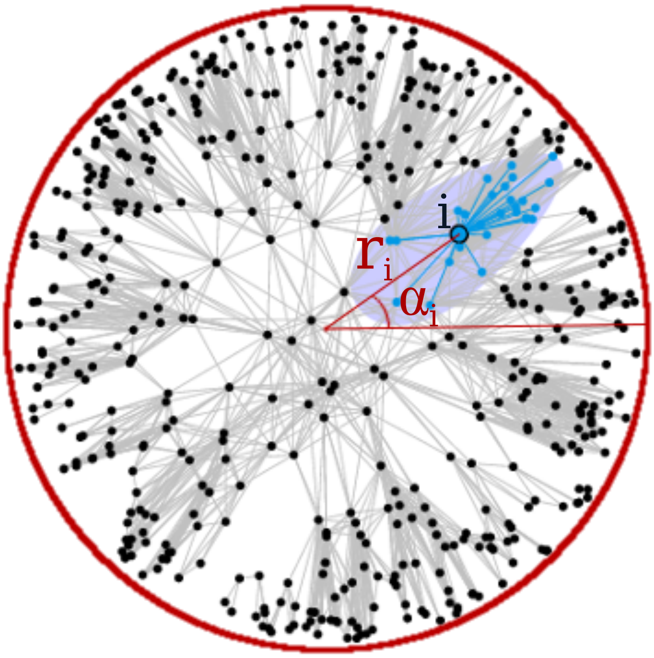

Figure A.1: Visualization of a network in the hyperbolic space. The skyblue area shows the basin of attraction of the node, defined by its radial coordination δ and angular coordination ν.

The geometric multiplex model is based on the (single-layer) network construction procedure of the newtonian $\mathbb{S}^1$[215] and hyperbolic $\mathbb{H}^2$[216] models. The two models are isomorphic and here we present the results for the $\mathbb{H}^2$ version. In the previous presented model of complex networks the nodes were equipped with a single characteristic, the degree. Here, instead, to each node are assign two characteristics corresponding to two coordinate in the hyperbolic space. The first is the *popularity*, directly interpretable as its degree and the second is the *similarity*. Similarity is an abstract feature that defines the affinity between two nodes: If the network describes the connections between people, similarity indicates how much the two people are actually *similar*, i.e. they listen the same music or they like the same movie director and so on.. If two people have little popularity, similarity plays a very important role in establishing connections.

The construction of a network of size N proceed firsts by assigning to each node $i = 1, \ldots, N$ its popularity and similarity coordinates r_i, α, specified by the radial and angular coordinate of the node, see Fig. A.1. Subsequently each pair of nodes i, j is connected with probability

$$p(x_{ij}) = 1/(1 + e^{\frac{1}{2T}(x_{ij} - R)}) \,, \tag{A.1}$$

where x_{ij} is the hyperbolic distance between the nodes and $R \sim \ln N$. The coordinates of the nodes define its basin of attraction. The connection probability $p(x_{ij})$ is the Fermi-Dirac distribution where the *temperature* parameter T_{GMM} controls the level of clustering in the network. The average clustering $\bar{c}$ is maximized at $T = 0$, linearly decreases to zero with $T \in [0, 1)$, and is asymptotically zero if $T > 1$. As $T \to 0$ the connection probability becomes the step function $p(x_{ij}) \to 1$ if $x_{ij} \leq R$, and $p(x_{ij}) \to 0$ if $x_{ij} > R$. It has been shown that the $\mathbb{S}^1$ and $\mathbb{H}^2$ models can build synthetic networks reproducing a wide range of structural characteristics of real networks, including power law degree distributions and strong clustering [215, 216]. The use of these models for the single-layer networks allows for radial and angular coordinate correlations across the different layers. The level of these correlations can be controlled by model parameters $\delta \in [0, 1]$ and $\nu \in [0, 1]$, without affecting the topological structure of the single layers. The radial correlations, related to the node's degree, increase with parameter δ—at $\delta = 0$ there are no radial correlations, while at $\delta = 1$ radial correlations are maximized. Similarly, the angular correlations increase with parameter ν—at $\nu = 0$ there are no angular correlations, while at $\nu = 1$ angular correlations are maximized. The radial, or similarity, correlation define the probability of links overlap across the layers. See [84] for details.

A.2 Supplementary information for Chapter 3

A.2.1 Final cooperation

In Fig. A.2 we show the final density of cooperators averaged over 50 realization of the system.

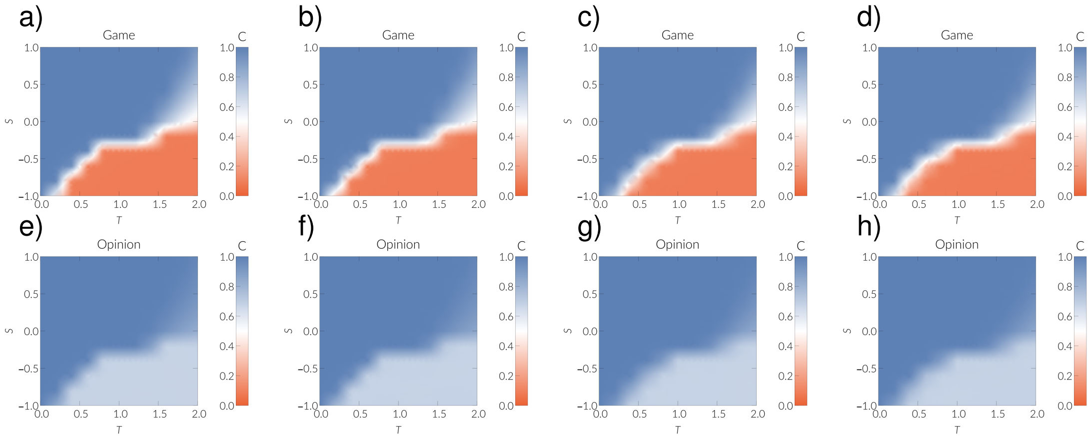

Figure A.2: Final density (after 5×10^5 rounds) of cooperation for different correlation values. **(a, e)** Uncorrelated ($g = \nu = 0$). **(b, f)** Radial correlations ($g = 0, \nu = 1$). **(c, g)** Angular correlations ($g = 1, \nu = 0$). **(d, i)** Angular and radial correlations ($g = 1, \nu = 0$). Game layer is shown in the top row **(a-d)**. Opinion layer is shown in the bottom row **(e-h)**.

A.2.2 Phase diagrams for numerical simulations

Fig. A.3 shows the probability to reach the harmony state from different initial conditions. The region in which the final state differs for the different initial conditions is the bistable region.

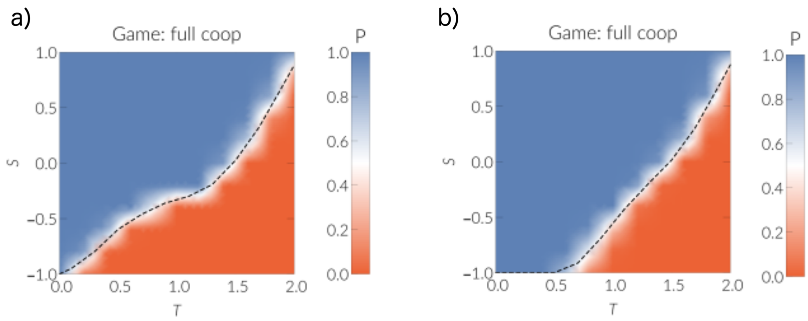

Figure A.3: Results for GMM multiplexes with $\gamma = 0.2$, $\beta = 0.7$, $N = 10000$ nodes, $g = 1$, and $\nu = 0$. **(a)** Probability to reach the harmony state starting with $C_{I,II} = 0.01$ and $g = 1, \nu = 0$. **(b)** The same for starting with $C_{I,II} = 0.99$.

A.2.3 Impact of the coupling constant γ

In Fig. A.4 we show the bifurcation diagram with the coupling strength γ as a control parameter. At a critical value $\gamma_c \approx 0.4$ the system undergoes a transcritical bifurcation. For $0 < \gamma < \gamma_c$ we have a stable mixed solution, which is particularly interesting. In the main text, we therefore fix γ in this range.

