Emergence of fast agreement in an overhearing population: the case of naming game

Suman Kalyan Maity∗ and Animesh Mukherjee†
Department of Computer Science and Engineering, Indian Institute of Technology, Kharagpur, India – 721302

Francesca Tria‡
Institute for Scientific Interchange (ISI), Viale Settimio Severo 65, 10133 Torino, Italy

Vittorio Loreto§
Physics Department, Sapienza University, Piazzale Aldo Moro 5, 00185 Rome, Italy and Institute for Scientific Interchange (ISI), Viale Settimio Severo 65, 10133 Torino, Italy

The Naming game (NG) describes the agreement dynamics of a population of $N$ agents interacting locally in pairs leading to the emergence of a shared vocabulary. This model has its relevance in the novel fields of semiotic dynamics and specifically to opinion formation and language evolution. The application of this model ranges from wireless sensor networks as spreading algorithms, leader election algorithms to user based social tagging systems. In this article, we introduce the concept of overhearing (i.e., at every time step of the game, a random set of $N\delta$ individuals are chosen from the population who overhear the transmitted word from the speaker and accordingly reshape their inventories). When $\delta = 0$ one recovers the behaviour of the original NG. As one increases $\delta$, the population of agents reaches a faster agreement with a significantly low memory requirement. Remarkably, the convergence time to reach global consensus scales as $\log N$ as $\delta$ approaches 1.

PACS numbers: 89.75.-k, 05.65.+b, 89.65.Ef

I. INTRODUCTION

The naming game (NG) [1] is a simple multi-agent model that employs local communications which leads to the emergence of shared communication scheme in a population of agents. The game is played by a group of agents in pairwise interactions to negotiate conventions, i.e., associations between forms (names) and meanings (for example individuals in the world, objects, categories, etc.). The negotiation of conventions is a process through which one of the agents (i.e., the speaker) tries to draw attention of the other agent (the so-called hearer) towards the external meaning by the production of a conventional form. For example, the speaker might be interested to make the hearer identify an object through the production of a name. The hearer may be able to express the proper meaning and the speaker-hearer pair meet a local consensus in which case we call it a “success”. The other side of the coin is the hearer producing a wrong interpretation in which case the hearer takes lesson from the meeting by updating its meaning-form association. Thus, on the basis of success and failure of the hearer in producing meaning of the name, both the interacting agents reshape their internal meaning-form association. Through successive interactions, the local adjustment of individual meaning-form association leads or should lead to the emergence of a global consensus.

The model represents one of the simplest example leading progressively to the establishment of human-like languages. It was expressly conceived to explore the role of self-organization in the evolution of language [2, 3] and it has acquired, since then, a paradigmatic role in the novel field of semiotic dynamics which studies how language evolves through invention of new words and grammatical constructions, adoption of new meaning for words.

Implementing the naming game with local broadcasts, serves as a model for opinion dynamics in large-scale autonomously operating wireless sensor networks. In [4], it is pointed out that NG can be used as a leader-election model among a group of sensors where one does not intend to disclose information as to who the leader is at the end of the agreement process. The leader is a trusted agent having possible responsibilities ranging from routing coordination to key distribution and the NG identifies the leader which is hardly predictable from outside resulting in highly secure systems.

The creation of shared classification schemes by the NG in a system of artificial and networked autonomous agents can also be relevant from a system-design viewpoint, e.g., for sensor networks [5, 6]. Imagine a scenario where mobile or static sensor nodes are deployed in a large spatially extended region exploring an unknown and
possibly hostile environment. One of the important tasks
would be to convey information to the agents about their
discoveries, in particular they should be able to agree on
the identification of the new objects with no prior clas-
sification scheme or language to communicate regarding
detecting and sensing objects. Since subsequent efficient
operation of the sensor network inherently depends on
unique object identification, the birth of a communica-
tion system among the agents is crucial at the explora-
tion stage after network deployment. Besides artificial
systems where it is obvious that the agreement has to
take place rapidly, it concerns social dynamics too [7].
In particular, as an example, one can think of the emer-
gence of shared lexicon inside social groups and communi-
ties. When a new concept is introduced, different people
name it differently. These words spread among the popu-
lation, competing against each other, until the choice
of one of them is taken and everybody uses the same
word [8–10]. This type of dynamics has become a broad
interest of social groups and communities with the in-
ception of user-based tagging systems (such as flickr.com
or del.icio.us) [11, 12], where users manage tags to share
information as well as the “likes” of Facebook [13] and Twitter [14].

The minimal NG has diverse applications in many
fields [4–6, 11–14]. Here we shall reshape the model in a
“multi-party” communication framework. In particu-
lar, this involves conversations between two parties and
plays a significant role in the formation of shared mental
model [15]. Parties involved in a multi-party dialogue
can assume roles other than the speaker/addressee roles
in traditional two-party communication. One of the most
important roles is that of the overhearer. Overhearing
involves monitoring the routine conversations of agents,
who know they are being overheard, to infer information
about the agents. The overhearers might then use such
information to assist themselves, assess their progress or
suggest advice to the others. When an agent “overhears
an interaction”, she receives information about something
that is not primarily addressed to him. For instance, one
can listen to a conversation between two friends with-
out being part of their dialogue. Multi-party discourse
analysis shows that overhearing is a required communica-
tion type to model group interactions and consequently
reproduces them among artificial agents [16]. Various
applications are known to employ the concept of over-
hearers [17–22]. Novick and Ward [17] have employed
overhearing to model interactions between pilots and air
traffic controllers. Kaminka et al. [18] have developed
a plan-recognition approach to overhearing in order to
monitor the state of distributed agent teams. Aiello et
al. [19] and Bussetta et al. [20, 21] have investigated an
architecture that enables overhearing, so that domain ex-
perts can provide advice to problem-solving agents when
necessary. Legras [22] has examined the use of overhear-
ing for maintaining organizational awareness. Recently,
Komarova et al. [23] have studied the effect of eavesdrop-
ping in the evolution of language.

Motivated by the above literature and diverse applica-
tions of overhearer, we review the naming game for the
emergence of a communication system in the presence of
overhearers and attempt to investigate its global prop-
erties. To the best of our knowledge, NG has not been
studied in this perspective of multi-party communication.
The basic activity of the overhearers in the naming game
is as follows: when a conversation between two parties
is going on, the third party (i.e., the overhearers) may
eavesdrop the conversation and reshape their meaning-
form association. As we shall see in this article that the
introduction of the concept of overhearing leads to much
faster convergence than traditional NG [1] coupled with
a low memory requirement per agent.

An alternative but closely related approach for opinion
(rumor) spreading has been introduced in [24] where the
authors investigated the problem on a fully connected
network of N agents and showed that the rumor spread-
takes O(log N) rounds. The same rumor spreading
problem has been studied on networks with conductance
ϕ in [25] and later thoroughly investigated and made
more efficient in [26]. In particular, the authors achieve a
tight bound on the number of rounds required in spread-
ing a rumor over a connected network of N nodes and
conductance ϕ which is O(\log^2 N). We shall outline a de-
tailed comparison between our approach and the above
literature later in this article.

The rest of the article is organized as follows. Section
II is devoted to the description of the basic naming game
model in the presence of overhearers. In section III, we
investigate the scaling relations of some important quan-
tities and provide analytical arguments to derive the rel-
vant exponents. In section IV, we discuss the state of
the art and compare our findings with [26]. Finally, con-
clusions are drawn in section V.

II. THE MODEL DEFINITION

The model consists of an interacting population of N
artificial agents observing a single object to be named,
i.e., a set of form-meaning pairs (in this case only names
competing to name the unique object) which is empty
at the beginning of the game (t = 0) and evolves dy-
amically in time. At each time step (t = 1, 2, . . . ) two
agents are randomly selected and interact: one of them
plays the role of speaker, the other one that of hearer. In
addition, a set of Nδ individuals are randomly selected
in each step who behave as overhearers. Note that δ is a
parameter of the model.

In each game the following steps are executed:

- The speaker transmits a name to the hearer. If
  her inventory is empty, the speaker invents a new
  name, otherwise she selects randomly one of the
  names she knows.
- If the hearer has the uttered name in her inventory,
  the game is a success, and both agents delete all
their names, but the winning one.

- If the hearer does not know the uttered name, the game is a failure, and the hearer inserts the name in her inventory.

- Each overhearer overhears the word uttered by the speaker; if the word is in her inventory, she removes all the words from her inventory except this word (i.e., treats the event as a success) else she adds this word in her inventory (i.e., treats the event as a failure).

Fig. 1 shows a hypothetical example illustrating the inventory update rules of the different agents in the model of NG with overhearers.