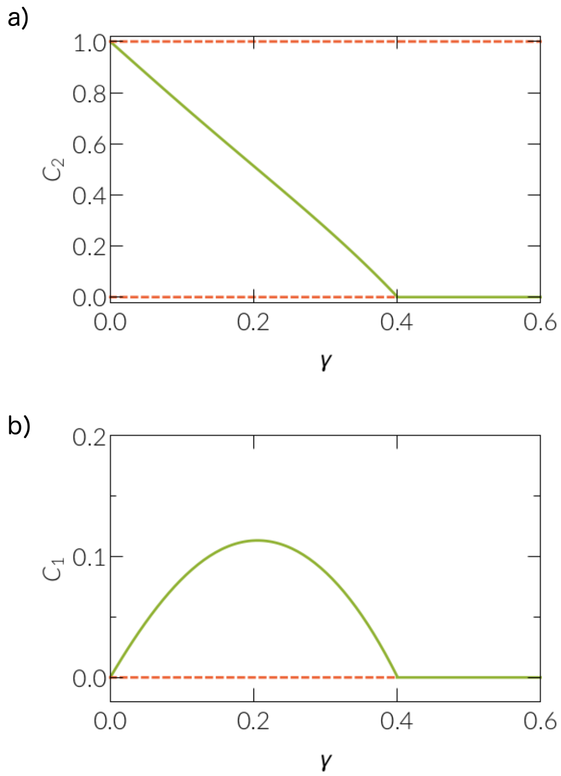

Figure A.4: Bifurcation diagram for the prisoner's dilemma ($T = 1.5, S = -0.5$) for $\beta = 0.7$ as a function of the control parameter β. Top: Density of cooperative attitude in the opinion layer. Bottom: Density of cooperators in the game layer.

A.3 Supplementary information for Chapter 4

A.3.1 Symmetric case for Eq. 4.2

When the commitment supports the old convention $\mathcal{O}$, eq. (1) takes the form:

$$\begin{aligned} \frac{1}{r}\mathcal{N}'(t) &= (1-c)\left[(1-\gamma)\,\mathcal{N}(t) + \mathcal{W}(t)\gamma E_{\mathcal{N}}\right] \\ \frac{1}{r}\mathcal{O}'(t) &= (1-c)\left[(1-\gamma)\,\mathcal{O}(t) + \mathcal{W}(t)\gamma E_{\mathcal{O}}\right] + c\mathcal{W}(t)\,. \end{aligned} \tag{A.2}$$

A.3.2 Spanish past subjunctive

In Spanish two equivalent forms exist to construct the past subjunctive : the one ending in $-ra$ and the one ending in $-se$ (as in *pensa-ra* and *pensa-se* âĂŸhad thoughtâĂŹ). The form $-se$ evolved from the Latin plusquamperfect subjunctive, while the form $-ra$ evolved from the Latin plusquamperfect indicative [217].

A.3.3 Spanish spelling reforms

We present the complete list of the 23 words examined in the Spanish spelling change case, grouped into their respective reforms [145, 146, 147, 148, 149, 150, 151].

- 1815 :
  - antiquario $\rightarrow$ anticuario (antiquarian)
  - quaderno $\rightarrow$ cuaderno (notebook)
  - quadro $\rightarrow$ cuadro (picture)
  - quando $\rightarrow$ cuando (when)
  - quanto $\rightarrow$ cuanto (how much)
  - quarto $\rightarrow$ cuarto (fourth)
  - quatro $\rightarrow$ cuatro (four)
  - quociente $\rightarrow$ cociente (quotient)
  - quota $\rightarrow$ cuota (quote)
  - quotidiano $\rightarrow$ cotidiano (daily)
  - Equador $\rightarrow$ Ecuador (Ecuador)
  - iniquo $\rightarrow$ inicuo (iniquitous)
  - obliquo $\rightarrow$ oblicuo (oblique)
- 1884

  - guion → guión (script)
  - truhan → truhán (rogue)
  - virey → virrey (viceroy)
  - vireina → virreina (viceroy's wife)
  - vireinato → virreinato (viceroyalty)

- 1911
  - ó → o (or)
  - á → a (to)

- 1954
  - dió → dio (it gave)
  - fué → fue (it was)
  - vió → vio (it saw)

A.3.4 American Spelling Important Moments

Important moment for the American spelling reforms

- 1806 Noah Webster published *'A Compendious Dictionary of the English Language'*
- 1828 First American Dictionary *'An American Dictionary of the English Language'*
- 1848 Alexander John Ellis published *'A Plea for Phonetic Spelling'*
- 1876 American Spelling Reform Association were founded and start to adopt the reforms
- 1883 The Chicago Tribune newspaper start to adopt the reforms
- 1906 The Simplified Spelling Board was founded and President of the United States Theodore Roosevelt signed an executive order imposing the use of reformed spelling in the official communications of the Congress.

- 1919 H.L. Mencken published the first edition of *The American Language*
- 1926 Henry Fowler published the first edition of *Dictionary of Modern English Usage*
- 1969 Harry Lindgren published *Spelling Reform: A New Approach*

A.3.5 British and American spelling conflict

It is important to mention that, in the spellings conflict case, the various acceptation of a term, such as singular or plural, or, for a verb, present, past.. ecc, were considered separately because they behave differently. As en example of the last phenomena we report in Fig. A.5 the evolution of the singular and the plural of the word “behavior/behaviour" (American/British spelling). The American spelling of the singular exceeds that of the British almost a century before the plural one.

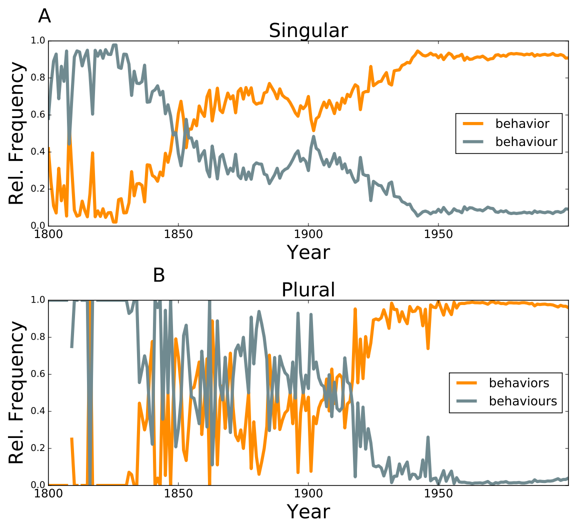

Figure A.5: (A) Evolution of “behavior", the American form and “behaviour", the bristish form. (B) Evolution of the plural “behaviors" and "“behaviour""

Resumen

Los temas tratados en esta tesis son todos parte del fenómeno más general del consenso social, específicamente, cómo florece y decae una convención social y qué motiva a las personas a ajustarse a ella. Es un hecho común notar que las personas que interactúan finalmente se vuelven más parecidas, construyendo esas convenciones sociales que subyacen a muchas relaciones sociales y económicas [6, 7, 8, 9]. Ejemplos van desde conducir por el lado derecho de la calzada, hasta el lenguaje, las reglas de cortesía o los juicios morales. Las convenciones pueden surgir gracias a la acción de alguna institución formal o informal, o mediante un proceso auto-organizado en el que el consenso a nivel global es la consecuencia involuntaria de individuos que se coordinan localmente [7, 10].

En el contexto de los comportamientos colectivos que pueden desarrollarse en los sistemas sociales, voy a tratar la dinámica de las opiniones, el fenómeno de la cooperación humana y la evolución de las normas sociales. La evolución dinámica de las opiniones y la norma social a menudo se cruzan con una descripción en términos de idiomas. De hecho, el fenómeno de la existencia de un lenguaje común se toma como un prototipo del proceso de consenso social.