III. RESULTS AND DISCUSSIONS

The basic quantities to be measured in the NG are the total number of words $N_u(t)$, defined as the sum of the inventory sizes of all the agents at the given time instance $t$, and the number of different words $N_d(t)$ present in the system at time $t$, telling us how many synonyms are present in the system at that time instance. The dynamics proceeds as illustrated in fig. 2(a) and 2(b). At the beginning both $N_u(t)$ and $N_d(t)$ grow linearly as the agents invent new words. As invention ceases, $N_d(t)$ reaches a plateau, i.e. a maximum number of distinct words. On the other hand, $N_u(t)$ keeps growing till it reaches a maximum at time $t_{\text{max}}$. The total number of words then decreases and the system reaches the convergence state at time $t_{\text{conv}}$. At convergence all the agents share the same unique word, so that $N_u(t_{\text{conv}}) = N$ and $N_d(t_{\text{conv}}) = 1$. It is observed that all the global quantities in the basic naming game [1] follow a power-law scaling as a function of the population size $N$. In particular, $t_{\text{max}} \sim N^\alpha$, $t_{\text{conv}} \sim N^\beta$, $N_{u_{\text{max}}} \sim N^\gamma$ where $\alpha \approx \beta \approx \gamma \approx 1.5$ for the original naming game ($\delta = 1$) on a fully connected graph topology.

We now focus on analytically estimating the scaling of (i) $N_{u_{\text{max}}}$, (ii) $t_{\text{max}}$ and (iii) $t_{\text{conv}}$ with $N$ in the presence of $N^\delta$ overhearers.

A. Scaling of $N_{u_{\text{max}}}$

![FIG. 2: Evolution of (a) the total number of words $N_u(t)$, (b) the number of different words present in the system, with time $t$ when the number of overhearers is $\eta N$ ($\delta = 1$ as in the original NG) with $\eta = 0.05$. Data refer to a population of $N = 50000$ agents. (c) Scaling of $N_{u_{\text{max}}}$ with $N$ for different values of $\delta$. (d) The figure expresses the relation of $\gamma$ vs $\delta$. Each point in the above curves represents the average value obtained over 100 simulation runs.](image)
is because at each time step only two agents can update their inventories, inventing in particular a new word if their inventories are empty. When $N^\delta$ overhearers are present the fraction of agents who can invent new words is reduced by a factor $N^\delta$. In this way the number of unique words in the system when the total number of words is close to the maximum is $\propto N/N^\delta = N^{1-\delta}$. Further, let us assume that each agent has on an average $cN^\alpha$ words in her inventory when the total number of words is close to the maximum. As in the original NG, $N_w^{max} \sim N^\gamma$ so that $\gamma = \alpha + 1$ holds here also. In the following, we shall attempt to find a relation between $\gamma$ and $\delta$. We can write the evolution equation of $N_w(t)$ as

$$\frac{dN_w(t)}{dt} \propto \left(1 - \frac{cN^\alpha}{N^1 - \delta}\right)N^\delta - \frac{cN^\alpha}{N^1 - \delta}cN^\alpha N^\delta$$

(1)

where the first term is related to unsuccessful games (increase in $N_w$ is proportional to $N^\delta$ times the probability of a single failure) and the second term is for successful games (decrease in $N_w$ is proportional to $cN^\alpha N^\delta$ times the probability of a single success). At maximum, $\frac{dN_w(t_{max})}{dt} = 0$ and therefore in the limit $N \to \infty$ the only relation possible is $\alpha = \frac{1}{3} - \frac{\delta}{2}$ which implies $\gamma = \frac{3}{2} - \delta$. When $\delta = 0, \gamma = 1.5$ we recover the original NG behaviour. In general, as one varies $\delta$ in the interval $[0, 1]$, $N_w^{max}$ varies as $N^\gamma$ where $\gamma \in [1, 1.5]$. The scaling of $N_w^{max}$ with $\delta$ for different values of $N$ is shown in fig. 2(c). In other words, for all values of $N$, $N_w^{max}$ monotonically decreases as $\delta$ increases and in the limit $\delta \to 1$ we have $N_w^{max} \to N$. This behaviour of $\gamma$ vs $\delta$ is confirmed by the simulation results shown in fig. 2(d).

### B. Scaling of $t_{max}$

We have to analyze the behaviour of the success rate in the beginning of the process in order to estimate the scaling relations for $t_{max}$. At early stages, most successful interactions involve agents which have already met in previous games. Thus, the probability of success is proportional to the ratio between the number of couples that have interacted before time $t$, which is $\propto tN^\delta(N - 1)/2$ and the total number of possible pairs is $N(N - 1)/2$. Thus, in the early stages, success rate $S(t) \propto \frac{\left(tN^\delta\right)^2}{N} = tN^{2(\delta - 1)}$. Note that if we put $\delta = 0$, we immediately recover $S(t) \propto t/N^2$ which is the case for the original NG. If $\delta \to 1$, we have $S(t) \propto t$, while if $\delta = \frac{1}{2}$ we have $S(t) \propto t/N$. Both these observations are validated by fig. 3(a) and 3(b) respectively for different values of $N$. With this information about $S(t)$ we can now easily estimate the value of $t_{max}$ by once again writing the evolution equation:

$$\frac{dN_w(t)}{dt} \propto \left(1 - tN^{2(\delta - 1)}\right)N^\delta - tN^{2(\delta - 1)}cN^\alpha(N^{1-\delta}/2)N^\delta$$