Un aspecto fundamental que debe tenerse en cuenta al modelar los sistemas sociales es la topología de la interacción que define, en términos generales, quién interactúa con quién. Varios estudios [11, 52, 60] demuestran cómo, aunque las reglas dinámicas son las mismas, varias topologías conducen a resultados diferentes, señalando cómo la estructura de la interacción es fundamental en la emergencia de comportamientos colectivos humanos específicos. La estructura de los sistemas sociales se representa a través del uso de herramientas de redes complejas, una representación matemática de un grupo de entidades interactuantes en el que la interacción entre los componentes es crucial en la aparición de estructuras organizadas y los comportamientos colectivos.

Varios modelos [11, 47, 48, 49] se han propuesto para describir el proceso de consenso social, siendo el *voter model* uno de los más simples y más estudiados [50]. El voter model es un modelo con dos estados equivalentes, primero introducido para describir la competencia de especies biológicas [50] y más tarde nombrado como el modelo de votante en ref. [51] para su interpretación inmediata en términos de votaciónes [11]. En el voter model, se llega a un estado final común como resultado de la imitación directa entre individuos. Por lo tanto, una sucesión de consensos locales se convierte en un consenso total.

Las dinámicas que conducen al surgimiento de la cooperación, en cambio, son capturadas por la teoría evolutiva de juegos (Evolutionary Game Theory) donde los individuos, al equilibrar los costos y los beneficios, pueden decidir conformarse o transgredir (cooperar o desertar). Los modelos de juego se basan en el mecanismo de adaptación porque los individuos necesitan adaptar su estrategia en función de cómo *funcionó* en relación con las estrategias elegidas por los demás. El equilibrio dinámico se alcanza cuando todos los individuos decidieron no cambiar su estrategia y no se da por hecho que todos converjan en la misma.

Parte de mi trabajo será extender los modelos de comportamiento colectivo antes mencionados a estructuras más complejas llamadas *Multiplex*. Una red multiplex [64, 80, 124, 218, 219] consiste en dos o más redes interconectadas que se encuentran en distintas capas. Las capas tienen el mismo número de nodos que están conectados con sus contrapartes en las capas y, en general, tienen una estructura de conectividad diferente dentro de las capas [124, 218]. El modelo de las redes multiplex introducido permite un enfoque más realista en el estudio de las interacciones individuales que se pueden comunicar a través de diferentes tipos de canales. Las redes multiplex se han utilizado para analizar los sistemas de transporte público [65, 66], difusión de la conciencia e infecciones [68, 69], la dinámica de las poblaciones ecológicas [70, 92] así como la evolución de las redes sociales [72, 73].

A continuación, presentaré los tres modelos desarrollados en mi investigación y los respectivos resultados. Estos modelos son el resultado de tres colaboraciones diferentes.

En el primer trabajo nos centramos en formular un modelo capaz de contemplar la coexistencia de opciones opuestas como una solución dinámica estable. El modelo se inspiró en un modelo de competición de idiomas, el modelo Abrams-Strogatz, en el cual los autores introducen la observación importante sobre cómo los idiomas en una sociedad bilingüe pueden tener un estatus social diferente [21]. La percepción de que un idioma tiene un mayor prestigio que el otro lleva eventualmente a la desaparición de este último. De todos modos, mientras que muchas lenguas minoritarias están muriendo, otras continúan coexistiendo con las de la mayoría. Nuestra propuesta es que una de las razones de esta coexistencia puede residir en el hecho de que se hablan diferentes idiomas en diferentes situaciones y que la percepción del estado de un idioma puede depender del contexto en el que se habla. En la práctica, cada individuo recibe presiones sociales conflictivas. Nuestro modelo se puede aplicar naturalmente a la descripción de la competencia entre dos opciones genéricas (es decir, opinión, idioma, convenciones ...).

Consideramos que los estados sin consenso, en el que ambas opciones sobreviven, pueden ser el resultado de la participación de individuos en distintas redes representadas como capas distintas de una red multiplex. Las interacciones sociales dentro de un contexto social dado (una capa) se denotan mediante enlaces dentro de la capa, mientras que los enlaces entre capas representan la tendencia a mantener la misma opción en todos los dominios. Aunque modelos similares [49, 73, 88] ya se han realizado en redes multiplex, la principal novedad de nuestro estudio radica en el hecho de que los individuos pueden tener diferentes opciones en diferentes capas. Esto naturalmente refleja que un individuo puede consentir con sus conexiones en un contexto social dado, pero en otro contexto puede tener una opinión o lenguaje diferente. Nuestra análisis muestra que esta última propiedad enriquece la dinámica del sistema y permite no solo un consenso global sobre la misma opción para ambas capas, sino también estados dinámicos activos de convivencia: se ha encontrado una nueva solución de campo medio donde ambas opiniones coexisten. Estos estados también se han encontrado en simulaciones numéricas, donde, sin embargo, los efectos de tamaño finito finalmente pueden llevar el sistema a un consenso. Además, examinamos tanto el impacto de la topología de las redes como las correlaciones entre capas en la dinámica mediante simulaciones numéricas con redes "Geometrical Multiplex". Encontramos que las altas correlaciones entre las capas promueven la coexistencia de diferentes grupos de individuos con el mismo estado en ambas capas. En términos de lenguaje, esto se puede interpretar como un escenario en el que hay diferentes grupos monolingües en un sistema globalmente bilingüe.

Con el primer modelo, encontramos que la red multiplex ha demostrado ser una excelente herramienta ya que permite la representación de diferentes dinámicas simultáneamente y un comportamiento más sofisticado de los individuos. Por lo tanto, hemos decidido aplicar la misma *filosofía* al estudio de dilemas sociales.

En el segundo modelo, analizamos la influencia de la dinámica de opinión en juegos estratégicos competitivos. La cooperación entre humanos es bastante común y estable incluso en situaciones donde tanto la teoría de juegos como los experimentos predicen el prevalecer de la defección. Una de las razones podría ser simplemente el hecho de que los individuos que participan en interacciones estratégicas también están expuestos a la influencia social y, en consecuencia, a la difusión de opiniones. Para dar cuenta de esta interacción presentamos un nuevo modelo de juego evolutivo donde las dinámicas del juego y las opiniones tienen lugar en diferentes capas de una red multiplex. Suponemos que la influencia social impacta a las acciones de los jugadores y, viceversa, las acciones en la capa del juego impacta las opiniones propagadas en el sistema.

Mostramos que el acoplamiento entre los dos procesos dinámicos puede conducir a la cooperación en escenarios donde la dinámica del juego predice la defección. Adicionalmente, consideramos que las capas que comprenden los sistemas múltiplex reales no son completamente independiente, pero muestran ciertas relaciones entre estructuras de capas. Estas relaciones afectan la dinámica para aumentar el nivel de cooperación y, en particular, encontramos un estado metaestable en el que los nodos que adoptan la misma estrategia se auto-organizan en grupos locales.

Los dos modelos expuestos tienen en común la interacción de dinámicas concurrentes y la posibilidad de que los individuos tomen diferentes estados en diferentes contextos, es decir, que sean inconsistentes. Esta novedad enriquece la dinámica con propiedades que no existen en los modelos de capa única correspondientes, como por ejemplo diferentes tipos de coexistencia entre estados. Además, la consistencia de los individuos en diferentes contextos no es un parámetro impuesto sino que surge espontáneamente de la dinámica solo bajo ciertas condiciones. En ambos modelos encontramos que las altas correlaciones entre las redes que componen el múltiplex promueven el comportamiento consistente de los individuos y la formación de grupos que comparten el mismo estado a través de las capas. Estos hallazgos sugieren que las correlaciones geométricas ocultas entre diferentes capas de redes múltiplex pueden alterar significativamente el comportamiento de la dinámica que está en la parte superior de las mismas, y por lo tanto, tales correlaciones deben tenerse en cuenta en futuras investigaciones sobre procesos dinámicos en redes multiplex [84].