(2)

FIG. 3: Success rate at the onset of the dynamics (a) Success rate $S(t) \propto t$ when no. of overhearers = $\eta N$ where $\eta = 0.05$. (b) $S(t) \propto t/N$ when $\delta = \frac{1}{2}$. All the curves have been generated averaging over 100 simulation runs.

FIG. 4: Scaling of $t_{max}$ with population size $N$. As one varies $\delta$, $t_{max}$ scales as $\frac{N^{3(1-\delta)/2}}{a + bN^{(\delta - 1)/2}}$ where $a$ and $b$ are some constants. Each data point of all the above curves represents averaged value taken over 100 simulation runs. The bold lines show the fit from the analytical results.

If we now impose $\frac{dN_w(t_{max})}{dt} = 0$, then in the limit $N \to \infty$ we have $t_{max} \propto \frac{N^{3(1-\delta)/2}}{a + bN^{(\delta - 1)/2}}$ where the de-
nominator is precisely a correction term with $a$ and $b$ as constants. Once again, for $\delta = 0$ we have $t_{\text{max}} \propto N^{3/2}$ thus recovering the original NG property. On the other hand, in the limit $\delta \to 1$, $t_{\text{max}}$ approaches $O(1)$. The results of the scaling of $t_{\text{max}}$ with $N$ for different values of $\delta$ are shown in fig 4.

C. Scaling of $t_{\text{conv}}$

The exponent for the convergence time, $\beta$, deserves a more intricate discussion, and we can only attempt to provide a naïve argument here. We concentrate on the scaling of the interval of time separating the peak of $N_\text{w}(t)$ and the convergence, i.e., $t_{\text{diff}} = (t_{\text{conv}} - t_{\text{max}})$, since we already have an argument for $t_{\text{max}}$. $t_{\text{diff}}$ is the time span required by the system to get rid of all the words but the one which survives in the final state.

If we adopt the mean field assumption that at $t = t_{\text{max}}$ each agent has on average $N_{\text{w}}^\text{max}/N \sim N^{-\frac{1}{2}}$ words, we see that, by definition, in the interval $t_{\text{diff}}$, each agent must have won at least once. This is a necessary condition to have convergence, and it is interesting to investigate the timescale over which this happens. Assuming that $N$ is the number of agents who did not yet have a successful interaction at time $t$, we have:

$$\bar{N} = N(1 - p_s p_w)^t$$

where $p_s$ is the probability of choosing a specific agent and $p_w = S(t)$ is the probability of a success. In this case, $p_s = \frac{1}{N}$ and $p_w = tN^{2(\delta - 1)}$. In order to estimate $t_{\text{diff}}$, we require the number of agents who have not yet had a successful interaction to be finite just before the convergence, i.e., $\bar{N} \sim O(1)$ and we consider $p_w(t_{\text{max}}) = t_{\text{max}} N^{2(\delta - 1)} = N^{(1-\delta)/2}$. In this way one gets:

$$t_{\text{diff}} \propto N^{\frac{3(1-\delta)}{2}} (a + b N^{-(1-\delta)/2}) \log N$$

The above scaling relation of $t_{\text{diff}}$ is well confirmed by the simulation results in fig 5.

Thus, when $\delta = 0$, and we ignore the correction, we recover the original NG case: $t_{\text{conv}} \propto N^{3/2} \log N$. On the other hand, in the limit $\delta \to 1$, we have $t_{\text{conv}} \to \log N$.

IV. RELATED WORK

Most previous studies in semiotic dynamics has focused on populations of agents in which all pairwise interactions are allowed, i.e., the agents are placed on the vertices of a fully connected graph. In statistical mechanics, this topological structure is commonly referred to as mean-field topology. In the original work on the minimal naming game model [1], Baronchelli et al. studied, numerically and analytically, the behavior of the mean-field model, providing theoretical arguments in order to explain the main properties of the global behavior of the population.