En el último trabajo de mi investigación, presentamos el primer análisis cuantitativo (que nosotros sepamos) del fenómeno del evolución de las normas, es decir, lo que sucede cuando una nueva norma social reemplaza a una norma existente. Si bien las posibles fuerzas que favorecen el cambio de normas, como las instituciones o los activistas, se han identificado hace mucho tiempo, poco se sabe acerca de cómo una población adopta una nueva convención, debido a las dificultades de encontrar datos representativos. Nos planteamos este problema al observar los cambios ocurridos a 2,365 normas ortográficas y léxicas en inglés y español a través del análisis de un gran corpus de libros publicados entre los años 1800 y 2008. Detectamos tres patrones marcadamente distintos en los datos, dependiendo de si el cambio de comportamiento resulta de la acción de un institución, de una autoridad informal o de un proceso espontáneo de evolución. Además proponemos un único modelo evolutivo capaz de capturar todos los comportamientos observados y mostramos que esto reproduce cuantitativamente los datos empíricos.

Resumiendo, esta tesis se desarrolla en torno a tres preguntas principales aún abiertas en el contexto del estudio de los comportamientos humanos colectivos: cómo es posible la coexistencia de convenciones (opiniones, idiomas, etc.) concurrentes; por qué la cooperación en sistemas reales es más común de lo que se predice; y cómo una norma inicialmente minoritaria puede suplantar a una mayoría. Incluso si el trabajo en el campo de los comportamientos humanos colectivos en realidad nunca puede llegar a la conclusión, he añadido algunas piezas, centrándose en el papel de algunos aspectos específicos. Para las primeras dos preguntas, hemos descubierto que el hecho de que los individuos participen simultáneamente en diferentes contextos sociales influye significativamente tanto en la dinámica de las opiniones como en la dinámica de los juegos. Para la última pregunta hicimos uno estudio masivo que mejora la comprensión y la descripción analítica de cómo una población sufre un cambio global de comportamiento, con implicaciones que van desde el estudio de la dinámica colectiva en grandes grupos hasta la economía y las ciencias sociales y políticas.

Bibliography

[1] Philip Ball. The physical modelling of society: A historical perspective. *Physica A*, 314(1):1–14, 2002.

[2] Tomas Hobbes and William B Molesworth. *The English Works of Thomas Hobbes of Malmesbury, volume II*. Number VOL. II in The English Works of Thomas Hobbes of Malmesbury. John Bohn, London, 1845.

[3] Murray Forsyth. Thomas Hobbes and the constituent power of the people. *Political Studies*, 29(2):191–203, 1981.

[4] Thomas Hobbes. *The Elements of Law, Natural and Politic*. University of Virginia Library, 2001.

[5] Immanuel Kant. *On History, ed. Lewis White Beck*. 1963.

[6] H Peyton Young. The evolution of social norms. *Annual Reviews of Economics*, 7(1):359–387, 2015.

[7] Paul R Ehrlich and Simon A Levin. The evolution of norms. *PLoS Biology*, 3(6):e194, 2005.

[8] Cristina Bicchieri. *The grammar of society: The nature and dynamics of social norms*. Cambridge University Press, 2005.

[9] Andrei Marmor. *Social conventions: From language to law*. Princeton University Press, 2009.

[10] Andrea Baronchelli. The emergence of consensus. *ArXiv e-prints arXiv:1704.07767*, 2017.

[11] Claudio Castellano, Santo Fortunato, and Vittorio Loreto. Statistical physics of social dynamics. *Reviews of Modern Physics*, 81(2):591, 2009.

[12] Ann Senghas and Marie Coppola. Children creating language: How Nicaraguan sign language acquired a spatial grammar. *Psychological Science*, 12(4):323–328, 2001.

[13] Susan Goldin-Meadow and Carolyn Mylander. Spontaneous sign systems created by deaf children in two cultures. *Nature*, 391(6664):279–281, 1998.

[14] Lewis Campbell. *The Life of James Clerk Maxwell*. London: Macmillan, 1882.

[15] William D Niven. *The Scientific Letters and Papers of James Clerk Maxwell*, volume 2. Dover Publications. Reprint of Cambridge University Press edition of 1890, New York, 1965.

[16] John C Turner. *Social influence*. Buckingham, England: Open University Press, 1991.

[17] Miller McPherson, Lynn Smith-Lovin, and James M Cook. Birds of a feather: Homophily in social networks. *Annual Review of Sociology*, 27(1):415–444, 2001.

[18] Armin Falk and Urs Fischbacher. A theory of reciprocity. *Games and Economic Behavior*, 54(2):293–315, 2006.

[19] Joyce Berg, John Dickhaut, and Kevin McCabe. Trust, reciprocity, and social history. *Games and Economic Behavior*, 10(1):122–142, 1995.

[20] Romualdo Pastor-Satorras, Claudio Castellano, Piet Van Mieghem, and Alessandro Vespignani. Epidemic processes in complex networks. *Reviews of Modern Physics*, 87(3):925, 2015.

[21] Daniel M Abrams and Steven H Strogatz. Linguistics: Modelling the dynamics of language death. *Nature*, 424(6951):900–900, 2003.

[22] David G Rand and Martin A Nowak. Human cooperation. *Trends in cognitive sciences*, 17(8):413–425, 2013.

[23] Urs Fischbacher, Simon Gächter, and Ernst Fehr. Are people conditionally cooperative? evidence from a public goods experiment. *Economics Letters*, 71(3):397–404, 2001.

[24] Jelena Grujić, Carlos Gracia-Lázaro, Manfred Milinski, Dirk Semmann, Arne Traulsen, J A Cuesta, Yamir Moreno, and Angel Sánchez. A comparative analysis of spatial prisoner's dilemma experiments: Conditional cooperation and payoff irrelevance. *Scientific Report*, 4:47718, 2014.

[25] Philip W Anderson et al. More is different. *Science*, 177(4047):393–396, 1972.

[26] Mark Newman. *Networks: an introduction*. Oxford University Press, 2010.

[27] Jacob Levy Moreno. Who shall survive?: A new approach to the problem of human interrelations. 1934.

[28] Stephen P Borgatti, Ajay Mehra, Daniel J Brass, and Giuseppe Labianca. Network analysis in the social sciences. *Science*, 323(5916):892–895, 2009.

[29] Vito Latora, Vincenzo Nicosia, and Giovanni Russo. *Complex Networks: Principles, Methods and Applications*. Cambridge University Press, 2017.

[30] Neo D Martinez. Artifacts or attributes? effects of resolution on the little rock lake food web. *Ecological monographs*, 61(4):367–392, 1991.

[31] Wayne W Zachary. An information flow model for conflict and fission in small groups. *Journal of Anthropological Research*, 33(4):452–473, 1977.

[32] Albert-László Barabási. Network science. *Cambridge University Press, Cambridge*, 2016.

[33] Michelle Girvan and Mark EJ Newman. Community structure in social and biological networks. *Proceedings of the national academy of sciences*, 99(12):7821–7826, 2002.

[34] Martin Rosvall and Carl T Bergstrom. An information-theoretic framework for resolving community structure in complex networks. *Proceedings of the National Academy of Sciences*, 104(18):7327–7331, 2007.

[35] Mark EJ Newman. Modularity and community structure in networks. *Proceedings of the national academy of sciences*, 103(23):8577–8582, 2006.

[36] Zachary Karate Club CLUB.

[37] Sergei N Dorogovtsev and José FF Mendes. *Evolution of networks: From biological nets to the Internet and WWW*. OUP Oxford, 2013.

[38] Aaron Clauset, Cosma Rohilla Shalizi, and Mark EJ Newman. Power-law distributions in empirical data. *Siam Review*, 51(4):661–703, 2009.

[39] Marián Boguñá, Romualdo Pastor-Satorras, Albert Díaz-Guilera, and Alex Arenas. Models of social networks based on social distance attachment. *Physical Review E*, 70(5):056122, 2004.

[40] Mark EJ Newman. Clustering and preferential attachment in growing networks. *Physical Review E*, 64(2):025102, 2001.

[41] Jeffrey Travers and Stanley Milgram. An experimental study of the small world problem. *Sociometry*, pages 425–443, 1969.

[42] Paul Erdos and Alfréd Rényi. On the evolution of random graphs. *Publications of the Mathematical Institute of the Hungarian Academy of Sciences*, 5(1):17–60, 1960.