The model is extensively studied apart from fully connected network, in regular lattices [27, 28]; small world networks [28-31]; random geometric graphs [28, 32, 33]; and static [34-36], dynamic [37], and empirical [38] complex networks. The final state of the system is usually a complete consensus [39], but stable polarized states can be reached introducing a simple confidence/trust parameter [40]. NG as defined in [1] is also modified in several ways [28, 32, 38, 40–49] and it represents the fundamental stepping stone of more complex models in computational cognitive sciences [50–53]. In [27], effects of topological embedding on the naming game dynamics is reported and it has been shown that the convergence process requires a memory per agent scaling as $N$ and lasts a time $N^{1+\frac{d}{2}}$ in dimension $d \leq 4$ (the upper critical dimension), while in mean field both memory and time scale as $N^2$. Thus, low dimensional lattices require more time to reach the consensus compared to mean-field but for a lower memory. In [34], for both the ER and BA network models, the convergence time $t_{\text{conv}}$ scales as $N^\beta$, where $\beta \approx 1.4$. In [31], Barrat et al. show that for small-world networks the convergence towards consensus is reached on a timescale of order $N^{\beta_{\text{SW}}}$, with $\beta_{\text{SW}} \approx 1.4 \pm 0.1$, close to the mean-field case ($N^2$) and this is in strong contrast with the $N^3$ behavior of purely one-dimensional systems. In particular, time to converge scales as $p^{1.4 \pm 0.1}$, which is...
consistent with the fact that for $p$ of order $\frac{1}{N}$ one should recover an essentially one-dimensional behavior with convergence times of order $N^3$. The small-world topology therefore allows to combine advantages from both finite dimensional lattices and mean-field networks.

There has been a long history in the area of rumor spreading which closely parallels the major concepts of the model investigated here. One of the benchmarks is the PUSH-PULL strategy introduced in [24] and then further extended and made incrementally more efficient in [25, 26]. The simple PUSH-PULL mechanism is as follows: at each round, a node that knows the rumor selects a random neighbor and forwards the rumor (PUSH), or if the node does not know the rumor selects a neighbor uniformly at random and asks for the information (PULL). This scheme informs all $N$ agents in a fully connected network in time $\log_3 N + O(\ln \ln N)$ with probability at least $1 - O(N^{-\alpha})$ where $\alpha > 0$. In [26], Chierichetti et al. investigated the rumor spreading in a connected network of $N$ nodes with conductance $\phi$. The conductance of a graph, a name borrowed from electrical networks, is a quantity that measures how well information spreads in the graph, its maximum value $\phi = 1$ being reached for a fully connected graph. In [26] it has been shown that the PUSH-PULL strategy broadcasts a message within $O(\log \frac{\log N}{\phi^3} \log N)$ rounds with a probability of $1 - o(1)$ which the authors claim to be a tight bound. Estimates regarding the amount of memory required per agent for the purpose of spreading have not been presented so far in the above literature. In general, since there is usually one rumor to be spread the memory estimate becomes trivial ($O(1)$). However, one can envisage a situation where new rumors need to be constantly invented in the population.

In our work, we theoretically as well as by means of simulations show that opinion spreading in a fully connected network (i.e., conductance $\phi = 1$) of $N$ nodes takes a $O(\log N)$ time to reach the global agreement with a maximum memory estimate of $N$ as $\delta \to 1$ which is comparable with the time requirement for the spreading of rumor in [24, 26]. It is important to stress that in our model the case of maximal conductance $\phi = 1$ is obtained on a fully connected graph only in the limit $\delta = 1$. Further, we point out that the model of NG in overhearing population can be recast for rumor spreading when constantly new rumors can be generated that should compete to spread in the whole population.

V. CONCLUSIONS AND FUTURE WORK

In this article, we have introduced the agreement dynamics of naming game to describe the convergence of population of agents on assigning a unique name to an object in the domain of multi-party communication. We have investigated the basic naming game model in an overhearing population and computed the scaling behaviour of the main global quantities: $N_{\max} \propto N^\gamma$ where the exponent $\gamma = \frac{3}{\alpha};$ $t_{\max} \propto N^\alpha$ where roughly the exponent $\alpha = \frac{3(1-\delta)}{2}$ and $t_{\text{conv}} \propto N^\alpha \log N$. In particular, we achieve a very fast agreement in the population with significantly low memory requirement. Moreover, we have also suggested that this model with overhearers can find relevant application in rumour spreading.

There could be many interesting future directions. First of all, it will be interesting to explore the model in a scenario where agents make their success update probabilistically as studied in [40]. The role of agent topology can also be one of the future perspectives. Different complex topologies could be studied where agents are embedded on more realistic networks. Furthermore, while in this article we have concentrated only on the study of the scaling properties of the system, performing a detailed analysis of the microscopic aspects of the dynamics could be another interesting topic for future research. One might also extend the idea of overhearers to the more complex tasks like categorization [50–53].