[43] Nino Boccara. *Modeling complex systems*. Springer Science & Business Media, New York, 2010.

[44] Duncan J Watts and Steven H Strogatz. Collective dynamics of âĂŸsmall-worldâĂŹnetworks. *Nature*, 393(6684):440–442, 1998.

[45] Réka Albert, Hawoong Jeong, and Albert-László Barabási. Internet: Diameter of the world-wide web. *Nature*, 401(6749):130–131, 1999.

[46] Albert-László Barabási and Réka Albert. Emergence of scaling in random networks. *Science*, 286(5439):509–512, 1999.

[47] Marco Patriarca and Teemu Leppänen. Modeling language competition. *Physica A*, 338(1):296–299, 2004.

[48] Adrián Carro, Raúl Toral, and Maxi San Miguel. Coupled dynamics of node and link states in complex networks: a model for language competition. *New Journal of Physics*, 18(11):113056, 2016.

[49] Arda Halu, Kun Zhao, Andrea Baronchelli, and Ginestra Bianconi. Connect and win: The role of social networks in political elections. *EPL (Europhysics Letters)*, 102(1):16002, 2013.

[50] Peter Clifford and Aidan Sudbury. A model for spatial conflict. *Biometrika*, 60(3):581–588, 1973.

[51] Thomas M L Richard A H. Ergodic theorems for weakly interacting infinite systems and the voter model. *The Annals of Probability*, 3(4):643–663, 1975.

[52] Krzysztof Suchecki, Víctor M Eguíluz, and Maxi San Miguel. Voter model dynamics in complex networks: Role of dimensionality, disorder, and degree distribution. *Phys. Rev. E*, 72:036132, Sep 2005.

[53] Claudio Castellano, Vittorio Loreto, Alain Barrat, Federico Cecconi, and Domenico Parisi. Comparison of voter and glauber ordering dynamics on networks. *Physical review E*, 71(6):066107, 2005.

[54] Federico Vazquez and Víctor M Eguíluz. Analytical solution of the voter model on uncorrelated networks. *New Journal of Physics*, 10(6):063011, 2008.

[55] Frantisek Slanina and Hynek Lavicka. Analytical results for the sznajd model of opinion formation. *The European Physical Journal B-Condensed Matter and Complex Systems*, 35(2):279–288, 2003.

[56] Withers C W J. Gaelic in Scotland, 1698âĂŞ1981: the geographical history of a language. *Northern Scotland*, 6 (First Series)(1):204–206, 1984.

[57] John William Aitchison and Harold Carter. *A geography of the Welsh language 1961-1991*. University of Wales Press, 1994.

[58] Giulio Cimini and Angel Sanchez. How evolutionary dynamics affects network reciprocity in prisonerââĆňâĎć s dilemma. *Journal of Artificial Societies and Social Simulation*, 18(2):1–22, 2015.

[59] Robyn M Dawes. Social dilemmas. *Annual review of psychology*, 31(1):169–193, 1980.

[60] Francisco C Santos, Jorge M Pacheco, and Tom Lenaerts. Evolutionary dynamics of social dilemmas in structured heterogeneous populations. *Proceedings of the National Academy of Sciences of the United States of America*, 103(9):3490–3494, 2006.

[61] Anxo Sánchez. Games on networks and cooperation: Models and experiments, 2010.

[62] Clara Granell, Sergio Gómez, and Alex Arenas. Competing spreading processes on multiplex networks: awareness and epidemics. *Physical Review E*, 90(1):012808, 2014.

[63] Vincenzo Nicosia and Vito Latora. Measuring and modeling correlations in multiplex networks. *Physical Review E*, 92(3):032805, 2015.

[64] Manlio De Domenico, Albert Solé-Ribalta, Emanuele Cozzo, Mikko Kivelä, Yamir Moreno, Mason A Porter, Sergio Gómez, and Alex Arenas. Mathematical formulation of multilayer networks. *Physical Review X*, 3(4):041022, 2013.

[65] Richard G Morris and Marc Barthelemy. Transport on coupled spatial networks. *Physical Review Letters*, 109:128703, Sep 2012.

[66] Maciej Kurant and Patrick Thiran. Layered complex networks. *Physical review letters*, 96(13):138701, 2006.

[67] Manlio De Domenico, Albert Solé-Ribalta, Elisa Omodei, Sergio Gómez, and Alex Arenas. Ranking in interconnected multilayer networks reveals versatile nodes. *Nature Communications*, 6:6868 EP, 2015.

[68] Clara Granell, Sergio Gómez, and Alex Arenas. Dynamical interplay between awareness and epidemic spreading in multiplex networks. *Physical Review Letters*, 111(12):128701, 2013.

[69] Federico Vazquez, M Ángeles Serrano, and Maxi San Miguel. Rescue of endemic states in interconnected networks with adaptive coupling. *Scientific reports*, 6:29342, 2016.

[70] Malbor Asllani, Daniel M Busiello, Timoteo Carletti, Duccio Fanelli, and Gwendoline Planchon. Turing patterns in multiplex networks. *Physical Review E*, 90(4):042814, 2014.

[71] Federico Battiston, Vincenzo Nicosia, Vito Latora, and Maxi San Miguel. Layered social influence promotes multiculturality in the axelrod model. *Scientific Reports*, 7, 2017.

[72] Kaj-Kolja Kleineberg and Marián Boguñá. Digital ecology: Coexistence and domination among interacting networks. *Scientific reports*, 5(10268), 2015.

[73] Peter Klimek, Marina Diakonova, Víctor M Eguíluz, Maxi San Miguel, and Stefan Thurner. Dynamical origins of the community structure of an online multilayer society. *New Journal of Physics*, 18(8):083045, 2016.

[74] Alessio Cardillo, Jesús Gómez-Gardenes, Massimiliano Zanin, Miguel Romance, David Papo, Francisco Del Pozo, and Stefano Boccaletti. Emergence of network features from multiplexity. *Scientific reports*, 3, 2013.

[75] Filippo Radicchi and Alex Arenas. Abrupt transition in the structural formation of interconnected networks. *Nature Physics*, 9(11):717–720, 2013.

[76] Manlio De Domenico, Vincenzo Nicosia, Alexandre Arenas, and Vito Latora. Structural reducibility of multilayer networks. *Nature communications*, 6:6864, 2015.

[77] Stanley Wasserman and Katherine Faust. *Social network analysis: Methods and applications*, volume 8. Cambridge University Press, 1994.

[78] David Krackhardt. Cognitive social structures. *Social networks*, 9(2):109–134, 1987.

[79] John F Padgett and Christopher K Ansell. Robust action and the rise of the medici, 1400-1434. *American Journal of Sociology*, 98(6):1259–1319, 1993.

[80] Mikko Kivelä, Alex Arenas, Marc Barthelemy, James P Gleeson, Yamir Moreno, and Mason A Porter. Multilayer networks. *Journal of complex networks*, 2(3):203–271, 2014.

[81] Federico Battiston, Vincenzo Nicosia, and Vito Latora. Structural measures for multiplex networks. *Physical Review E*, 89(3):032804, 2014.

[82] The social media matrix from the pew research center. *http://www.pewinternet.org/*, 2015.

[83] Mark E Dickison, Matteo Magnani, and Luca Rossi. *Multilayer social networks*. Cambridge University Press, 2016.

[84] Kaj-Kolja Kleineberg, Marián Boguñá, M. Ángeles Serrano, and Fragkiskos Papadopoulos. Hidden geometric correlations in real multiplex networks. *Nature Physics*, 12(11):1076–1081, jul 2016.

[85] Marián Boguñá, Romualdo Pastor-Satorras, and Alessandro Vespignani. Absence of epidemic threshold in scale-free networks with degree correlations. *Physical Review Letters*, 90(2), jan 2003.

[86] Kaj-Kolja Kleineberg and Marián Boguñá. Digital ecology: Coexistence and domination among interacting networks. *Scientific Report*, 5:10268, 2015.

[87] Kaj-Kolja Kleineberg, Marian Boguna, M. Ángeles Serrano, and Fragkiskos Papadopoulos. Hidden geometric correlations in real multiplex networks: multidimensional communities, trans-layer link prediction, and efficient navigation. 2016.

[88] Marina Diakonova, Vincenzo Nicosia, Vito Latora, and Maxi San Miguel. Irreducibility of multilayer network dynamics: the case of the voter model. *New Journal of Physics*, 18(2), 2016.

[89] Walter Quattrociocchi, Guido Caldarelli, and Antonio Scala. Opinion dynamics on interacting networks: media competition and social influence. *Scientific Reports*, 4:4938, 2014.

[90] J Gómez-Gardenes, Irene Reinares, Alex Arenas, and Luis Mario Floría. Evolution of cooperation in multiplex networks. *Scientific Reports*, 2:620, 2012.

[91] Luis Mario Floría, Carlos Gracia-Lázaro, Jesus Gómez-Gardenes, and Yamir Moreno. Social network reciprocity as a phase transition in evolutionary cooperation. *Physical Review E*, 79(2):026106, 2009.

[92] Nikos E Kouvaris, Shigefumi Hata, and Albert Díaz-Guilera. Pattern formation in multiplex networks. *Scientific Reports*, 5:10840, 2015.

[93] Valerio Gemmetto and Diego Garlaschelli. Multiplexity versus correlation: the role of local constraints in real multiplexes. *Scientific Reports*, 5, 2015.

[94] Michael Szell, Renaud Lambiotte, and Stefan Thurner. Multirelational organization of large-scale social networks in an online world. *Proceedings of the National Academy of Sciences*, 107(31):13636–13641, 2010.

[95] Luc Steels. Language as a complex adaptive system. In *Parallel Problem Solving from Nature PPSN VI*, pages 17–26. Springer, 2000.

[96] René Appel and Pieter Muysken. *Language contact and bilingualism*. Amsterdam University Press, 2006.

[97] Joshua A Fishman. *Reversing language shift: Theoretical and empirical foundations of assistance to threatened languages*, volume 76. Multilingual matters, 1991.

[98] Joshua A Fishman. *Readings in the Sociology of Language*. Walter de Gruyter, 1968.

[99] Fishman J A. Bilingualism in the barrio. *The Modern Language Journal*, 53(3):151–151, 1969.

[100] Fishman J A. Who speaks what language to whom and when? *La Linguistique*, 1(2):pp. 67–88, 1965.

[101] Fishman J A. Bilingualism with and without diglossia; diglossia with and without bilingualism. *Journal of social issues*, 23(2), 1967.

[102] Joshua A Fishman. *Language and ethnicity in minority sociolinguistic perspective.* Multilingual Matters, 1989.

[103] Jan-Petter Blom and John J Gumperz. Social meaning in linguistic structure: Code-switching in norway. *The Bilingualism Reader*, pages 111–136, 2000.

[104] Howard Giles, Justine Coupland, and Nikolas Coupland. *Contexts of accommodation: Developments in applied sociolinguistics*. Cambridge University Press, 1991.

[105] Charles A Ferguson. Diglossia. *word*, 15(2):325–340, 1959.

[106] Juan Pablo Pinasco and Liliana Romanelli. Coexistence of languages is possible. *Physica A: Statistical Mechanics and its Applications*, 361(1):355–360, 2006.

[107] Marco Patriarca, Xavier Castelló, JR Uriarte, Víctor M Eguíluz, and Maxi San Miguel. Modeling two-language competition dynamics. *Advances in Complex Systems*, 15(03n04):1250048, 2012.

[108] Federico Vazquez, Xavier Castelló, and Maxi San Miguel. Agent based models of language competition: macroscopic descriptions and order–disorder transitions. *Journal of Statistical Mechanics: Theory and Experiment*, 2010(04):P04007, 2010.

[109] Xavier Castelló, Víctor M Eguíluz, MAXI SAN MIGUEL, Lucía Loureiro-Porto, Riitta Toivonen, J Saramäki, and K Kaski. Modelling language competition: bilingualism and complex social networks. In *The Evolution of Language: Proceedings of the 7th International ConferencE Singapore: World Scientific Publishing Co*, pages 59–66. Citeseer, 2008.

[110] Riitta Toivonen, X Castelló, VM Eguíluz, Jari Saramäki, K Kaski, and M San Miguel. Broad lifetime distributions for ordering dynamics in complex networks. *Physical Review E*, 79(1):016109, 2009.

[111] Joan-Maria Esteban and Debraj Ray. On the measurement of polarization. *Econometrica: Journal of the Econometric Society*, pages 819–851, 1994.

[112] Jelena Grujić, Torsten Röhl, Dirk Semmann, Manfred Milinski, and Arne Traulsen. Consistent strategy updating in spatial and non-spatial behavioral experiments does not promote cooperation in social networks. *PLoS One*, 7(11):e47718, 2012.

[113] Jelena Grujić, Constanza Fosco, Lourdes Araujo, José A Cuesta, and Angel Sánchez. Social experiments in the mesoscale: Humans playing a spatial prisoner's dilemma. *PloS one*, 5(11):e13749, 2010.

[114] Anatol Rapoport and Albert M Chammah. *Prisoner's dilemma: A study in conflict and cooperation*, volume 165. University of Michigan Press, 1965.

[115] Jeffrey A Fletcher and Michael Doebeli. A simple and general explanation for the evolution of altruism. *Proc. R. Soc. Lond. B*, 276(1654):13–19, 2009.

[116] Jennifer Jacquet, Christoph Hauert, Arne Traulsen, and Manfred Milinski. Shame and honour drive cooperation. *Biol. Lett.*, 7:899âĂŞ901, 2011.

[117] Martin A Nowak. Five rules for the evolution of cooperation. *Science*, 314(5805):1560–1563, dec 2006.

[118] Martin A. Nowak and Karl Sigmund. Evolution of indirect reciprocity by image scoring. *Nature*, 393(6685):573–577, jun 1998.

[119] Dirk Helbing and Wenjian Yu. The outbreak of cooperation among success-driven individuals under noisy conditions. *Proceedings of the National Academy of Sciences*, 106(10):3680–3685, 2009.

[120] Robert Boyd, Herbert Gintis, Samuel Bowles, and Peter J Richerson. The evolution of altruistic punishment. *Proceedings of the National Academy of Sciences*, 100(6):3531–3535, 2003.

[121] Daniele Vilone, José J. Ramasco, Angel Sánchez, and Maxi San Miguel. Social and strategic imitation: the way to consensus. *Sci. Rep.*, 2:686, 2012.

[122] Daniele Vilone, José J. Ramasco, Angel Sánchez, and Maxi San Miguel. Social imitation versus strategic choice, or consensus versus cooperation, in the networked prisoner's dilemma. *Phys. Rev. E*, 90:022810, 2014.

[123] María Pereda. Evolution of cooperation under social pressure in multiplex networks. *Physical Review E*, 94(3):032314, 2016.

[124] Jesús Gómez-Gardenes, Manlio De Domenico, Gerardo Gutiérrez, Alex Arenas, and Sergio Gómez. Layer–layer competition in multiplex complex networks. *Phil. Trans. R. Soc. A*, 373(2056):20150117, 2015.

[125] Joan T Matamalas, Julia Poncela-Casasnovas, Sergio Gómez, and Alex Arenas. Strategical incoherence regulates cooperation in social dilemmas on multiplex networks. *Scientific Report*, 5, 2015.

[126] Marta D Santos, Sergey N Dorogovtsev, and José F.F. Mendes. Biased imitation in coupled evolutionary games in interdependent networks. *Scientific Report*, 4:4436, 2014.

[127] Zhen Wang, Lin Wang, Attila Szolnoki, and Matjaž Perc. Evolutionary games on multilayer networks: a colloquium. *EPJ B*, 88(5):124, 2015.

[128] Kaj-Kolja Kleineberg, Lubos Buzna, Fragkiskos Papadopoulos, Marián Boguñá, and M. Ángeles Serrano. Geometric correlations mitigate the extreme vulnerability of multiplex networks against targeted attacks. *Phys. Rev. Lett.*, 118:218301, 2017.

[129] György Szabó and Gábor Fáth. Evolutionary games on graphs. *Phys. Rep.*, 446(4-6):97–216, jul 2007.

[130] Ross Cressman and Yi Tao. The replicator equation and other game dynamics. *Proc. R. Soc. Lond.*, 111(Supplement 3):10810–10817, 2014.

[131] Dirk Helbing. A stochastic behavioral model and a microscopic foundation of evolutionary game theory. *Theor. Decis.*, 40(2):149–179, mar 1996.

[132] Thomas Milton Liggett. *Stochastic interacting systems: contact, voter, and exclusion processes*. Grundlehren der mathematischen Wissenschaften. Springer-Verlag, Berlin, New York, 1999.

[133] György Szabó and Csaba Tőke. Evolutionary prisoner's dilemma game on a square lattice. *Phys. Rev. E*, 58:69–73, 1998.

[134] F. C. Santos and J. M. Pacheco. Scale-free networks provide a unifying framework for the emergence of cooperation. *Phys. Rev. Lett.*, 95:098104, 2005.

[135] Francisco C Santos and Jorge M Pacheco. Scale-free networks provide a unifying framework for the emergence of cooperation. *Physical Review Letters*, 95:9, 2005.

[136] Francisco C Santos, Jorge M Pacheco, and Tom Lenaerts. Evolutionary dynamics of social dilemmas in structured heterogeneous populations. *Proceedings of the National Academy of Sciences*, 103(9):3490–3494, 2006.

[137] Attila Szolnoki, Matjaž Perc, and Zsuzsa Danku. Towards effective payoffs in the prisoner's dilemma game on scale-free networks. *Physica A: Statistical Mechanics and its Applications*, 387(8-9):2075–2082, mar 2008.

[138] Jesús Gómez-Gardenes, M Campillo, LM Floría, and Yamir Moreno. Dynamical organization of cooperation in complex topologies. *Phys. Rev. Lett.*, 98:108103, 2007.

[139] Kaj-Kolja Kleineberg. Spatial clustering and heterogeneity in evolutionary games on structured populations: a unified framework. *arxiv: 1704.00952*, 2017.

[140] Kaj-Kolja Kleineberg. Evolutionary games on scale-free multiplex networks. *arxiv: 1705.06972*, 2017.

[141] Karine Nyborg, John M Anderies, Astrid Dannenberg, Therese Lindahl, Caroline Schill, Maja Schlüter, W Neil Adger, Kenneth J Arrow, Scott Barrett, Stephen Carpenter, et al. Social norms as solutions. *Science*, 354(6308):42–43, 2016.

[142] Jean-Baptiste Michel, Yuan Kui Shen, Aviva Presser Aiden, Adrian Veres, Matthew K Gray, Joseph P Pickett, Dale Hoiberg, Dan Clancy, Peter Norvig, Jon Orwant, et al. Quantitative analysis of culture using millions of digitized books. *science*, 331(6014):176–182, 2011.

[143] Erez Lieberman, Jean-Baptiste Michel, Joe Jackson, Tina Tang, and Martin A Nowak. Quantifying the evolutionary dynamics of language. *Nature*, 449(7163):713–716, 2007.

[144] Mitchell G. Newberry, Christopher A. Ahern, Robin Clark, and Joshua B. Plotkin. Detecting evolutionary forces in language change. *Nature*, 2007.

[145] María Luisa Calero Vaquera and José Andrés de Molina Redondo. *Historia de la gramática española:(1847-1920)*. Gredos, 1986.

[146] Palomo Olmos B. Rendimiento funcional de las nuevas normas de prosodia y ortografia de la real academia española de la lengua (1959). *Cauce, 1992,(14-15): 171-219*, 1992.

[147] Claudio Wagner. Andrés bello y la gramática castellana latinoamericana. *Revista Documentos Lingüísticos y Literarios UACh*, (29), 2016.

[148] Mercedes Quilis Merín. La academia literaria i zientífica de instruczion primaria: defensa razonada (y apasionada) de su ortografía filosófica en 1844. 2014.

[149] Santiago Alcoba. Ortografía y drae. algunos hitos en la fijación léxica y ortográfica de las palabras. *Español Actual*, 88:11–42, 2007.

[150] RAE. Nuevo tesoro lexicogrÃąfico de la lengua espaÃśola (NTLLE). *Española (NTLLE):< http://ntlle. rae. es/ntlle*, 2017.

[151] Julio Casares. La academia y las âĂŸnuevas normas. *BRAE XXXIV*, 1954.

[152] Incorporated Merriam-Webster. Merriam-webster.com. *https://www.merriam-webster.com*, 2017.

[153] www.tysto.com/uk-us-spelling list.html. Comprehensive* list of american and british spelling differences. 2017.

[154] Es.wiktionary.org. Categoría:ES:Verbos. *Available at: https://es.wiktionary.org/wiki/Categoría:ES:Verbos*, 2017.

[155] RAE. El adverbio solo y los pronombres demostrativos, sin tilde. *Consultas lingÃijÃ■sticas. Available at: http://www.rae.es*, 2017.

[156] Bruno Gonçalves, Lucía Loureiro-Porto, José J Ramasco, and David Sánchez. The fall of the empire: The americanization of english. *arXiv preprint arXiv:1707.00781*, 2017.

[157] Charles E Osgood, Thomas A Sebeok, John W Gardner, John B Carroll, Leonard D Newmark, Susan M Ervin, Sol Saporta, Joseph H Greenberg, Donald E Walker, James J Jenkins, et al. Psycholinguistics: a survey of theory and research problems. *The Journal of Abnormal and Social Psychology*, 49(4p2):i, 1954.

[158] William Croft. *Explaining language change: An evolutionary approach*. Pearson Education, 2000.

[159] Richard A Blythe and William Croft. S-curves and the mechanisms of propagation in language change. *Language*, 88(2):269–304, 2012.

[160] Fakhteh Ghanbarnejad, Martin Gerlach, José M Miotto, and Eduardo G Altmann. Extracting information from s-curves of language change. *Journal of The Royal Society Interface*, 11(101):20141044, 2014.

[161] Roger Lass. *Historical linguistics and language change*, volume 81. Cambridge University Press, 1997.

[162] John H Vivian. Spelling an end to orthographical reforms: Newspaper response to the 1906 Roosevelt simplifications. *American Speech*, 54(3):163–174, 1979.

[163] Donald George Scragg. *A history of English spelling*, volume 3. Manchester University Press, 1974.

[164] I Martin Rodrigo, D Moran, M De la Fuente, and S Doria. Sólo/solo: la tilde que enfrenta a la RAE con los escritores. *ABC Cultura*, 2014.

[165] RAE. OrÃ■genes y fines de la rae. *Available at: http://www.rae.es/la-institucion/historia/origenes*, 2017.

[166] RAE. Estatutos y reglamento de la real academia española. *Available at: http://www.rae.es/la-institucion/organizacion/estatutos*, 2017.

[167] María Teresa Pochat. Historia de la real academia española. *Olivar*, 2(2):249–253, 2001.

[168] RAE Real Academia Española. *Ortografía de la lengua española*. Espasa, 2010.

[169] E Alvarez Mellado. Sólo y la tilde de la nostalgia. *eldiario*, 2017.

[170] Matías Guzmán Naranjo. The se-ra alternation in spanish subjunctive. *Corpus Linguistics and Linguistic Theory*, 13(1):97–134, 2017.

[171] Ilpo Kempas. Sobre la variación en el marco de la libre elección entre cantara y cantase en el español peninsular. *Moenia*, 17:243–264, 2011.

[172] Antonio Briz Gómez. *Corpus de conversaciones coloquiales*. Oralia (Madrid). Arco Libros, 2002.

[173] Malte Rosemeyer and Scott A Schwenter. Entrenchment and persistence in language change: the spanish past subjunctive. *Corpus Linguistics and Linguistic Theory*, 2017.

[174] Brian Weinstein. Noah webster and the diffusion of linguistic innovations for political purposes. *International Journal of the Sociology of Language*, 1982(38):85–108, 1982.

[175] Richard L Venezky. *The American way of spelling: The structure and origins of American English orthography*. Guilford Press, 1999.

[176] Richard E Hodges. A short history of spelling reform in the united states. *The Phi Delta Kappan*, 45(7):330–332, 1964.

[177] Robert Eugen Zachrisson. Four hundred years of english spelling reform. *Studia Neophilologica*, 4(1):1–69, 1931.

[178] Axel Wijk. Regularized english. *Educational Research*, 3(2):157–159, 1961.

[179] David Micklethwait. *Noah Webster and the American dictionary*. McFarland, 2005.

[180] List of words with the double uk/us spelling. *Available at: https://gist.github.com/robertaamato/ca7d173d5f90902d4a79f81a02fbb9d1*, 2017.

[181] Geoffrey Leech. *Change in contemporary English: A grammatical study*. Cambridge University Press, 2009.

[182] List of cases of substitution of british forms with american ones. *Available at: https://gist.github.com/robertaamato/39d79ce03f11ffcefcad5fe6b9ae0ab3*, 2017.

[183] Hosam Mahmoud. *Pólya urn models*. CRC press, 2008.

[184] List of irregular spanish verbs with the corresponding imperfect subjunctive form in '-ra' and '-se'. *Available at: https://gist.github.com/robertaamato/99f646670e9f8d0f261807a0185503be*, 2017.

[185] List of regular spanish verbs with the corresponding imperfect subjunctive form in '-ra' and '-se'. *Available at: https://gist.github.com/robertaamato/4cd3682cdab72bce0019c0e5b356f121*, 2017.

[186] Kaj-Kolja Kleineberg and M Boguna. Evolution of the digital society reveals balance between viral and mass media influence. *Phys. Rev. X*, 4:031046, 2014.

[187] András Gulyás, József J. Bíró, Attila Kőrösi, Gábor Rétvári, and Dmitri Krioukov. Navigable networks as nash equilibria of navigation games. *Nat. Commun.*, 6:7651, jul 2015.

[188] Kai P Spiekermann. Sort out your neighbourhood. *Synthese*, 168(2):273–294, 2009.

[189] Jose A. Cuesta, Carlos Gracia-Lázaro, Alfredo Ferrer, Yamir Moreno, and Angel Sánchez. Reputation drives cooperative behaviour and network formation in human groups. *Sci. Rep.*, 5:7843, jan 2015.

[190] David M Kreps and Robert Wilson. Reputation and imperfect information. *Journal of economic theory*, 27(2):253–279, 1982.

[191] Kaj-Kolja Kleineberg and Marián Boguñá. Competition between global and local online social networks. *Sci. Rep.*, 6:25116, 2016.

[192] Kaj-Kolja Kleineberg. Evolutionary games on scale-free multiplex networks. *arXiv preprint arXiv:1705.06972*, 2017.

[193] Byungjoon Min, Su Do Yi, Kyu-Min Lee, and K-I Goh. Network robustness of multiplex networks with interlayer degree correlations. *Physical Review E*, 89(4):042811, 2014.

[194] Farshad Kooti, Haeryun Yang, Meeyoung Cha, P Krishna Gummadi, and Winter A Mason. The emergence of conventions in online social networks. In *ICWSM*, 2012.

[195] Damon Centola and Andrea Baronchelli. The spontaneous emergence of conventions: An experimental study of cultural evolution. *Proceedings of the National Academy of Sciences*, 112(7):1989, 2015.

[196] Barbara Becker and Gloria Mark. Constructing social systems through computer-mediated communication. *Virtual reality*, 4(1):60–73, 1999.

[197] Cristina Bicchieri and Yoshitaka Fukui. The great illusion: Ignorance, informational cascades, and the persistence of unpopular norms. *Business Ethics Quarterly*, 9(1):127–155, 1999.

[198] Michela Del Vicario, Alessandro Bessi, Fabiana Zollo, Fabio Petroni, Antonio Scala, Guido Caldarelli, H Eugene Stanley, and Walter Quattrociocchi. The spreading of misinformation online. *Proceedings of the National Academy of Sciences*, 113(3):554–559, 2016.

[199] Sebastián Valenzuela. Facebook, Twitter, and youth engagement: A quasi-experimental study of social media use and protest behavior using propensity score matching. 2014.

[200] Daniel L Hartl, Andrew G Clark, and Andrew G Clark. *Principles of population genetics*, volume 116. Sinauer associates Sunderland, 1997.

[201] Anne Kandler and Stephen Shennan. A non-equilibrium neutral model for analysing cultural change. *Journal of Theoretical Biology*, 330:18–25, 2013.

[202] Jierui Xie, Sameet Sreenivasan, Gyorgy Korniss, Weituo Zhang, Chjan Lim, and Boleslaw K Szymanski. Social consensus through the influence of committed minorities. *Physical Review E*, 84(1):011130, 2011.

[203] Pushpendra Singh, Sameet Sreenivasan, Boleslaw K Szymanski, and Gyorgy Korniss. Accelerating consensus on coevolving networks: The effect of committed individuals. *Physical Review E*, 85(4):046104, 2012.

[204] D Mistry, Q Zhang, N Perra, and A Baronchelli. Committed activists and the reshaping of status-quo social consensus. *Physical Review E*, 92(4):042805, 2015.

[205] Xiang Niu, Casey Doyle, Gyorgy Korniss, and Boleslaw K Szymanski. The impact of variable commitment in the naming game on consensus formation. *Scientific Reports*, 7, 2017.

[206] Alberto Antonioni and Alessio Cardillo. Coevolution of synchronization and cooperation in costly networked interactions. *Physical review letters*, 118(23):238301, 2017.

[207] Cristian Danescu-Niculescu-Mizil, Robert West, Dan Jurafsky, Jure Leskovec, and Christopher Potts. No country for old members: User lifecycle and linguistic change in online communities. In *Proceedings of the 22nd international conference on World Wide Web*, pages 307–318. ACM, 2013.

[208] Sharon Peperkamp and Emmanuel Dupoux. Reinterpreting loanword adaptations: the role of perception. In *Proceedings of the 15th international congress of phonetic sciences*, volume 367, page 370, 2003.

[209] Elizabeth Levy Paluck, Hana Shepherd, and Peter M Aronow. Changing climates of conflict: A social network experiment in 56 schools. *Proceedings of the National Academy of Sciences*, 113(3):566–571, 2016.

[210] Robert B Cialdini and Noah J Goldstein. Social influence: Compliance and conformity. *Annu. Rev. Psychol.*, 55:591–621, 2004.

[211] Robert B Cialdini and Melanie R Trost. Social influence: Social norms, conformity and compliance. 1998.

[212] Damon Centola. The spread of behavior in an online social network experiment. *science*, 329(5996):1194–1197, 2010.

[213] Everett M Rogers. *Diffusion of innovations*. Simon and Schuster, 2010.

[214] Sinan Aral and Dylan Walker. Identifying influential and susceptible members of social networks. *Science*, page 1215842, 2012.

[215] M. Ángeles Serrano, Dmitri Krioukov, and Marián Boguñá. Self-Similarity of Complex Networks and Hidden Metric Spaces. *Physical Review Letters*, 100(7):078701, 2008.

[216] Dmitri Krioukov, Fragkiskos Papadopoulos, Maksim Kitsak, Amin Vahdat, and Marián Boguñá. Hyperbolic geometry of complex networks. *Phys. Rev. E*, 82(3):036106, September 2010.

[217] Joseph Michael Wilson. The-ra and-se verb forms in mexico: a diachronic examination from non-literary sources. *Tesis Doctoral*, 1983.

[218] Albert Solé-Ribalta, Manlio De Domenico, Nikos E Kouvaris, Albert Díaz-Guilera, Sergio Gómez, and Alex Arenas. Spectral properties of the laplacian of multiplex networks. *Phys. Rev. E*, 88:032807, Sep 2013.

[219] Alessio Cardillo, Jesús Gómez-Gardenes, Massimiliano Zanin, Miguel Romance, David Papo, Francisco Del Pozo, and Stefano Boccaletti. Emergence of network features from multiplexity. *Scientific reports*, 3:1344, 2013.